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New Fuzzy Probabilistic Aggregation Operator in the Information System Implementation Management Problem

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ABSTRACT. In this article a new generalization of the probabilistic averaging operator –Associated Fuzzy Probabilistic Averaging (As-FPA) operator is used in the MADM problem of information system implementation management. Experts evaluations as arguments of the aggregation operator are described by triangular fuzzy numbers (TFN). Some propositions on the correctness of generalization are presented. © 2016 Bull. Georg. Natl. Acad. Sci.

Key words: averaging aggregation operators; associated probabilities; body of evidence; fuzzy decision making; information system implementation management

In some MADM problems the decision making person (DMP) has a collection $D = \{d_1, d_2, \dots, d_m\}$ of possible uncertain alternatives from which he/she must select one or perform ranking of decisions by some experts preference relation values. The result, associated with this problem, is a variable of attributes or criteria affecting the decision procedure. This variable is usually called the state of nature, which affects the expert evaluations, DMP's preferences or subjective activities. This variable is assumed to take its values from some set $S = \{s_1, s_2, \dots, s_n\}$ (set of criteria or factors). As a result the DMP knows that if he/she selects d_i and the state of nature assumes the value s_j then his/her evaluation is a_{ij} (values of fuzzy variable). The objective of the decision is to select the "best" alternative and get the biggest payoff. But the selection procedure becomes more difficult. In this case each alternative can be seen as corresponding to a row vector of possible payoffs. To make a choice the DMP must compare these vectors, a problem which generally does not lead to a compelling solution. Our focus will be on the construction of aggregation operator (function) in fuzzy probabilistic environment that can take a row vector of possible payoffs and convert it into a single value.

Preliminary concepts and motivation

We say that for TFNs $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ $\tilde{a} > \tilde{b}$ if $a_2 > b_2$ and if $a_2 = b_2$ then $\tilde{a} > \tilde{b}$ if $\frac{a_1 + a_3}{2} > \frac{b_1 + b_3}{2}$ otherwise $\tilde{a} = \tilde{b}$. The set of all nonnegative TFNs ($a_i \geq 0$) is denoted by Ψ^+ [1]. Note that on the lattice Ψ^+ $1_{\Psi^+} = (1, 1, 1)$ and $0_{\Psi^+} = (0, 0, 0)$. The latest notion of inequality induces the total ordering t on the lattice Ψ^+ and we shall say that $\tilde{a} \geq_t \tilde{b}$ iff $\tilde{a} > \tilde{b}$ or $\tilde{a} = \tilde{b}$. We define the operations of max and min based on the total ordering of t . We say that $\max_t \{\tilde{a}; \tilde{b}\} = \tilde{a}$ and $\min_t \{\tilde{a}; \tilde{b}\} = \tilde{b}$ iff $\tilde{a} \geq_t \tilde{b}$.

We use elements of the Theory of a Body of Evidence [2]. This Theory is based on two dual fuzzy measures: Belief and Plausibility measures. Belief and Plausibility measures can be characterized by the set function: $m: 2^S \rightarrow [0; 1]$, which is required to satisfy two conditions: (a) $m(\emptyset) = 0$, (b) $\sum_{B \in 2^S} m(B) = 1$.

This function is called a Basic Probability Assignment (BPA). For each set $B \in 2^S$, the value $m(B)$ expresses the proportion that all available and relevant evidence supporting the claim that a particular element of S , whose characterization in terms of relevant attributes is deficient, belongs to the set B . This value $m(B)$, pertains solely to one set B ; it does not imply any additional claims regarding subsets of B . If there is some additional evidence supporting the claim that the element belongs to a subset of B , say $B_1 \subseteq B$, it must be expressed by another value $m(B_1)$. If $m(B) > 0, B \subseteq S$, then B is called a focal element. Let $F = \{B_1, \dots, B_q\}$ be the set of all focal elements. The pair $\langle F, m \rangle$ is called a Body of Evidence.

