საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, ტ. 10, №2, 2016
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 10, no. 2, 2016

Hydrology

Investigation of Stability of Long One-Dimensional Waves in Debris and Water Flows

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ABSTRACT. Instability in uniform motion flows occurs when continues waves take over dynamic waves. In those cases primary uniform motion of the flow looses its stability and waves appear on free surface of the flow. The issue of prediction of waves occurrence on free surface are discussed in the paper. The calculations are made for both, cohesive debris flows (non Newtonian liquid) and water flows (Newtonian liquid) in order to control and secure stability of ecological situation in the bed and surrounding medium of water flow. *© 2016 Bull. Georg. Natl. Acad. Sci.*

Key words: debris flow, water flow

Instability in uniformly moving cohesive debris and water flows [1,2] occurs when continuous waves overtake dynamic waves C_1. In this case primary uniform motion will be unstable which is realized by occurrence of the wave with significant amplitude on free surface of the wave flow, i.e.,

$$V_b > V + C_1 \qquad (1)$$

where V is an average velocity on the live cross-section at uniform motion rate.

Cohesive Debris Flows

Huge hyperconcentrated with sediments (cohesive, structural) debris flows are mainly formed is in erosion incisions, representing the whole system of beds of upper mountain water flows which as a result of continuous rock destructions and their motions from the above are filled with broken mass being exposed to the wind and crushed under the influence of different natural factors. As a result of the like phenomena mud mass mixes with broken rock mass and fills all the cavities. Being ready in erosion incisions debris mixture is in hyperconcentrated with sediments (cohesive) state and one of the reasons, such as shower, intensive snow melting, the occurrence of underground waters, will make the debris flow surge down the slopes on grabbing on the way stone pieces, trunks of the trees, etc., forming tremendous debris flow with powerful destruction force [1,3].

Hyperconcentrated alluvial debris flow includes 80÷90% (by mass) hard material and 10÷20% of water (in viscous state) Density of such mixture is 1.8÷2.3 t/m^3, moving medium presents plastic mudstone conglomerate.

Let us consider continuous waves at the motion of progressive flow with constant discharge on the way. Naturally, the discharge of cohesive debris flow

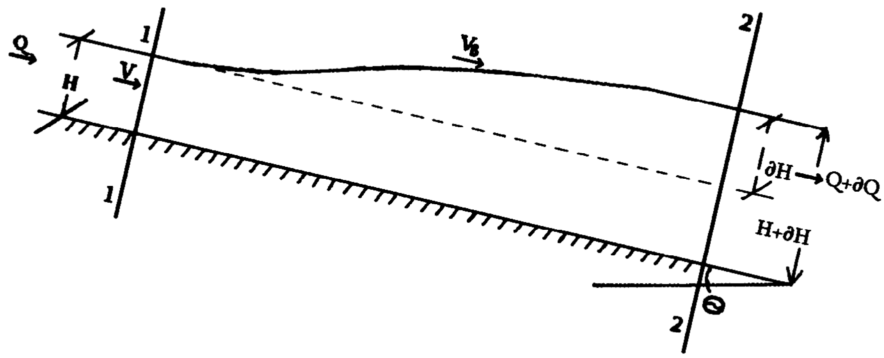

Fig. 1. Scheme of calculation of debris flow continues wave with constant discharge along the way.

at stationary regime of the motion depends on the depth H.

The velocity of continuous wave V_b passing through the control section lines 1÷1 and 2÷2 (Fig. 1) can be determined from the conditions of continuity; in this case, we have the following equality:

$$Q-\omega V_b=Q+\partial Q-V_b(\omega+\partial\omega), \qquad (2)$$

where Q is the flow discharge in the section line 1÷1; $Q+\partial Q$ – flow discharge in the section line 2÷2; ω – live cross section in the section line 1÷1, V_b – velocity of the spread of continuous wave.

From (2) it follows [1]:

$$V_b=\frac{\partial Q}{\partial\omega}. \qquad (3)$$

Given that

$$Q=V\omega, \qquad (4)$$

then, taking into account (4) instead of (3) we have:

$$V_b=\frac{\partial(\omega V)}{\partial\omega}=V+\omega\frac{\partial V}{\partial\omega}. \qquad (5)$$

From (5) it turns out that continuous wave velocity is greater, than the average velocity on the flow cross-section by the value

$$\omega\frac{\partial V}{\partial\omega}.$$

Discharge of the cohesive debris flow at the uniform motion mode [1]:

$$Q=\frac{BgiH^3}{\nu_c}f(\beta), \qquad (6)$$

where $\nu_c=\mu_c/\rho_c$ - the kinematic viscosity of the cohesive debris flow; μ_c - the dynamic viscosity of the cohesive debris flow; ρ_c - density of cohesive debris flow.

$$f(\beta)=\frac{\beta}{2}(\beta^2-1)+\frac{1}{3}(1-\beta^3), \qquad (7)$$

where $\beta=\dfrac{h}{H}$ is relative depth;

h - the depth of the structural part of the flow core;

B - the width of the flow;

g - the acceleration of gravity force.

