საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, ტ. 10, №4, 2016
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 10, no. 4, 2016

Cybernetics

Long-Term Inventory Control Problem for Cascade Systems

Jimsher Giorgobiani

Muskhelishvili Institute of Computational Mathematics, Georgian Technical University, Tbilisi, Georgia

(Presented by Academy Member M. Salukvadze)

ABSTRACT. Present work deals with multi-phase models of inventory control for cascade systems. Situation is similar to that of queuing theory, the inflows are used to fulfill various types of demands. Three models are introduced. The first two models are relatively easy and fit well the control of hydro-energy system. They are implemented by means of dynamic programming. The third model is general, cascade may include some types of enterprises together with their storages. In the case of homogenous productions the model can be applied for the control of cascade hydropower stations. This model is the problem of linear programming. In all cases optimality criteria is the maximum profit. *© 2016 Bull. Georg. Natl. Acad. Sci.*

Keywords: inventory control, mathematical model, mathematical programming, dynamic programming, optimality criteria

While analyzing and modeling the production management problems it is important to apply the principles and methods of various brunches of operations research, in particular the general concepts and mathematical models of the inventory control theory. This sphere is of a great variety due to the reason that it reflects the factors, together with its technical and economic features, production, demand, supply-storing, marketing etc.

Many problems of inventory control can be posed as the problem of queuing theory. Most frequently they are associated in the case of random flow control problem. The main issue of the queuing theory is the “handling” of inflow, while in inventory control theory this is the “usage” of flow for the demand fulfillment. There is not only a formal similarity between these two theories, but methodological as well [1].

Flow control with one base from the view-point of control theory was first treated in the 60-ies of previous century [2, 3]. The models with the series of bases through the conveyer regime are considered completely in the monographs [4, 5].

Here we consider the flow control in the cascade systems, where each “base” of inventory serves the enterprise of some profile. These enterprises use some part of the flow (resource) according to the queue and earn their income in this respect. The models for the cascade systems fit many problems of hydro and heat energy, oil production, sales and many other spheres of economy. Cascade systems are important in hydro energy. This natural problem is well discussed in the monograph [5], where together with the engineering tools the balancing equations, reflecting the essence of the problem are also given. Paper [6] deals with the original method of accumulation of the energy in cascade systems – hydrogen, derived in the process of electrolyze of water is used in the peak regime. Optimal control problem of a hydro-cascade is given in [7], where the scheme of solution for the case of one common reservoir is obtained by means of the dynamic programming method, while the case of multiple (consecutive) reservoirs is solved under one natural assumption. Paper [8] suggests the general scheme of control of energy stations. The authors of this work aside from the classical models [9, 10] discuss our work [7] as well.

Our problems are of multistage (dynamic) and multi-production type and their linear programming models will be considered. On the stages the inflow will be assumed determined (will be equal to the statistical or forecasted mean of the random variable). Stochastics may be involved due to the conventional standard form. Generally the volume of the flow on the stage can be considered as the quantity of the purchase order, which is not fulfilled exactly and the statistical distribution is known.

Let us introduce some notations:

T - scheduling horizon consisting of the stages $t = 1, 2, .., T$;

N - number of enumerated bases (enterprises), $i = 1, 2, \ldots, N$;

$\overline{Q_i}$ - inventory capacity of i-th base;

I_i - capacity of i-th enterprise (maximum resource that the enterprise is capable to admit at one stage);

Q_i^t - storage of i-th base in the beginning of stage t;

u_i^t - resource utilized by the i-th enterprise at stage t;

v_i^t - resource stored by the i-th enterprise at stage t;

$u_i^t + v_i^t$ - resource used by the i-th enterprise at stage t;

r_i^t - demand on i-th enterprise at stage t;

c_i^t - profit from the unit of i-th production at stage t;

p_i^t - penalty for the shortage (deficit);

$W_i(u)$ - quantity of production received from the resource u at i-th base $W_i(u) = \propto_i u$;

$\overline{W}_i^t(u)$ - profit from the resource of quantity u at stage t, $\overline{W}_i^t(u) = c_i^t \cdot W_i(u)$;

β - cost of the unit resource;

u_i^t, v_i^t - variables, their number is $2NT$.

