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Cybernetics

Open Queuing System for Two Parallel Maintenance Operations as Mathematical Model for Dependability and Performance Analysis

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ABSTRACT. In the paper a multi-component redundant system with unreliable, repairable units is considered. Two types of maintenance operations are performed in the system: 1) the replacement of the failed active unit by the redundant one; 2) the repair of the failed unit. The open exponential queuing model for the system's dependability and performance analysis is constructed in the form of infinite system of ordinary linear differential equations. In steady state it is reduced to the infinite system of linear algebraic equations. At present the system is still being investigated. *© 2016 Bull. Georg. Natl. Acad. Sci.*

Key words: maintenance, replacement, repair, queuing model.

In modern reliability theory the performance (effectiveness) analysis of the complex systems with unreliable, repairable components is one of the most topical directions in the field [1,2].This is exactly the system level of investigation, unlike the component (equipment) level of classical reliability theory.

Performance analysis is related to systems, for which one is not able to formulate the "all or nothing" serviceable or non-serviceable type of failure criterion. Effectiveness characterizes the system ability to perform its main functions even with partial capacity. Failures of some or even majority of the system components lead only to a gradual degradation of the system ability to perform its functions (operations). Actually one deals with such indices like "partial availability", "partial system down time". These types of notions are used to describe multi-component systems, e.g., global terrestrial systems, computer, telecommunication and transportation networks, gas and oil distribution systems, power systems, defense systems, etc. or the so-called systems with embedded "functional redundancy", where the optional ways to perform system tasks exist [1,2].

While studying the mentioned systems, traditional mathematical models of classical renewal theory, reliability theory and queuing theory in many cases proved to be unsuitable, and an urgent necessity for the construction and investigation of completely new types of models for them arose [1-9].

In classical reliability theory and practice the equipment reliability provision was the main direction. That is why in its framework the replacement problem for single-unit systems was studied so thoroughly. Also the repairman problem has been studied thoroughly mainly for such complex systems, which are reducible to simple two state failure criterion: serviceable and non-serviceable (The same is "all or nothing" with Ushakov, "on" or "off" with Barlow, "active" or "inactive", "good" or "bad" with Epstein, "up" or "down" with Gertsbach [1,2, 12-16].

In the models related to the replacement problem of single-unit systems the time required to make a replacement mainly was considered to be zero. Even in the cases, where replacement time was non-negligible, the replacement problem of a single-unit system was described by alternating renewal process, which caused no difficulties [12-16].

While analyzing complex systems the replacement time was not taken into consideration [14].

As a matter of fact, in traditional cases of redundancy, active and redundant units (as a rule) were territorially concentrated at the same place and the replacement of the failed active unit with a redundant one meant the latter switched over, which was often automatically performed and its duration was negligibly small.

In modern networks of the above type, however, redundant units are not directly attached (linked) to active ones. They are placed at specific storages and may be located at the distance of tens, hundreds and sometimes thousands of kilometers away from the active units. Therefore, the delivery time of the redundant unit to the place of the failed active one is quite essential.

At the same time, in practical cases due to various reasons before the start of the delivery operation of the redundant unit some time passes, which is often many times greater, than the delivery time itself. In addition, the replacement operation, apart from the delivery of the redundant unit to the active unit place, includes other sub-operations, whose execution is necessary in order the redundant unit to continue the active unit functions. Under such circumstances the mean replacement time is not insignificant and often reaches 20-40% of the mean repair time. Moreover, the replacement operation, as a rule, is performed not by a repair facility, but by special replacement channel. Therefore, the replacement of the failed active unit by the redundant one quite naturally becomes an independent maintenance operation [3-9].

Subject of the Study and its Initial Mathematical Description.

The investigation subject of this paper is a multi-component redundant system with unreliable, repairable units. The system consists of identical active and redundant units, their numbers are m and n, respectively. The redundant units are designated for permanent replacement of the main components in case of their failure. It is supposed that for the normal operation of the system, the serviceability of all active units is desired. However, if their number is less, then the system continues to function, but with lower economic effectiveness.

The total failure rate of all active units is ±. The redundant ones do not subject to failures. A failed active unit is replaced by a serviceable redundant one, if there is an available unit in the system. In the opposite case the replacement will be performed after the availability of the redundant unit. The failed units are repaired, become identical with the new ones and pass to the group of redundant units. The system has one replacement facility and one repair facility. The replacement time and repair time are random variables with distribution functions F and G, respectively. When maintenance facilities are busy, requests for replacement or repairs are queued. Service discipline is FCFS (first come, first served). As we see, in a natural way we have a queuing system with two types of maintenance operations: replacement and repair. We consider here the case, where m is a large number (in practice it might be tens, hundreds, thousands and more), and we suppose that we have an infinite source of requests and get an open queuing system for two parallel maintenance operations: replacements and repairs.

