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Physics

Why do Quarks Have Fractional Charges?

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ABSTRACT. The quark model has a long history. The notion of quarks was introduced on the basis of unitary symmetry $SU(3)$ for hadrons. This tradition lasts in the modern books on particle physics. We want to mention that quark's fractional charge follows only from the compound nature of hadrons without any references to some symmetry. The constituent quarks must be fermions, and because charge is additive physical quantity, the sum of quarks' charge must be equal to the total charge of hadron. Normalization is chosen on charges of proton and neutron. The obtained linear system of equations is simply solvable and the solutions give the true values for quarks' charge. Therefore, this approach seems to be much more simple and do not require consideration of some methods of group theory. We consider this approach more simple and reasonable for student understanding. *© 2017 Bull. Georg. Natl. Acad. Sci.*

Key words: quarks, quarks charges, standard model

"Sunt igutur solida primordia simplicitate"
/ Lucretius Corus, "De Rerun Natura"I.548/

The standard model [1] is the most successful theory of particle physics. Its all ingredients are already observed on experiments. In spite of this, there remain some questions on which the standard model has no direct answers. One of them is the question of quarks' charge. More precisely, standard model does not answer the question – why do quarks have fractional charges.

As is well known, quarks were introduced by Gell-Mann and Zweig [2,3] in the framework of flavor $SU(3)$ symmetry. Now quark masses are so different that speaking about some flavor symmetry for them is, probably, exaggerated. Nevertheless for lightest quarks some broken symmetries, such as isotopic $SU(2)$ or unitary $SU(3)$, are still accessible. The phenomenological achievements of these symmetries are well known. Among them the prediction of quarks by Gell-Mann and Zweig was the most important in the framework of $SU(3)$ as a fundamental representation of this group, namely triplet.

There are 8 generators in this group, only two of them commute with each other, i.e. the rank of this group is 2 and its tensorial irreducible representations are characterized by two quantum numbers, which are the third component of isospin T_3 and the hypercharge Y. Their values in the irreducible representation $D(p,q)$ are following:

$$\begin{aligned} T &= \frac{1}{2}(p - p_3 + q - q_3) \\ T_3 &= \frac{1}{2}(p_1 - p_2 - q_1 + q_2) \\ Y &= \frac{1}{3}(p - q) - p_3 + q_3 \end{aligned} \tag{1}$$

where p and q are the numbers of upper and lower indices of irreducible tensorial representation, and p_i, q_1 $(i = 1,2,3)$ - numbers of one, two or three among them.

The eightfold way of Gell-Mann and Ne'eman [4] corresponds to representations in which $(p-q)$ is exactly divisible by 3 or the symmetry group is effectively $SU(3)/Z_3$. In this approach the course is standing to well-known empirical relation of Gell-Mann and Nishijima [5,6] for hadron charges

$$Q = T_3 + \frac{Y}{2}. \tag{2}$$

The hypercharge quantum number Y is fixed inside the baryonic octet by the requirement $Y(proton) = +1$. After that for proton and for all other particles the true values of charge Q follow. But, the charge operator itself is not the generator of the considered group. Therefore, relation (2) still remains empirical. Only in the frame of grand unified groups, such as minimal $SU(5)$, all operators of equation (2) are generators.

But today we know that this symmetry is not a component part of the standard model. Therefore, when quarks are considered, the strategy is that all their properties derived previously outside the standard model, are implied automatically.

As regards the standard model, here the situation is the following: The model has two branches – quantum chromodynamics (QCD - the theory of quarks and gluons) and electroweak (Glashow-Salam-Weinberg theory). In both theories for leptons and quarks the "needed" prescriptions of electric charges are used usually to be valid. But if leptons' electric charges are established experimentally, the same is not true for quarks. While the experimental verification of so called "Gottfried sum rule" [7] inclined to favor $1/3e$ for average quark charge inside the nucleon, in our opinion, it is not enough - for the theory more strong arguments are desired. There is no theoretical background for this, except the magic word –"*consist*".

Below we'll see that there is a very apparent and simple way to obviate a difficulty if we remember that in QCD quarks are "confined" inside hadrons, and bosons consist of quark–antiquark pairs, whereas baryons – of three quarks. As regards of the Pauli principle it is settled by the additional quantum number – color [8-10]. Nowadays the experimental data show that according to electroweak sector quarks are encountered in three generations of $SU(2)_L \times U(1)_Y$ gauge group. Below we take attention first of all to the first generation, so-called "ordinary quarks", which traditionally are denoted by u and d symbols.

Consider a nucleon – proton and neutron, which have electric charges +1 and 0, respectively. Let us suppose that until we don't know their internal structure, i.e. we do not know in advance how many u and d quarks are inside the nucleon out of 3 valence quarks. Let the respective numbers in proton and neutron be α and β. Clearly $\alpha, \beta \le 3$. Using the additivity of charge let us construct the relations for proton and neutron charges (in units of elementary charge E)

$$\alpha Q_1 + (3-\alpha) Q_2 = 1, \qquad \text{for protons} \tag{3}$$

$$\beta Q_1 + (3-\beta) Q_2 = 0, \qquad \text{for neutrons,} \tag{4}$$

where quark charges are denoted by $Q_{1,2}$ .We have also used that the sum of coefficients (constituents) are 3. This information should be enough for determining both α and β. The solutions of the Eqs. (1),(2) are

$$Q_1 = \frac{(3-\beta)}{\Delta}, \qquad Q_2 = -\frac{\beta}{\Delta} \tag{5}$$

where Δ is the discriminant of this simultaneous system of these equations,

$$\Delta = \alpha(3-\beta) - \beta(3-\alpha) = 3(\alpha-\beta) \tag{6}$$

Now remember that α and β are to be integer numbers. Clearly they are not equal to each others, $\alpha \neq \beta$, as is evident from (3) and (4), their values can be only 1 or 2. The solutions for these numbers would be