Specific values $f(\beta)$ can be taken from Table 1.

In the bed with rectangular cross-section the average flow velocity is

Table 1. Specific values of $f(\beta)$

$\beta=\frac{h}{H}$	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$f(\beta)$	0.333	0.283	0.234	0.187	0.14	0.1	0.069	0.04	0.018	0.0

$$V = \frac{q}{H}, \tag{8}$$

where q is the discharge per unit width flow.

From (3) and (6) it follows:

$$V_b = \frac{\partial Q}{\partial \omega} = \frac{dV}{dH} = \frac{3giH^2}{v_c} f(\beta), \tag{9}$$

while solving the task without specific error we can assume that $\frac{f(\beta)}{v_c} \cong const$

Using (8), we obtain:

$$V = \frac{Q}{\omega} = \frac{q}{H} = \frac{giH^2}{v_c} f(\beta). \tag{10}$$

Comparing (9) and (10) we have:

$$V_b = 3V. \tag{11}$$

Thus, it turns out that continuous wave velocity is three times greater, than the average flow velocity along the cross section of the flow velocity.

Since the cohesive debris flow mixture differs from the water, has the property of the so-called "static shift stress" [1,3], which corresponds to the shift in start of the motion, then the "dynamic" shift stress is a relative concept and expresses the part of the tangent stress (not depending on velocity) while moving.

Due to the above, the cohesive debris mixture at a certain depth does not move, even on sloping surface, that is, does not flow down, so unlike water the dependence of Lagrange $C_1 = \sqrt{gH}$ [4] for non-Newtonian liquids (including cohesive debris flow) should be expressed as follows [1]:

$$C_1 = \sqrt{gH \cos \theta_1} \tag{12}$$

where θ_1 is a limit value of sloping plane, in which the debris flow mixture of certain depth and given consistency starts to move; at the same angle of inclination of the bottom of the water source the debris flow, reaching certain depth less than at the motion, stops moving.

Therefore, the dependence (12) characterizes the dynamic wave in the cohesive debris flow that includes the part of stress, which is necessary to overcome, the so-called, bias resistance motion.

Substituting (9), (10), (12) into (1) and taking into account that $i = Sin\theta$, we obtain the necessary condition for the instability in the form of inequality:

$$\frac{3gSin\theta H^2}{v_c} f(\beta) > \frac{gSin\theta H^2}{v_c} f(\beta) + \sqrt{gHCos\theta_1}$$

or

$$3 > 1 + \frac{v_c \sqrt{gHCos\theta_1}}{gSin\theta H^2 f(\beta)} \tag{13}$$

Dependence (13) characterizes the condition of the instability of one-dimensional long waves in cohesive debris flow with positive gradient of the bottom, when traffic flow is conditioned by gravity force.

Water Flows

Erosion processes are particularly intense in mountain and foothill conditions, where they often reach catastrophic proportions, washing away several dozens of tons of soil per hectare during a year [2].

During the movement of the liquid effluent shallow depth along the slope often the wave motion occurs, contributing to the intensification of erosion processes.

Waves in the watercourses and on the slopes of landscapes carry changes of the main hydraulic and hydrological parameters of runoff, both continuously and in steps.

Taking into account that the width of the slope B is usually much greater, than the depth H of the flow, i.e. $B >> H$, it is possible to see the drain on flat sloping surface of 1 m wide. This is possible because there is only wave propagation in the direction of translational motion of the flow. Then from (5) it follows:

$$V_b = V + H\frac{dV}{dH}.$$

Designating flow runoff in the alignment of 1-1, through $q = \frac{dQ}{dH}\left(\frac{M^2}{cek}\right)$, then (3) takes the form:

$$V_b = \frac{dq}{dH}. \qquad (14)$$

The flow moves along the slope in uniform turbulent motion regime (before the waves occur) and is described by the Chezy formula $Q = \omega C\sqrt{Ri}$, where C - Chezy coefficient, $R = \frac{\omega}{\chi}$ - hydraulic radius, χ - wetted perimeter, i - the gradient of flat surface slope.

