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Physics

Landau-Kelly Representation of Statistical Thermodynamics of Quantum Plasma and Magnetic String Waves

Levan Tsintsadze*, Grigol Peradze*, Nodar Tsintsadze**

** Faculty of Exact and Natural Sciences, Andronikashvili Institute of Physics, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*
*** Academy Member, Faculty of Exact and Natural Sciences, Andronikashvili Institute of Physics, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*

ABSTRACT. We studied thermodynamic quantities of the quantum plasma in strong magnetic field.In this case, the distribution function becomes anisotropic, due to strong magnetic field. First, we consider non-degenerate quantum, Landau and Kelly distribution function. It was found that adiabatic equation is similar to the adiabatic equation for Maxwell's distribution function. Using Kelly's distribution for degenerate Fermi gas, parallel and perpendicular components of pressure were derived. It was found that perpendicular component never becomes zero and three-dimensional system always stays three dimensional. Also, investigation of low-frequency electromagnetic waves in the case of Kelly's distribution gave new dispersion relation for waves propagating across the magnetic field which we can call "magnetic string waves". *© 2017 Bull. Georg. Natl. Acad. Sci.*

Key words: *quantum plasma, thermodynamics, low-frequency waves, Landau-Kelly distribution, Kelly distribution*

Quantum plasmas are subject of increasing interest due to their potential applications in modern emerging technologies, [1] e.g metallic and semiconductor nanostructures which include metallic nano particles, metal clusters, thin films, spintronics, nanotubes, quantum wells and quantum dots, nanoplasmonic devices, quantum X-ray free electron lasers, etc.

In the case of the degenerate Fermi gas, the shape of Fermi surface provides information about the physical properties of a plasma. Fermi surface is conveniently considered spherical by considering isotropic momentum distribution attributed to the Fermi gas particles. A lot of literature is available that describes various aspects of linear and nonlinear propagation characteristics of different electrostatic or electromagnetic modes in the context of isotropic Fermi surfaces [2]. However, it is well known that there do exist certain situations where the concept of spherical symmetry of Fermi surface is no more valid, even in a collisionless regime of a Fermi gas [3]. In the presence of a magnetic field, the momentum in parallel and perpendicular directions will be different. A precise study in such scenarios demands elongated or even cylindrical Fermi surfaces [4].

References [5-7] studied a quantum Weibel instabilities, but considered anisotropy for thermal electron only and proposed the self-generation of a magnetic field due to the Weibel instability.

Tsintsadze and Tsintsadze [8, 9] developed a new type of quantum kinetic equations for Fermi particles of various spices and subsequently obtained a set of hydrodynamic equations describing a quantum plasma. These dispersion properties of electrostatic oscillations were discussed later. Based on this studies, the investigation of linear and nonlinear ion acoustic waves in quantum plasmas as well as ion acoustic solitary structures has attracted substantial attention [10-14].

Effects of Landau quantization on the longitudinal electric wave characteristic in a quantum plasma are considered in Ref. [15]. Novel branches of longitudinal waves are found, which have no analogies without Landau quantization. Using Ref. [15], an effect of trapping in a degenerate quantum plasma in the presence of Landau quantization was considered in Ref. [16].

Our understanding of the thermodynamics of a Fermi quantum plasma, which is of great interest due to its important application in astrophysics [17, 18-21] has recently undergone some appreciable theoretical progress.

The influence of strong magnetic field on the thermodynamic properties of a medium is an important issue in supernovae and neutron stars, the convective zone of the sun. The early pre-stellar period of evolution of the universe. A wide range of new phenomena arises from the magnetic field in the Fermi gas. Such as the change of shape of the Fermi sphere and thermodynamics (De Haas - Van Alphen [22] and Shubnikov [23] effects). Quite recently an adiabatic magnetization process was proposed in Ref. [24] for cooling the Fermi electron gas to ultra-low temperatures.