Definition 1 [2]. Let m be a BPA on S . The plausibility measure Pl associated to m is given by

$$Pl(A) = \sum_{B \in F: A \cap B \neq \emptyset} m(B) \quad \forall A \in 2^S, \quad (1)$$

and the Belief measure Bel associated to m is given by

$$Bel(A) = \sum_{B \in F: B \subseteq A} m(B) \quad \forall A \in 2^S. \quad (2)$$

As known a fuzzy measure is a capacities of order two and therefore Bel and Pl are dual fuzzy measures [1]. We denote Bel or Pl by g .

Now on the new fuzzy probabilistic generalization of the finite Choquet Averaging Operator [1, 3].

Definition 2. The Fuzzy Probabilistic Averaging (FPA) operator on the lattice Ψ^+ with respect to some probability distribution p is defined by the additive sum: If $a_i \in \Psi^+; i = 1, \dots, m$, then

$$FPA_p(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \sum_{i=1}^n p_i \tilde{a}_i. \quad (3)$$

Definition 3. Let we have a fuzzy measure g on 2^S and a fuzzy variable of expert evaluations $\tilde{a}: S \Rightarrow \Psi^+$ such that $\tilde{a}(s_i) \equiv \tilde{a}_i \in \Psi^+, i = 1, 2, \dots, n$. Then the aggregation

$$FCA_g(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \sum_{j=1}^n p_j \tilde{a}_{(j)}, \quad (4)$$

where

$$p_j = g(\{s_{i(1)}, \dots, s_{i(j)}\}) - g(\{s_{i(1)}, \dots, s_{i(j-1)}\}), \quad g(\{s_{i(0)}\}) \equiv 0,$$

is called a finite Fuzzy Choquet Averaging (FCA) operator. In the proceeding $i(\cdot)$ is index function such that

$\tilde{a}_{i(j)}$ is the j th largest of the $\{\tilde{a}_i\}_{i=1}^n$ in the sense of the total ordering t .

It is obvious that $FCA_g(\tilde{a})$, $\tilde{a} = (\tilde{a}_1, \dots, \tilde{a}_n)$ is some generalization of the probabilistic averaging operator $FPA_P(\tilde{a})$. The FCA value with respect to a fuzzy measure g coincides with the FPA operator value with respect to a probability measure P that depends only on g and the ordering of the values of \tilde{a} :

Proposition 1. The value of FCA operator with respect to a probabilistic measure P coincides with the value of FPA operator:

$$FCA_P(\tilde{a}) = FPA_P(\tilde{a}), \forall \tilde{a} : S \Rightarrow \Psi^+. \tag{5}$$

Following the Definition 3 the maximum number of probability distributions in FCA coincides with the number of possible orderings or permutations in a set with n elements, that is, $n!$. Thus, it makes sense to associate the $n!$ probabilities to each fuzzy measure, provided that they are deduced from this fuzzy measure through the different possible orderings. In general, the possible orderings of the elements of S are given by the permutations of a set with n elements, which form the group S_n . Now we consider a definition of associated probabilities induced by a fuzzy measure on the group S_n .

Definition 4 [4]. The probability functions P_σ defined by

$$\begin{aligned} P_\sigma(s_{\sigma(1)}) &= g(\{s_{\sigma(1)}\}), \dots, \\ P_\sigma(s_{\sigma(i)}) &= g(\{s_{\sigma(1)}, \dots, s_{\sigma(i)}\}) - g(\{s_{\sigma(1)}, \dots, s_{\sigma(i-1)}\}), \dots, \\ P_\sigma(s_{\sigma(m)}) &= 1 - g(\{s_{\sigma(1)}, \dots, s_{\sigma(n-1)}\}), \\ g(\{s_{\sigma(0)}\}) &\equiv 0 \end{aligned} \tag{6}$$

for each $\sigma = (\sigma(1), \sigma(2), \dots, \sigma(n)) \in S_n$, are called the associated probabilities and the Associated Probability Class (APC) - $\{P_\sigma\}_{\sigma \in S_n}$ of a fuzzy measure g .