1. If $\alpha = 1$, then $\beta = 2$, i.e. in this case $\Delta = -3$ and $Q_1 = -\frac{1}{3}$, $Q_2 = \frac{2}{3}$ (7)

2. If $\alpha = 2$, then $\beta = 1$, i.e. in this case $\Delta = 3$ and $Q_1 = \frac{2}{3}$, $Q_2 = -\frac{1}{3}$ (8)

We see that the charges of ordinary quarks are reproduced. The choice among above two solutions is arbitrary (the principal result is a fractional character). It reduces to the nomination, which is a proton and which - a neutron, the consequence of old isotopic symmetry, known in nuclear physics. The accepted prescription corresponds to the second solution, i.e. to the identification

$$Q(u) = 2/3, \qquad Q(d) = -1/3. \tag{9}$$

For their antiparticles, evidently, the opposite sign must be taken. As regards the other quarks, their charges may be determined in the same way – simply we can take attention on the quark content of known particles and use the above derived solutions. The simplest way will be to choose a baryon, which consists of 2 ordinary quarks and 1 new one, or mesons with known quark or antiquark, and new ones. For example, take $\Lambda^0 = (uds)$, we should have for its charge

$$Q(u) + Q(d) + Q(s) = 0. \tag{10}$$

Then, it follows

$$Q(s) = -1/3 \tag{11}$$

For the other flavored quarks we can take, for example, mesons $D^+ = (c\tilde{d})$ and $B^+ = (u\tilde{b})$. Using obtained values, we determine

$$Q(c) = 2/3, \qquad Q(b) = -1/3. \tag{12}$$

The only exception from this simple rule could be the top quark, which is not inside some hadrons.

For it one can use the symmetry between generations, or some observed decay mode, for example, $t \to b + W^+$. This would give $Q(t) = 2/3$.

In conclusion, we are convinced that the fractional character of quark's charge may be established very easily without any reference to the flavor group $SU(3)$, though it must be understood that this symmetry had played a *decisive historical* role for the introduction of the conception of quarks.

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ფიზიკა

რატომ აქვთ კვარკებს წილადი მუხტი?

ა. ხელაშვილი

აკადემიის წევრი, ივანე ჯავახიშვილის სახ. თბილისის სახელმწიფო უნივერსიტეტი, მაღალი ენერგიების ფიზიკის ინსტიტუტი; საქართველოს საპატრიარქოს წმიდა ანდრია პირველწოდებულის სახ. ქართული უნივერსიტეტი, თბილისი, საქართველო

კვარკების მოდელს აქვს ხანგრძლივი ისტორია. კვარკების ცნება შემოტანილი იყო ჰადრონებისთვის უნიტარული სიმეტრიის საფუძველზე. ეს ტრადიცია გრძელდება დღესაც ნაწილაკთა ფიზიკის თანამედროვე წიგნებში. ჩვენ გვინდა შევნიშნოთ, რომ კვარკების მუხტის წილადოვნება გამომდინარეობს მხოლოდ ჰადრონების შედგენილი ბუნებიდან ყოველგვარი სიმეტრიის მოხსენიების გარეშე. შემადგენელი კვარკები უნდა იყვნენ ფერმიონები, და რადგან მუხტი ადიტიური ფიზიკური სიდიდეა, კვარკების მუხტების ჯამი ტოლი უნდა იყოს ჰადრონის მუხტისა. მუხტის ნორმირებას ვახდენთ პროტონისა და ნეიტრონის მუხტებზე. მიღებული წრფივ განტოლებათა სისტემა მარტივად ამოხსნადია და მისი ამოხსნები იძლევა კვარკების მუხტის სწორ მნიშვნელობებს. ამიტომ, ეს მიდგომა გამოიყურება გაცილებით მარტივად და არ მოითხოვს ჯგუფთა თეორიის რაიმე მეთოდების გამოყენებას. მიგვაჩნია, რომ ეს მიდგომა უფრო მისაღებია სტუდენტებისათვის მისი სიმარტივის გამო. ცხადია, რომ შემდგომი დინამიკის საკითხები განხილული უნდა იყოს კვანტური ქრომოდინამიკის ჩარჩოებში. მაგრამ ეს უკანასკნელი არ არის კვარკების სახეობების სიმეტრია, არამედ არის ფერის სიმეტრია თითოეული სახეობის კვარკისათვის, როდესაც კვარკებს შორის ურთიერთქმედებას განაპირობებენ ფერადი გლუონები, რომელთა რაოდენობა არის რვა. როგორც ცნობილია, თეორიულად დადგენილია კვარკებს შორის პოტენციალის ყოფაქცევა. ასეთი პოტენციალი, რომელიც მოტივირებულია გლუონის პროპაგატორის ინფრაწითელი ყოფაქცევით, წარმოადგენს წრფივად ზრდადი და კულონური ტიპის პოტენციალთა კომბინაციას და უზრუნველყოფს მიზიდვას ფერით ნეიტრალურ მდგომარეობებში.

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