It should be noted that the diamagnetic effect has a purely quantum nature and in the classical electron gas, it is absent because in a magnetic field the Lorentz's force $\frac{e}{c}\vec{v}\times\vec{H}$ acts on a particle in the perpendicular direction to a velocity $\vec{v}$, so it cannot produce work on the particle. Hence, its energy does not depend on the magnetic field. However, as was shown by Landau: the situation radically changes in the quantum mechanical theory of magnetism. The point is that under the action of a constant magnetic field the electrons rotate in circular orbits in a plane perpendicular to the field $\vec{H}_0\left(0,0,H_0\right)$. Therefore, the motion of the electrons can be resolved into two parts: one along the field, in which the longitudinal component of energy is not quantized, $E_{\|} = p_{\|}^2/2m_e$, and the second, quantized [25] in a plane perpendicular to $\vec{H}_0$ (the transverse component). Thus, in the non-relativistic case, the net energy of the electron in a magnetic field without taking into account its spin is $E\left(p_{\|},l\right) = p_{\|}^2/2m_e + \hbar\omega_{c0}\left(l+1/2\right)$, where m_e is the electron rest mass and $\mu_{c0} = \frac{|e|H_0}{m_e c}$ is the cyclotron frequency of the electron.

If a particle has a spin, the intrinsic magnetic moment of the particle interacts directly with the magnetic field. The correct expression for the energy is obtained by adding an extra term «$\vec{H}_0$, corresponding to the energy of the magnetic moment «, in the field $\vec{H}_0$. Hence, the electron energy levels £$_e^{l,\delta}$ are determined in the non-relativistic limit by the expression:

$$£_e^{l,\delta} = \frac{p_\parallel^2}{2m_e} + \left(2l+1+¢\right)«_B H_0 \,, \tag{1}$$

where l is the orbital quantum number (l=0,1,2,3,...), $¢$ is the operator to the z component of which describes the spin orientation $\vec{s} = \frac{1}{2}\vec{¢}$ $\left(¢ = \pm 1\right)$ and $«_B = \frac{|e|\hbar}{2m_e c}$ is the Bohr magneton.

From the expression (1) one sees that the energy spectrum of electrons consists of the lowest Landau level $l = 0, ¢ = -1$ and pairs of degenerate levels with opposite polarization $¢ = 1$. Thus each value with $l \neq 0$ occurs twice and that with $l = 0$ once. Therefore, in the non-relativistic limit $£_e^{l,¢}$ can be rewritten as:

$$£_e^{l,¢} = £_e^{l} = \frac{p_\parallel^2}{2m_e} + \hbar \mu_{c0} l \,, \tag{2}$$

where $\hbar$ is the Plank constant divided by 2®.

Thermodynamics of Landau-Kelly Distribution Function

We investigated Thermodynamic quantities of the quantum plasma using the two type distribution function: one is non-degenerate quantum, Landau and Kelly distribution function and the second one is Kelly's distribution function for degenerate Fermi gas.

First, we consider non-degenerate electron gas in the strong magnetic field. As was shown by L. Landau [4] (Page 90) and Don. C. Kelly. [26] that, for particles executing small oscillations about some equilibrium positions (as we say, to an oscillator) the distribution function of Landau-Kelly statistics has the form:

$$f_0^{\,lk} = exp\left(-\frac{p_\parallel^2}{2m_e T} - \frac{p_\perp^2}{2m_e £_\perp}\right), \tag{3}$$

where $£_\perp = \frac{\hbar \mu_{c0}}{2}\left|\coth\frac{\hbar\mu_{c0}}{2T}\right|$, T is the temperature in energetic units, H_0 is an external magnetic field, $\coth x$ is the hyperbolic cosine function.