The following results are evident and valid for every fuzzy measure:

Proposition 2. If $P_\sigma, \sigma \in S_m$ are the associated probabilities of a fuzzy measure g on S , then for every variable $\tilde{a} : S \rightarrow \Psi^+$ it holds

$$\min_{\sigma \in S_n} FPA_{P_\sigma}(\tilde{a}) \leq_t FCA_g(\tilde{a}) \leq_t \max_{\sigma \in S_n} FPA_{P_\sigma}(\tilde{a}). \tag{7}$$

Proposition 3. A necessary and sufficient condition for a pair of dual fuzzy measures (g_*, g^*) to be lower and upper capacities of order two (the belief and plausibility measures, respectively) is that for every variable $\tilde{a} : S \rightarrow \Psi^+$ it holds

$$FCA_{g_*}(\tilde{a}) = \min_{\sigma \in S_n} FPA_{P_\sigma}(\tilde{a}), \quad FCA_{g^*}(\tilde{a}) = \max_{\sigma \in S_n} FPA_{P_\sigma}(\tilde{a}). \tag{8}$$

Now we use the presented results on the fuzzy measure for the generalizations of the FPA operator.

Associated Probabilities of a Fuzzy Measure in the Generalization of the FPA Operator

In previous section the FCA were defined along with their probability representations by associated probability class (APC) $\{P_\sigma\}_{\sigma \in S_m}$, where the number of probability distributions on S is equal to $k = n!$.

We have k values of FPA operator for a variable $a - \{PA_{P_\sigma}(a)\}_{\sigma \in S_n}$, where

$$FPA_{P_\sigma}(\tilde{a}) = \sum_{i=1}^n \tilde{a}_i P_\sigma(s_i), \quad \sigma \in S_n. \tag{9}$$

The main idea of our generalization is the following: *We will focus on the use of $n!$ probabilistic averaging (9) in the new fuzzy FPA operator, instead of one probabilistic averaging $FPA_p(\tilde{a}) = \sum \tilde{a}_i p_i$, as a more usual extension of this operator from minimum $FPA_{\min}(\tilde{a}) = \min_{\sigma \in S_n} FPA_{p_\sigma}(\tilde{a})$ to the maximum $FPA_{\max}(\tilde{a}) = \max_{\sigma \in S_n} FPA_{p_\sigma}(\tilde{a})$ associated probabilistic averaging values. The choice of the associated probabilistic averaging values will depend on DMP's activities when the DMP can manipulate it according to his/her degree of optimism or pessimism. Therefore, a new operator will be as a function of associated probabilistic averaging values. More exactly: Let $\tilde{M} : (\Psi^+)^k \Rightarrow \Psi^+$ ($k = n!$) be some averaging aggregation function.*

Definition 5. An associated fuzzy probabilistic averaging operator As-FPA of dimension n is mapping $As-FPA : (\Psi^+)^n \Rightarrow \Psi^+$, that has an associated probability class $\{P_\sigma\}_{\sigma \in S_n}$ of a fuzzy measure $g : 2^S \Rightarrow [0, 1]$, according to the following formula:

$$As-FPA_{\tilde{M}}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \tilde{M}\left(FPA_{P_{\sigma_1}}(\tilde{a}), FPA_{P_{\sigma_2}}(\tilde{a}), \dots, FPA_{P_{\sigma_k}}(\tilde{a})\right), \quad (10)$$

Analytical properties of the As-FPA operator for general fuzzy measure g and different averaging aggregation function \tilde{M} are proved but omitted here. Now we consider concrete As-FPA operators: As-FP Amin if $M=Min$, As-FP Amax, if $M=Max$, As-FP Amean if $M=Mean$.