The request for the replacement arises due to the failure of the active unit. The same event generates the request for repair. Thus, the necessity of two parallel service operations arise.

To this day, neither in the reliability theory, nor in the queuing theory the above problems were investigated. At the same time modern research methods of Markov and semi-Markov processes make it possible to construct and analyze such models in the framework of the mathematical theory of reliability and queuing theory [10, 11].

During the last 10-12 years experts of the Georgian Technical University (GTU) achieved notable success in this direction [3-9].

Namely, the queuing systems of the above type, where in the arriving stream of homogenous events the demands for two parallel service operations arose, were first introduced by GTU experts and have not been considered by the other authors yet.

The Mathematical Model.

In this section we construct and investigate the mathematical model for the case $m=\infty$ where n is arbitrary. The replacement and repair times have exponential distribution functions with parameters λ and μ, respectively.

To describe the considered system we introduce the random processes, which determine the states of the system at the time t;

$i(t)$ – the number of units missed in the group of active units;

$j(t)$ – the number of non-serviceable (failed) units in the system.

Denote:

$$P(i,j,t)=\mathbb{P}\{i(t)=i;j(t)=j\},\ \ i=\overline{1,m};\ \ j=\overline{0,n+i}$$

Proceeding in the usual way, we can set up the basic difference equations, which relate the probability of being in a certain state at time to the probabilities of being in variousstates at time $t+\Delta t$. From these difference equations we obtain the infinite systems of ordinary linear differential equations.

For $n>0$ we have

$$\begin{cases} \dfrac{dP(0,0,t)}{dt}=-\alpha P(0,0,t)+\lambda P(1,0,t)+\mu P(0,1,t) \\ \dfrac{dP(i,0,t)}{dt}=-\alpha(\alpha+\lambda)P(i,0,t)+\lambda P(i+1,0,t)+\mu P(0,1,t), \quad 0<i<\infty, \\ \dfrac{dP(0,j,t)}{dt}=-\alpha(\alpha+\mu)P(0,j,t)+\lambda P(1,j,t)+\mu P(0,j+1,t), \quad 0<j<n, \\ \dfrac{dP(0,n,t)}{dt}=-(\alpha+\mu)P(0,n,t)+\beta P(0,n-1,t)+\lambda P(1,n,t), \\ \dfrac{dP(i,n+i,t)}{dt}=-\alpha(\alpha+\mu)P(i,n+i,t)+\alpha P(i-1,n+i-1,t)+\lambda P(i+1,n+i,t), \quad 0<i<\infty, \\ \dfrac{dP(i,j,t)}{dt}=-\alpha(\alpha+\lambda+\mu)P(i,j,t)+\alpha P(i-1,j-1,t)+\lambda P(i+1,j,t)+\mu P(i,j+1,t), \\ \qquad\qquad 0<i<\infty, 0<i<j+1. \end{cases} \tag{1}$$

For $n = 0$ we have:

$$\begin{cases} \dfrac{dP(0,0,t)}{dt} = -\alpha P(0,0,t) + \lambda P(1,0,t), \\ \dfrac{dP(i,0,t)}{dt} = -(\alpha+\lambda) P(i,0,t) + \lambda P(i+1,0,t) + \mu P(i,1,t), \quad 0 < i < \infty, \\ \dfrac{dP(i,i,t)}{dt} = -(\alpha+\mu) P(i,i,t) + \alpha P(i-1,i-1,t) + \lambda P(i+1,i,t), \quad 0 < i < \infty, \\ \dfrac{dP(i,j,t)}{dt} = -(\alpha_0+\lambda+\mu) P(i,j,t) + \alpha_0 P(i-1,j-1,t) + \lambda P(i+1,j,t) + \mu P(i,j+1,t), \\ \qquad\qquad 0 \le i < +\infty, \ j \le i < +\infty, \\ P(i,j,t) = 0, \ \text{if } i < j, \text{ or } i < 0, \text{ or } j < 0. \end{cases} \tag{2}$$

It can be proved that for these systems the limit of $P(i,j,t)$, as $t = \to \infty$ exists for all i, j, if $\alpha < \lambda$ and $\alpha < \mu$.