We note that in the magnetic field, the condition for absent degeneracyis:

$$£_{Fe} << T_e^{\frac{1}{3}} £_\perp^{\frac{2}{3}} \,.$$

The equilibrium density of electrons is defined as:

$$n_e = \frac{2}{\left(2®\hbar\right)^3}\int d\vec{p} f_0^{\,lk} \,. \tag{4}$$

Here the factor 2 is on account of the electron spin, $d\vec{p} = 2®\, p_\perp dp_\perp dp_\parallel$.

Substituting the Landau-Kelly's distribution function Eq. (3) into Eq. (4), we obtain such expression for the density of electrons:

$$n_e = 2\left(\frac{m_e}{2®\hbar^2}\right)^{3/2} T_e^{\frac{1}{2}} £_\perp \,. \tag{5}$$

Let us present the asymptotic expression of Eq. (5) for small $x = \frac{\hbar \mu_c}{2T}$:

$$n_e = \left(\frac{m_e}{2\text{®}\hbar^2}\right)^{2/3} T_e^{\frac{3}{2}} = n_0 \tag{6}$$

and for large *x*:

$$n_e = 2\left(\frac{2}{2\text{®}\hbar^2}\right)^{2/3} T_e^{\frac{1}{2}} \hbar\mu_{ce} = n_0 \frac{\hbar\mu_{ce}}{T} >> n_0 \,. \tag{7}$$

Now we will estimate the magnetic field at the low temperatures. $\hbar\mu_{ce} >> 2K_B T$ or $H_0 >> 2*10^4 T$. From here we can see for the temperature 1-10 degree, $H_0 > 6*10^4 - 10^5 G$.

The mean kinetic energy for one electron is defined in the form:

$$<\pounds>= \frac{2}{\left(2\text{®}\hbar\right)^3 n_e} \int_{-\infty}^{\infty} dp_{\|} \int_0^{\infty} 2\text{®}\, dp_{\perp}\, p_{\perp} \left(\frac{p_{\|}^2}{2m_e} + \frac{p_{\perp}^2}{2m_e}\right) f_0 = \pounds_{\perp} + \frac{k_B T_e}{2} \,. \tag{8}$$

Now we define the specific heat for the electron gas, as:

$$C_V = \frac{\partial <\pounds>}{\partial T} = k_B \left(\frac{1}{2} + \frac{x^2}{\sinh x^2}\right), \tag{9}$$

where $\sinh x$ is the sine hyperbolic function, the expression of which in power series:

$$\sinh x = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!} \tag{10}$$

for $x = \frac{\hbar\mu_{c0}}{2k_B T} << 1$, in the such small magnetic field the specific heat slightly decreases than $3/2k_B$, i.e.

$$C_V = \frac{3}{2} k_B \left(1 - \frac{2}{9} x^2\right). \tag{11}$$

For the strong magnetic field $\hbar\mu_c > k_B T$, we obtain the specific heat in a form:

$$C_V = k_B \left(\frac{1}{2} + \left(\frac{\hbar\mu_{c0}}{k_B T}\right)^2 e^{-\frac{\hbar\mu_{c0}}{k_B T}}\right). \tag{12}$$

For the calculation of the entropy per particle by Landau-Kelly's distribution function, we use well-known expression $S = -\frac{k_B}{n_e} \int d\vec{p} f \ln f$ which leads to the result:

$$S = k_B \ln \frac{\left(2\text{®} m_e\right)^{3/2} T^{1/2} \pounds_{\perp}}{n_e} \,. \tag{13}$$

Since for an adiabatic process $dS = 0$, from Eq. (13) we obtain the adiabatic equation:

$$\frac{T^{1/2} \pounds_{\perp}}{n_e} = \text{Const.} \tag{14}$$

First, in the case of weak magnetic field, i.e. $k_B T > \hbar\mu_c$, we have the following adiabatic equation:

$$\frac{T^{3/2}}{n_e}\left(1+\frac{1}{12}\left(\frac{\hbar\mu_c}{k_B T}\right)^2\right)=\text{Const.} \tag{15}$$

Next, in the case of strong magnetic field, i.e. $\hbar\mu_c >> k_B T$, which is a more interesting case the adiabatic equation reads:

$$\frac{T^{1/2}H}{n}=\text{Const.} \tag{16}$$

We want to emphasize that the same adiabatic equation Eq. (16) was found in our publication [24], in the same approximation. There we used Maxwell's distribution function with energy $£ = p_{\|}^2/2m+\hbar\mu_c l$ (where l is the orbital quantum number $l = 0,1,2,3,...$).