Proposition 4. Let \tilde{M} be the Min_t operator and $\tilde{a} : S \rightarrow \Psi^+$ be some fuzzy variable, then As-FPA operator may be written as:

$$As-FPA \min(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \min_t \left(\sum_{\sigma \in S_m}^n \tilde{a}_i P_\sigma(\tilde{a}_i) \right), \quad (11)$$

and if fuzzy measure g is a lower capacity of order two, then the As-FP Amin operator coincides with FCA_g operator:

$$As-FPA \min(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = FCA_g(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n). \quad (12)$$

Proposition 5. Let \tilde{M} be the Max_t operator, then As-FPA operator may be written as:

$$As-FPA \max(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = \max_t \left(\sum_{\sigma \in S_m}^n \tilde{a}_i P_\sigma(\tilde{a}_i) \right), \quad (13)$$

and if fuzzy measure g is an upper capacity of order two, then the As-FP O W Amax operator coincides with FCA_g operator:

$$As-FPA \max(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = FCA_g(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n). \quad (14)$$

Proposition 6. Let \tilde{M} be any averaging aggregation function $\tilde{M} : (\Psi_0^+)^{n!} \Rightarrow \Psi_0^+$ and in As-FPA operator a fuzzy measure g be a probability measure P . Then As-FPA and FPA operators coincide.

$$As-FPA_{\tilde{M}}(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n) = FPA_P(\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n). \quad (15)$$

Other recent results on the fuzzy probabilistic aggregations in uncertain environment see in [1, 5-7].

Decision Making Approach Regarding the Problem of Choosing Students Project's best Version for the Implementation

We analyse an illustrative example of the use of the As-FPA operator in a fuzzy decision-making problem regarding the choosing of the students' group project for implementation. The author of this work has an experience of working with graduate students pursuing a master's degree in 'intelligent information systems', in which students work on group projects, involving the evolution, control, engineering and management of

Table 1. Fuzzy Decision Making Matrix – evaluations of students

| D / S | s_1 | s_2 | s_3 | s_4 | s_5 | s_6 | s_7 |
|---------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| d_1 | (0.2, 0.4, 0.5) | (0.5, 0.6, 0.7) | (0.5, 0.7, 0.8) | (0.5, 0.6, 0.7) | (0.4, 0.5, 0.6) | (0.2, 0.4, 0.5) | (0.5, 0.6, 0.7) |
| d_2 | (0.5, 0.6, 0.7) | (0.6, 0.8, 0.9) | (0.5, 0.8, 0.9) | (0.6, 0.7, 0.8) | (0.5, 0.6, 0.7) | (0.6, 0.8, 0.9) | (0.6, 0.7, 0.8) |
| d_3 | (0.2, 0.3, 0.4) | (0.7, 0.9, 1.0) | (0.2, 0.4, 0.5) | (0.6, 0.7, 0.8) | (0.1, 0.3, 0.5) | (0.6, 0.7, 0.8) | (0.5, 0.6, 0.7) |

simulation models for study of complex systems. The students always create several versions of the project for implementation, because usually it is very hard to figure out the role of each student in the group and their utility. In addition, we have to take into account the fact that each student is working in several groups. After studying various versions of the project, we have the possibility to consider the levels of competency of each student concerning the implementation of the project and evaluate each student by compatibility levels for each given version of the project.

In one of such cases, we were dealing with the estimation of the financial state of a certain business organization. The estimation of the linguistic variable is represented by several fuzzy terms, which represent the output of a fuzzy control system. The input information was the objective-statistical data – linguistic variables, which influence the financial state of the organization. After analysing the problem, we found out that the number of input linguistic variables was 14. Their fuzzification was performed, and the students elaborated three versions of the project of constructing a system for the same input and output information (d_1). The fuzzy logic rules, corresponding to knowledge base, and the decision support system must be built using the MatLab Fuzzy-Logic Toolbox (d_2). The fuzzy rules, knowledge base, architecture and interface will be developed using the programming language C#. (d_3) The body of the control system – the transaction between input and output variables – will be developed using fuzzy relations and their compositions, and corresponding software also will be developed using C#.