Denote $P(i,j) = \lim\limits_{t\to\infty} P(i,j,t)$. Letting $t = \to \infty$ in (1) and (2) we obtain an infinite system of linear algebraic equations with respect to $P(i,j)$.

For $n > 0$ (together with normalizing condition) we have:

$$\begin{cases} \alpha P(0,0) = \lambda P(1,0) + \mu P(0,1), \\ (\alpha+\mu) P(i,0) = \lambda P(i+1,0) + \mu P(i,1), \qquad 0 < i < \infty, \\ (\alpha+\mu) P(0,j) = \lambda P(1,j) + \mu P(0,j+1), \qquad 0 < j < n, \\ (\alpha+\mu) P(0,n) = \beta P(0,n-1) + \lambda P(1,n), \\ (\alpha+\mu) P(i,n+i) = \alpha P(i-1,n+i-1) + \lambda P(i+1,n+1), \quad 0 < i < \infty, \\ (\alpha+\lambda+\mu) P(i,j) = \alpha P(i-1,j-1) + \lambda P(i+1,j) + \mu P(i,j+1), \\ \qquad\qquad 0 < i < \infty, 0 < j < n+i, \\ \sum\limits_{i=0}^{\infty} \sum\limits_{j=i}^{n+i} P(i,j) = 1; 0 < i < \infty, 0 < j < n+i. \end{cases} \tag{3}$$

For $n = 0$ we have (together with normalizing condition):

$$\begin{cases} aP(0,0) = \lambda P(1,0), \\ (a+\lambda) P(i,0) = \lambda P(i+1,0) + \mu P(i,1), \quad 0 < i < \infty, \\ (a+\mu) P(i,i) = \alpha P(i-1,i-1) + \lambda P(i+1,i), \quad 0 < i < \infty, \\ (\alpha+\lambda+\mu) P(i,j) = \alpha P(i-1,j-1) + \lambda P(i+1,j) + \mu P(i,j+1), \quad 0 \le j \le i < \infty, \\ P(i,j) = 0, \ \text{if } i < j, \text{ or } i < 0, \text{ or } j < 0, \\ \sum\limits_{i=0}^{\infty} \sum\limits_{j=0}^{i} P(i,j) = 1. \end{cases} \tag{4}$$

After finding the probabilities $P(i, j)$, it is easy to calculate all the steady-state dependability and performance measures for the considered system.

The results of the investigation of the systems (3) and (4) will be published in the nearest future.

Conclusions.

The present paper is the first in scientific publications to discuss an open queuing system for two parallel service operations.

In general, while constructing and investigating the mathematical model of the discussed system the main difficulties, as always, are caused by types of functions F and G.

When one of the functions F and G is arbitrary, and the second one is exponential, it is quite possible to construct the semi-Markov model, the investigation of which, certainly, will be a complex problem, but we suppose it will be more or less surmountable.

When these functions are both arbitrary, it is possible to construct a model of Markov renewal type. The problem of its investigation is still unclear.

In our case both of them are exponential. This case is the simplest one and we have obtained the classical Markov model.

Note that the study and solution of infinite system of equations, as a rule, is a very complex problem, often unsurmountable. But the matrices of our systems (3) and (4) are highly sparse and this gives us a chance to advance in their investigation.

Namely, the problem of existence and uniqueness of the solution was investigated. Also, the numerical algorithms were developed, making it possible to find the approximate solution by means of finite arithmetical operations. Finally, the error of the approximate solution was estimated.

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კიბერნეტიკა

ორი პარალელური მომსახურების ღია სისტემა როგორც საიმედოობისა და ეფექტიანობის ანალიზის მათემატიკური მოდელი

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წარმოდგენილ ნაშრომში განხილულია მრავალკომპონენტიანი დარეზერვებული სისტემა, რომელიც შედგება არასაიმედო, აღდგენადი ელემენტებისგან. ამ სისტემაში სრულდება მომსახურების ორი პარალელური ოპერაცია: 1) მტყუნებული ელემენტის ჩანაცვლება სარეზერვოთი; 2) მტყუნებული ელემენტის აღდგენა. აგებულია რიგების ღია ექსპონენტური მოდელი საკვლევი სისტემის საიმედოობისა და ეფექტიანობის ანალიზისათვის. ის წარმოადგენს ჩვეულებრივ წრფივ დიფერენციალურ განტოლებათა უსასრულო სისტემას. მისგან სტაციონარულ მდგომარეობაში მიღებულია წრფივ ალგებრულ განტოლებათა უსასრულო სისტემა. ამჟამად მიმდინარეობს ამ სისტემის გამოკვლევა.

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