Thus the Maxwell's distribution function, which is quite different than Landau-Kelly's distribution function, both gives similar expression of the adiabatic equation.

Thermodynamics of Kelly Distribution Function

Next we consider degenerate Fermi electron gas. To describe the state of Fermi particles Don. C. Kelly has derived the distribution function [26]

$$f_{\sim}^{k}=\frac{2}{(2®\hbar)^3}e^{-\frac{P_x^2+P_y^2}{m_{\sim}\hbar\mu_{c\sim}}}\sum_{l=0}\frac{(-1)^l L_l\left(2\frac{P_x^2+P_y^2}{m_{\sim}\hbar\mu_{c\sim}}\right)}{e^{\frac{£_l-«_{\sim}}{T}}+1} \tag{17}$$

Where suffix α stands for the particle species, $w_{\sim}^2=\frac{P_{\perp}^2}{m_{\sim}\hbar\mu_{c\sim}}=\frac{P_x^2+P_y^2}{m_{\sim}\hbar\mu_{c\sim}}$, $L_l(x)$ is the Laguerre polynomial of order l [27], for which such condition exist: $2(-1)^l\int e^{-w^2}L_l(2w^2)wdw=1$. The chemical potential $«_{\sim}$ is defined by the normalization condition:

$$n_{\sim}=2\int d\vec{p}f_{\sim}^{k}\left(\vec{p}_{\perp},p_{\|}\right). \tag{18}$$

Here the factor 2 is on account on the particle spin.We note that such distribution function was independently derived by Zilberman [28].

Kelly's distribution function represents hybrid distribution function. In a plane perpendicular to the magnetic field $\vec{H}$, Kelly's distribution function is Boltzmann, but along the magnetic field distribution function is Fermion.

To evaluate the density from the expression Eq. (18), we shall consider gas at the temperature limit $\left|l\hbar\mu_{ce}-«\right|>>T$. In this case the Fermi distribution function is in a good approximation described by the Heaviside step function $H\left(«-£^l\right)$, from which follows $«=£_{Fe}=£^l=\frac{P_{\|}^2}{2m_e}+l\hbar\mu_{ce}$. This allowsus to integrate Eq. (18) by $p_{\|}=\pm\sqrt{2m_e}\left(£_{Fe}-l\hbar\mu_{ce}\right)^{1/2}$.

The last expression reads that the summation along l is limited by the condition $£_{Fe}>\hbar\mu_{ce}$, so that

$l_{max} = \frac{\varepsilon_{Fe}}{\hbar\omega_{ce}}$. When $\varepsilon_{Fe} < \hbar\omega_{ce}$, l must be zero.

First, we consider the lowest Landau level. $l = 0, \sigma = -1$ (see Eq. (1)). In this case the Kelly's distribution function is:

$$f_{\alpha}^{k}\left(\vec{p}_{\perp}, p_{\parallel}\right) = \frac{2e^{-w_{\alpha}^{2}}}{\left(2\pi\hbar\right)^{3}} \frac{1}{exp^{\frac{p_{\parallel}^{2}/2m_{\alpha} - \mu_{\alpha}}{T_{\alpha}}} + 1}. \tag{19}$$

At $T = 0$, the Kelly's distribution function Eq. (19) reads:

$$f_{\alpha}^{k}\left(\vec{p}_{\perp}, p_{\parallel}\right) = \frac{2e^{-w_{\alpha}^{2}}}{\left(2\pi\hbar\right)^{3}} H\left(\mu_{\alpha} - p_{\parallel}^{2}/2m_{\alpha}\right). \tag{20}$$