Due to the fact that the problem is considered with flat width of 1 m, then Chezy formula takes the form $q = HC\sqrt{Hi}$ or

$$q = CH^{1,5}\sqrt{i}. \qquad (15)$$

Let $K = C\sqrt{i} = const$. Then (15) takes the form:

$$q = KH^{1,5}. \qquad (16)$$

If we take into account that for uniform motion $V = \frac{q}{H}$, then we shall have:

$$V_b = 1.5KH^{0,5}. \qquad (17)$$

It should be taken into account that

$$V = \frac{q}{H} = \frac{KH^{1,5}}{H} = KH^{0,5}. \qquad (18)$$

Comparing (15) and (16) we obtain:

$$V_b = 1.5V, \qquad (19)$$

i.e., continuous wave velocity is by one and a half time more, than the average velocity along the live cross-section of the flow at a uniform motion regime.

Dependence (19) points to the need of taking into account the availability of the wave-like flow traffic on the slopes to quantify the intensity of soil erosion.

The velosity of slope runoff determines the force action on the particles, aggregates, soil separations at their isolation, as well as the conveying ability of solid particles of the soil flow. For prediction of the critical velocity, at which the erosion process begins, currently a number of methods are spread. The mostly known are [5-10].

Dynamic wave velocity C_1 can be determined by Lagrange's formula [4]:

$$C_1 = \sqrt{gH}. \qquad (20)$$

Then, taking into account that continuous waves overtake the dynamic waves, the original uniform motion along the slope is unstable, that is realized in the advent of waves with significant amplitude on the free surface slope runoff i, e., (1).

Such waves can be clearly detected even on the sloping streets during heavy rain, even in the streets with minor deviations at small depths of runoff.

Substituting into (1) dependences (17), (18) and (20). we can obtain criteria correlation for prediction of waves on the free surface slope runoff with the following inequality:

$$C > 2\sqrt{\frac{g}{i}}. \qquad (21)$$

According to the Academician N. N. Pavlovsky [4], the coefficient in (the metric system) $C = \frac{1}{n}H^{Y}$, where n is the coefficient of the slope roughness; $Y = 1.5\sqrt{n}$ is index of the level at $H < 1$. Then minimal depth of slope runoff, at which the occurrence of the waves on the free surface of the flow is possible, will be:

$$H > \sqrt[Y]{2n\sqrt{\frac{g}{i}}}\ \text{m}. \qquad (22)$$

Continuous waves will carry the corresponding values of runoff depths, while each wave will propagate with its velocity in accordance with (14). If at the initial moment of the flow formation t=0 at X=0 then from this moment the waves corresponding to all values of H begin to spread.

The account of the occurrence the wave-like motion of slope runoff in the known classical methods of calculation [5÷10 etc.] has some difficulties, which we do not consider in this paper.

Thus, the forecast of the occurrence of the waves on the free surface slope runoff should be evaluated on the dependences for cohesive debris flow (13) and for the water flows (21) or (22).

ჰიდროლოგია

წყლის და ღვარცოფულ ნაკადებში გრძელი ერთგანზომილებიანი ტალღების გამოკვლევა

ო. ნათიშვილი

აკადემიის წევრი, საქართველოს მეცნიერებათა ეროვნული აკადემია, თბილისი

თანაბარი სიჩქარით მოძრავი (როგორც ნიუტონური, ასევე არანიუტონური) ნაკადი კარგავს თავის პირველად (სტაციონარულ, დამყარებულ) წონასწორობას იმ შემთხვევაში, როდესაც გრძელი ერთგანზომილებიანი უწყვეტი ტალღის სიჩქარის რიცხვითი მნიშვნელობა გადააჭარბებს ცოცხალ კვეთში ნაკადის საშუალო და დინამიკურ სიჩქარეთა ჯამს.

ნაშრომში მოყვანილია საანგარიშო დამოკიდებულებები როგორც წყლისათვის (ნიუტონური სითხე), ასევე ბმული ღვარცოფისათვის (არანიუტონური სითხე) გრძელი ერთგანზომილებიანი უწყვეტი ტალღის, ცოცხალი კვეთის საშუალო და დინამიკურ სიჩქარეთა საანგარიშო გამოსახულებანი. დადებითი ქანობის მქონე (როგორც ბუნებრივ, ასევე ხელოვნურ) კალაპოტების შემთხვევებში ზემოთ მითითებულ გამოსახულებათა ურთიერთკავშირი (უტოლობის ფორმით) საშუალებას იძლევა წინასწარ ვიმსჯელოთ არსებული მდგრადი პირობების შესაძლო დარღვევის შესაძლებლობაზე (პროგნოზზე), რითაც, შესაბამისი ღონისძიებების გატარებით, თავიდან ავიცილებთ ეკოლოგიური წონასწორობის დარღვევას ან შევარბილებთ მის უარყოფით ზეგავლენას გარემოზე.

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Received February, 2016