Thus three versions $D = \{d_1, d_2, d_3\}$ of the project were created in which seven students participated, say $S = \{s_1, s_2, \dots, s_7\}$. All seven of them participated in the development of all three versions, but in different subgroups, as often happens in engineering and management of simulation modelling. They created four groups (hereinafter called focal elements);

1. A_1 – The group for problem analysis, gathering input data, its initial processing and construction of the conceptual model.
2. A_2 – The group for conceptual model validation and software development.
3. A_3 – The software verification and testing group.
4. A_4 – The management group.

Table 2. Aggregation results

| $D / \text{Agg. Operator}$ | $g = \text{Bel}$ $As - \text{FPAmin}$ | $g = \text{Bel}$ $As - \text{FPAmax}$ | $g = \text{Bel}$ $As - \text{FPAmean}$ | FDA^* | FDA^* |
|----------------------------|--|--|---|--------------------|--------------------|
| d_1 | (0.20, 0.40, 0.50) | (0.50, 0.66, 0.76) | (0.46, 0.54, 0.64) | (0.20, 0.40, 0.50) | (0.50, 0.64, 0.73) |
| d_2 | (0.53, 0.63, 0.73) | (0.60, 0.80, 0.90) | (0.56, 0.72, 0.82) | (0.53, 0.63, 0.73) | (0.60, 0.80, 0.90) |
| d_3 | (0.25, 0.39, 0.53) | (0.61, 0.72, 0.88) | (0.40, 0.72, 0.82) | (0.25, 0.39, 0.53) | (0.40, 0.72, 0.82) |

Table 3. Ordering of alternatives

| N | Aggregation Operator | Result |
|---|---------------------------------|---------------------------|
| 1 | $As - FPA \min(g = Bel)$ | $d_2 \succ d_1 \succ d_3$ |
| 2 | $As - FPA \max(g = Bel)$ | $d_2 \succ d_3 \succ d_1$ |
| 3 | $As - FPA \text{mean}(g = Bel)$ | $d_2 \succ d_3 \succ d_1$ |
| 4 | FDA^* | $d_2 \succ d_1 \succ d_3$ |
| 5 | FDA^* | $d_2 \succ d_3 \succ d_1$ |

The students were divided into subgroups in the following way: $A_1 = \{s_1, s_3, s_4\}$, $A_2 = \{s_3, s_4, s_5, s_6\}$, $A_3 = \{s_1, s_2, s_6, s_7\}$, $A_4 = \{s_4, s_6, s_7\}$. We assigned the following weights to subgroups: $m(A_1) = 0.2$, $m(A_2) = 0.4$, $m(A_3) = 0.1$, $m(A_4) = 0.3$.

So, we built the body of evidence $\langle F, m \rangle$, where F - is a set of focal elements – subgroups A_1, A_2, A_3, A_4 and m is a BPA on the F . Based on the theory of body of evidence, we create dual fuzzy measures of uncertainty: plausibility measure and belief measure. These measures as uncertainty measures will be used in the As-FPA operator.

After some time, the students presented all three variants of the project (d_1, d_2, d_3). We had to choose the best one with the objective of optimal realization of the problem. We had to evaluate the utilities of the students concerning each version. Therefore, we had to study the projects in detail and to consider students' competence and knowledge in given topics, the quality and reliability of the realization of project, the ability to work in groups, etc.

The results of the evaluation process were as follows (the results are normalized in nonnegative triangular fuzzy numbers from the interval $[0, 1]$ (see Table 1)).