Here $H(x)$ is the Heaviside step function and $\mu_{\alpha} = \frac{p_F^2}{2m_{\alpha}}$. Substituting distribution function Eq. (20) into Eq. (18) we obtain the expression of density:

$$n_{\alpha} = \frac{m_{\alpha}\hbar\omega_{c\alpha}\, p_F}{\pi^2\hbar^3}, \tag{21}$$

which is true for the Lowest Landau level $(l = 0)$, i.e. this expression is associated with the Pauli paramagnetism and self-energy of particles. If we suppose that, the density of electrons is constant, then from Eq. (21) follows an important statement, namely, that the Fermi momentum decreases along with the increase of a magnetic field. So that a pancake configuration of the Fermi energy thins.

Now we calculate the mean energy of the particles at the lowest Landau level, $l = 0$ by Kelly's distribution function Eq. (17).

$$<\varepsilon> = <\varepsilon_{\perp}> + <\varepsilon_{\parallel}> = \frac{2}{n_e}\int d\vec{p}_{\perp}\int_{-\infty}^{\infty} d\vec{p}_{\parallel}\left(\frac{p_{\parallel}^2}{2m} + \frac{p_{\perp}^2}{2m}\right) f_{0e}^{k}. \tag{22}$$

The result of calculation is:

$$<\varepsilon> = \frac{\hbar\omega_c}{2} + \frac{\varepsilon_F}{3}\left(1 + \frac{\pi^2}{6}\left(\frac{k_B T}{\varepsilon_F}\right)^2\right). \tag{23}$$

We now define the specific heat for the Fermi gas, $C_V = \left(\frac{\partial <\varepsilon>}{\partial T}\right)_V$.

We know that in all temperature regions a metal consists of two subsystems: a crystalline Lattice of ions and a free electron gas. Therefore, the specific heat of metal can be presented as a sum of two items:

$$C_V = C_V^{lat} + C_V^{e}. \tag{24}$$

where C_V^{lat} is the specific heat of the lattice and for:

$$\Theta_D << T \qquad C_V^{lat} = 3k_B N, \tag{25}$$

$$\Theta_D >> T \qquad C_V^{lat} = \frac{12\pi^4}{5} k_B N \left(\frac{T}{\Theta_D} \right)^3 , \tag{26}$$

where Θ_D is the Debye characteristic temperature, N is the total number of particles.

C_V^e is specific heat for free electron isotropic gas. For $T >> T_F$

$$C_V^e = \frac{3}{2} k_B N \tag{27}$$

and for $T << T_F$

$$C_V^e = \frac{\pi^2}{2} K_B N \left(\frac{T}{T_F} \right), \tag{28}$$

where $T_F = \dfrac{\left(3\pi^2\right)^{2/3} \hbar^2 n_e^{2/3}}{2 m_e k_B}$.

Comparison between C_V^{lat} and C_V^e show us that for the temperatures $T \geq 1$ degree C_V^{lat} is always more than C_V^e.

As was shown by L.N. Tsintsadze [15], strong magnetic field leads to reduce the Fermi energy:

$$\varepsilon_F = k_B T_F = \beta \left(\frac{n}{H} \right)^2 , \tag{29}$$

where $\beta = \dfrac{\pi^4 \hbar^4 c^2}{2 m_e e^2}$.

In our case the specific heat follows from the expression of Eq. (23).

$$C_v^e = \frac{\pi^2}{9} k_B N \left(\frac{k_B T}{\varepsilon_F} \right) \tag{30}$$

where ε_F is defined by Eq. (29).

Therefore, when magnetic field increases, i.e. in this case Fermi energy ε_F decreases, This leads to the increase of the specific heat.