Below we present mathematical models of the problems of various difficulties. The first two problems are concrete for clarity, though they may fit other cases as well. These models are of dynamic programming format. Third model is general and is of linear programming type; in fact this is one of the dynamic problems of production smoothing.

Problem 1. First consider one simple example, which is not rare in hydrology and which has stimulated the idea of control of general cascade systems.

Assume the river flow is used to generate electric power, for water supply and melioration. We have one hydro station with reservoir (possibly cascade of stations), and one more reservoir below for water supply and melioration.

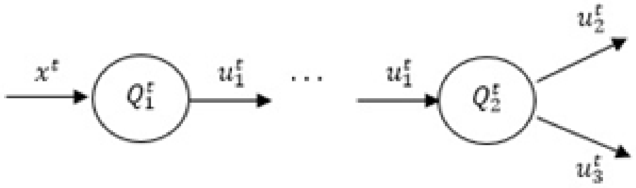

Fig. 1. The scheme of successive parallel type

Conditions:

$$\forall t: u_1^t \le \min\left[Q_1^t + x^t, I\right], \qquad u_2^t + u_3^t \le min\left[Q_2^t + u_{1,}^t\ \overline{Q_2}\right]. \tag{1}$$

Here I - is the maximum waste of water of the hydro station at one stage. Administrative costs at each stage for any decision are assumed constant. Thus, the optimal policy is the one, which maximizes the difference between the profit of electric energy and the losses tied with shortage. Hence, the profit at one stage is

$$F\left(Q_1^t, Q_2^t\right) = \overline{W}\left(u_1^t\right) - \left(\overline{u}_2^t - u_2^t\right)\cdot p_2 - \left(\overline{u}_3^t - u_3^t\right)\cdot p_3, \tag{2}$$

where $\overline{u}_2^t$ and $\overline{u}_3^t$ correspond to the norms of water supply, p_2 and p_3 are the losses (penalty) for the unite of shortage, respectively.

Taking into account (1) and (2) we apply the optimality principle of dynamic programming. Denote $f_t\left(Q_1^t, Q_2^t\right)$ the income for optimal policy from stage t to stage T (including), when at the beginning of stage t resources of water in reservoirs are equal to Q_1^t and Q_2^t:

$$f_t\left(Q_1^t, Q_2^t\right) = \max_{\substack{u_2^t \le \overline{u}_2^t; \\ u_3^t \le \overline{u}_3^t \\ u_2^t + u_3^t \le Q_2^t + u_1^t}} \begin{Bmatrix} W\left(u_1^t\right) - \left(\overline{u}_2^t - u_2^t\right)\cdot p_2 - \left(\overline{u}_3^t - u_3^t\right)\cdot p_3 + \\ f_{t+1}\left(Q_1^t + x^t - u_1^t, Q_2^t + u_1^t - u_2^t - u_3^t, \cdots\right) \end{Bmatrix}. \tag{3}$$

At first glance the problem is three-dimensional, but due to (1), it is clear that the problem can be reduced to the choice of u_1 and u_2 while u_3 is chosen automatically. Besides, it should be also noted that $\overline{u}_2^t$ is zero in most of the cases (melioration is seasonal).