Obviously, the comparison of the project's versions $d_i, i = 1, 2, 3$ and selection of the best one by their compatibility levels are impossible. Using the data presented in Table 6 and the definition of the As-FPA operator we calculate the values of the As-FPA operator for every possible alternative and different mean aggregation function $\tilde{M} \in \{mean, \max_i, \min_i\}$. For these calculations, a software that is dynamical by all of its input parameters was implemented. Aggregation results calculated by the software are presented in Table 2.

For comparing the aggregation results the generalizations of the Dempster's lower and upper averaging (Fuzzy Dempster's Averaging (FDA)) operators are used for the lattice Ψ^+ :

$$FDA_*(\tilde{a}_1, \dots, \tilde{a}_m) = \sum_{B \in F} m(B) \cdot (\min_{s_i \in B} \{\tilde{a}_i\}) \quad \text{and}$$

$$FDA^*(\tilde{a}_1, \dots, \tilde{a}_m) = \sum_{B \in F} m(B) \cdot (\max_{s_i \in B} \{\tilde{a}_i\})$$

Following Proposition 11 the AS-FPA operators' values calculated with respect to Pl and Bel measures coincide. Therefore, only results based on the Bel measure are presented in Table 2. In Table 3 the alternatives ordered by the values of the As-FPA and FDA operators are presented.

It is easy to see that the alternative d_2 or the second version of the project is preferable over other versions. As the decision, students were instructed to implement this version of the project.

Conclusion

In this work our focus is directed on the construction of new fuzzy probabilistic averaging operator – As-FPA. A new generalization is presented with respect to associated probability class (APC) of a fuzzy measure and induced by the finite Fuzzy Choquet integral. The example regarding the problem of choosing the best version of the students' project for implementation was also presented. By using the As-FPA operator the optimal decision is found.

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ახალი ფაზი-ალბათური აგრეგირების ოპერატორი ინფორმაციული სისტემის იმპლემენტაციის მენეჯმენტის ამოცანაში

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ი. ჯგერბილიძის სახ. თბილისის სახელმწიფო უნივერსიტეტის ზუსტ და საბუნებისმეტყველო მეცნიერებათა ფაკულტეტი, თბილისი

(წარმოდგენილია აკადემიის წევრის ა. ფრანგიშვილის მიერ)

ინფორმაციული სისტემების იმპლემენტაციის მენეჯმენტში მნიშვნელოვანი ადგილი უკავია მრავალფაქტორული და მრავალექსპერტული გადაწყვეტილებების მიღების ტექნოლოგიების დანერგვას, როცა შეფასებები შესაძლო ალტერნატივებზე წარმოდგენილია ექსპერტთა მიერ ფაზი-ცვლადების მნიშვნელობების სახით (ჩვენს შემთხვევაში სამკუთხა ფაზი-რიცხვებით). ნაშრომში ნაჩვენებია ახალი აგრეგირების მეთოდოლოგია, როდესაც გასაშუალოების აგრეგირების ოპერატორში ალბათური განაწილება შეცვლილია შესაძლებლობითი განაწილებით. წარმოდგენილია მტკიცებულებები განზოგადების კორექტულობაზე, როდესაც ცნობილი შოკეს ინტეგრალი წარმოდგენს ავტორთა მიერ განზოგადებული აგრეგირების ოპერატორის კერძო შემთხვევას. აღნიშნულ პრობლემაში ახალი აგრეგირება გამოყენებულია გადაწყვეტილების მხარდამჭერი სისტემის სახით, როდესაც ფაქტორებზე ობიექტური მონაცემები არ არსებობს ალბათური განაწილების შესაფასებლად და ექსპერტული მონაცემები ერთადერთი წყაროა ამოცანაში წარმომოხილვი განუზღვრელობის მოსახსნელად. ამოცანის რეალიზაციის მიზნით შექმნილია შესაბამისი პროგრამული უზრუნველყოფა. შედეგები ილუსტრირებულია პრაქტიკულ მაგალითზე.

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