We obtained the above expressions: Eqs. (23) - (30) in the limit $\hbar\omega_c > \varepsilon_F = \mu$. Now we suppose that $\hbar\omega_c = \varepsilon_F$. So, in this case the orbital quantum number can be only $l = 0,1$.

In this limit the Kelly's distribution function looks as:

$$f_0^k = \frac{2e^{-w^2}}{(2\pi\hbar)^3} \left(\frac{1}{\exp \dfrac{p_\parallel^2 / 2m_e - \mu}{k_B T} + 1} - \frac{L_1\left(\dfrac{2p_\perp^2}{m\hbar\omega_c} \right)}{\exp \dfrac{p_\parallel^2}{k_B T} + 1} \right), \tag{31}$$

where $L_1\left(\dfrac{2p_\perp^2}{m\hbar\omega_c} \right) = 1 - \dfrac{2p_\perp^2}{m\hbar\omega_c}$.

In such a case from the last term of Eq. (31) follows that $T \neq 0$.

Using the anisotropic distribution function Eq. (31) we obtain the expression of the electron density:

$$n_e = \frac{m\mu_c p_f}{\text{®}^2\hbar^2}\left(1+0.5\sqrt{\frac{k_B T}{\pounds_F}}\right). \tag{32}$$

For the mean kinetic energy of per particle

$$< \pounds > = < \pounds_\perp > + < \pounds_\parallel > = \frac{5}{6}\hbar\mu_c + \frac{1}{3}\hbar\mu_c\sqrt{\frac{k_B T}{\hbar\mu_c}}. \tag{33}$$

Following the equation (33), for the specific heat we obtain:

$$C_V = \frac{\partial < \pounds >}{\partial T} = \frac{k_B}{6}\sqrt{\frac{\hbar\mu_c}{k_B T}}. \tag{34}$$

To get the expression of the specific heat Eq. (34) we supposed, that $\hbar\mu_c \propto \pounds_F >> k_B T$, but the temperature here can not be zero. We can rewrite relation as $\hbar\mu_c \propto \pounds_F = \text{¥}\left(\frac{n}{H}\right)^2$, at $n \propto 10^{21} \quad 1/cm^3$.

Therefore the specific heat Eq. (34) can be called anomalous.

Parallel and Perpendicular Components of Pressure

We now derive the perpendicular component of the pressure using the Kelly distribution function Eq. (20) for electrons for the lowest Landau level ($l = 0, \text{¢} = -1$) for the temperature $T = 0$.

$$P_{\perp e} = \frac{1}{3}\int d\vec{p}\,\frac{\left(p_x^2 + p_y^2\right)}{m_e} f_e^k\left(\vec{p}_\perp, p_\parallel\right). \tag{35}$$

After simple integration of Eq. (35) we obtain

$$P_{\perp e} = \frac{1}{3}\hbar\mu_{ce} n_e, \tag{36}$$

where n_e is the density defined by Eq. (21).

$$P_{\perp e} = \frac{1}{3}\hbar\mu_{ce} n_e = \left(P_{\perp 0} + \frac{H_0^2}{8\text{®}}\right)\left(\frac{n}{n_0}\right)^2$$

$$P_{\perp 0} = \frac{1}{3}\hbar\mu_{c0} n_0 \quad \nabla = 2\frac{n}{n_0}\left(\frac{\hbar\mu_{co}}{3} + \frac{H_0^2}{8\text{®} n_0}\right).$$

At the temperatures lower than the degeneracy temperature, $T_F = \left(\frac{n}{H_0}\right)^2$ (where $= \frac{\text{®}^4\hbar^4 c^2}{2m_e e^2}$ [15]) from Eq. (18) and Eq. (19) for the density of electrons follows the expression:

$$n_e = \frac{m_e\hbar\mu_{ce} p_F}{\text{®}^2\hbar^3}\left(1 - \frac{\text{®}^2}{24}\left(\frac{T}{T_F}\right)^2\right). \tag{37}$$

In this case n_e in Eq. (36) is governed by Eq. (37).