Problem 2. Consider "renewable" homogenous cascade system - the cascade of hydro power plants with N stations, when the reservoir exists only with the first station and there is an extra flow ϑ_i^t between (i-1) and i stations. Under these conditions i-th station (i>1) during the period (t, t+1) utilizes $u_i^t = \min\left[I_i; u_1^t + \vartheta_1^t + \vartheta_2^t + \cdots + \vartheta_i^t\right]$ water and produces $W_i^t\left(u_i^t\right)$ energy with corresponding price $\overline{W}_i^t = W_i^t \cdot c^t$. If as above the optimality criteria is the difference between the profit of electric energy and the losses due to the shortage then the profit at stage i is

$$F_t\left(Q^t, u_1^t\right) = \sum_{i=1}^{N} \overline{W}_i\left[\min\left(I_i; u_1^t + \sum_{j=1}^{i}\vartheta_j^t\right)\right] + \left\{\sum_{i=1}^{N} W_i\left[\min\left(I_i; u_1^t + \sum_{j=1}^{i}\vartheta_j^t\right)\right] - r^t\right\}\cdot p^t$$

and due to the optimality criteria

$$f_t\left(Q^t\right) = \max_{0 \le u_i^t \le \min\left[I, Q^t + x^t\right]} \left\{F_t\left(Q^t, u_1^t\right) + f_{t+1}\left(Q_1^t + x^t - u_1^t\right)\right\}. \tag{4}$$

Note that as in the first problem we can consider reservoirs with some stations in this case as well. Though evidently this can be done for the case of 2-3 reservoirs.

Problem 3. Consider the general multi-production problem when each base utilizes u_i^t part of existing resources, keeps v_i^t part in the storage and transmits the rest to the successive base (enterprise). The bases generally produce the different production. Each of them receives the profit c_i^t from the unit resource and in total $\bar{W}_i^t = c_i^t \cdot u_i^t$ at a stage. This process consists of *T* stages and the existing in the storages resources can be used on the consecutive stages. Using the above notations situation at stage *t* can be given in the Table:

Table 1. Multistep regulation of flow

Q Base	Resources at stage t	Utilized	Storage	Remaining resources
1	$v_1^{t-1} + x^t$	u_1^t	v_1^t	$v_1^{t-1} + x^t - (u_1^t + v_1^t)$
2	$v_1^{t-1} + v_2^{t-1} + x^t - \left(u_1^t + v_1^t\right)$	u_2^t	v_2^t	$v_1^{t-1} + v_2^{t-1} + x^t - \sum_{i=1}^{2}\left(u_i^t + v_i^t\right)$
...	...	...	...	...
k	$\sum_{i=1}^{k} v_i^{t-1} + x^t - \sum_{i=1}^{k-1}\left(u_i^t + v_i^t\right)$	u_k^t	v_k^t	$\sum_{i=1}^{k} v_i^{t-1} + x^t - \sum_{i=1}^{k}\left(u_i^t + v_i^t\right)$
...	...	...	...	...
N	$\sum_{i=1}^{N} v_i^{t-1} + x^t - \sum_{i=1}^{N-1}\left(u_i^t + v_i^t\right)$	u_N^t	v_N^t	$\sum_{i=1}^{N} v_i^{t-1} + x^t - \sum_{i=1}^{N}\left(u_i^t + v_i^t\right)$

After the stage *N* there may remain some unused resources:

$$\sum_{i=1}^{N} v_i^{T-1} + x^T - \sum_{i=1}^{N} u_i^T .$$

Due to the above table we can represent our problem in the form of linear programming. The variables u_i^t and v_i^t $\left(i = \overline{1,N}, t = \overline{1,T}\right)$ are the sought variables. For each *i* and *t* they should fulfill the technical and technological conditions of the problem. First of all the variables are bounded:

$$0 \le u_i^t \le I_i, 0 \le v_i^t \le \overline{Q_i}, \forall i,t. \tag{5}$$

The balance across the stages between the used resources should be in accordance:

$$u_i^t + v_i^t \le \sum_{k=1}^{i} v_k^{t-1} + x^t - \sum_{k=1}^{i-1}\left(u_k^t + v_k^t\right). \tag{6}$$

If the optimality criteria is the maximum of total income and the production costs are assumed constant, then the target function is given as the difference between the production income and the shortage penalty:

$$L\left(u_i^t, v_i^t\right) = \sum_{t=1}^{T}\sum_{i=1}^{N} \propto_i u_i^t \cdot c_i^t - \sum_{t=1}^{T}\sum_{i=1}^{N}\left(r_i^t - \alpha_i u_i^t\right) p_i^t. \tag{7}$$

Thus, our problem will be posed analytically: find the values of the variables u_i^t and v_i^t, which fulfill the conditions (5), (6) and maximize the target function (7).

Finally, it should be noted that the elaborated models are adequate to the posed problems and their numerical implementation is quite easy.

კიბერნეტიკა

მარაგთა გრძელვადიანი მართვა კასკადურ სისტემებში

ჯ. გიორგობიანი

ნიკო მუსხელიშვილის სახელობის გამოთვლითი მათემატიკის ინსტიტუტი, საქართველოს ტექნიკური უნივერსიტეტი, თბილისი, საქართველო

(წარმოდგენილია აკადემიის წევრის, მ. სალუქვაძის მიერ)

ნაშრომში განხილულია მარაგთა მართვის მრავალფაზიანი მოდელები კასკადური სისტემებისთვის. სიტუაცია მსგავსია მასობრივი მომსახურების — შემომავალი ნაკადი გამოიყენება სხვადასხვა სახის მოთხოვნილებათა დასაკმაყოფილებლად. შემოთავაზებულია სამი მოდელი, რომელთაგან პირველი ორი შედარებით მარტივია და კარგად მიესადაგება ჰიდროენერგეტიკული სისტემების მართვას. მოდელები რეალიზდება დინამიკურ დაპროგრამებაზე დაყრდნობით. მესამე მოდელი ზოგადია — კასკადის შემადგენლობაში შეიძლება იყოს სხვადასხვაგვარი საწარმო თავიანთი საცავებით. ერთგვაროვან საწარმოთა შემთხვევაში მოდელი გამოდგება კასკადური ჰიდროსადგურების სამართავად. მოდელი წრფივი დაპროგრამების ამოცანაა. ოპტიმალობის კრიტერიუმად ყველგან აღებულია მოგების მაქსიმუმი.

REFERENCES:

1. *Buchan J. and Koenigsberg E*. (1963) Scientific inventory management. Prentice-Hall, Inc., New Jersey.
2. *Romanovskii I. V.* (1964) Teoria veroiatnostei i Primenenia. **9**, 1: 167-174 (in Russian). English translation: *Romanovskii I. V.* Theory of Probability and its Applications. **9**, 1: 149-156.
3. *Giorgobiani J. A*. (1964) Soobshchenie Akad. Nauk. Gruzii. **35**, 1: 17-21(in Russian).
4. *Hanssmann F.* (1962) Operation Research in Production and Inventory Control. John Wiley and Sons, Inc., New York.
5. *Cvetkov E.V., Alyabuesheva T.M., Parfenov L.G.* (1984) Optimal'nye rezhimi elektricheskix stantsii v energeticheskikh sistemakh. M. (in Russian).
6. *Di Lu., Bende Wang et al* (2015) Applied Energy. **137**, C: 56-63.
7. *Giorgobiani J.A., Nachkebia M.D., Lodwick W.A*. (2008) Dynamic model of smoothing problem in Water Power Systems. UCD/CCM (University of Colora-do, Denver, Center for Computational Mathematics). Report 272, arXiv: 0807.0641v1 [math. OC], Denver, Co.
8. *Ukwejeh J. M., Anyaka B. O.* (2013) IOSR Journal of Electrical and Electronics Engineering (IOSR-JEEE). **7**, 1: 101-104.
9. *Little J.D.C.* (1955) Operations Research. **3**, 2: 187-197.
10. *Koopmans T. C.* (1958) In: Proc. International Conference on Operations Research, Bristol, England: John Wright and Sons:193-227.

Received August, 2016