Its obvious from Eq.(36) that at $l = 0$, $P_\perp$ is not zero.

Next, for the parallel component of the pressure, in the same case, i.e. $l = 0$ and $T = 0$, we obtain:

$$P_{\|e} = \frac{1}{3} * 2\int d\vec{p}\, \frac{p_\|}{m_e} f_e^k . \tag{38}$$

Use of Eq.(20) in Eq.(38) yields:

$$P_{\|e} = \propto \left(\frac{n_e}{H} \right)^2 n_e , \tag{39}$$

where $\propto = \dfrac{\circledR^4 \hbar^4 c^2}{9 m_e e^2}$.

Magnetic String Waves

Now we consider low-frequency electromagnetic waves propagating across constant magnetic field $\vec{B}_0$. We suppose that $\vec{B}_0 = B_0 \vec{z}$ and the electric field $\vec{E} = E\vec{y}$, but we let the wave vector $\vec{k} = k\vec{x}$. Thus, there are waves propagating normal to magnetic field $\vec{B}_0$.

Let us set up the equations of magnetohydrodynamics for conditions such that all dissipative processes may be neglected.

In our case a set of equations are:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{1}{mn} \frac{\partial}{\partial x} \left(P_\perp + \frac{H^2}{8\circledR} \right) + \frac{\hbar^2}{2 m_e m_i} \frac{\partial}{\partial x} \frac{1}{\sqrt{n}} \frac{\partial^2}{\partial x^2} \sqrt{n} , \tag{40}$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} nu = 0 , \tag{41}$$

$$\frac{\partial B}{\partial t} + \frac{\partial}{\partial x} uH = 0 , \tag{42}$$

where the perpendicular component of preasure is defined by Eq. (36).

From Eqs. (41) and (42) follows the “ Frozen in” condition, which means that the magnetic lines of forces are “Frozen in” to the conducting fluid and are thereby constrained to move with the fluids.

$$\frac{H}{n} = \frac{H_0}{n_0} = const . \tag{43}$$

Using Eq. (43) and substituting B into Eq. (40), we obtain:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = -\frac{1}{mn} \frac{\partial}{\partial x} \left[\left(\frac{\hbar \mu_c n_0}{3} + \frac{H_0^2}{8\circledR} \right) \left(\frac{n}{n_0} \right)^2 \right] + \frac{\hbar^2}{2 m_e m_i} \frac{\partial}{\partial x} \frac{1}{\sqrt{n}} \frac{\partial^2}{\partial x^2} \sqrt{n} . \tag{44}$$

After linearization of Eqs. (41) and (44) and assuming plane wave solution $e^{i(kx - \mu t)}$, we will get dispersion:

$$\mu^2 = \left(C_{st}^2 + V_A^2 \right) k^2 + \mu_q^2 , \tag{45}$$

where $\mu_q = \dfrac{\hbar k^2}{2\sqrt{m_e m_i}}$ is quantum oscillation frequency, $V_A = \dfrac{B}{\sqrt{4\circledR\, m_i n_0}}$ is the Alfven velocity and

$C_{st} = \sqrt{\frac{2}{3}\frac{\hbar\mu_{c0}}{m_i}}$ is string velocity.

Those waves with dispersion relations Eq. (45) are new waves, which we can call magnetic string waves.

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ლ. ცინცაძე*, გ. ფერაძე*, ნ. ცინცაძე**

** ივანე ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტი, ზუსტ და საბუნებისმეტყველო მეცნიერებათა ფაკულტეტი, ე. ანდრონიკაშვილის ფიზიკის ინსტიტუტი, თბილისი, საქართველო.*
***აკადემიის წევრი, ივანე ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტი, ზუსტ და საბუნებისმეტყველო მეცნიერებათა ფაკულტეტი, ე. ანდრონიკაშვილის ფიზიკის ინსტიტუტი, თბილისი, საქართველო.*

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