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Mechanics

Analysis of the Internal Forces Caused by Seismic P and S Waves and Geostatic Load in a Circular Tunnel

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ABSTRACT. The problems of the action of arbitrarily propagated tension/compression (P) and shear (S) seismic waves on a "rock-support system", being under loading of geostatic gravitational forces are dealt with in this paper. The rock mass is assumed to be a homogenous, isotropic linearly deformable medium, simulated applying the apparatus the theory of elasticity. The tunnel liner is analyzed by methods of strength of materials. It is assumed that the lining is installed after the excavation at some distance from the tunnel face. Both cases of full-slip and no-slip contact conditions for rock-support interface are considered. An analytical apparatus is proposed for determination of internal forces in the tunnel structure undergoing geostatic pressures and affected by seismic P and S-waves. The numerical examples show that the maximum stresses in the underground structure imposed by a longitudinal wave may exceed that of a transverse wave; the sum of the stresses after superposition of static and seismic fields of stress differs greatly from same by shear seismic waves. Therefore ignoring one of them will decrease the accuracy of calculation and at last will reduce the safety of the designed structure. *© 2018 Bull. Georg. Natl. Acad. Sci.*

Key words: tunnels, calculation, geostatic loads, seismic impact, superposition, contact conditions

In modern literature [1-5] the evaluation procedures for transverse response of tunnel structures was carried out using either simplified analytical methods, or more complex refined methods, depending on the degree of complexity of the soil-structure system, subsurface conditions, the seismic hazard level, and the importance of the structures. In both cases of the simplified or refined methods, calculations of the structure are carried out only on impact of transverse waves originating common shear deformation. Such a model derived by Wang [3] may be accurate for free-field conditions, i.e., for tunnels in large overburden areas where shear waves transfer the most portion of seismic energy and only partially can reflect the actual stress-strain state of a tunnel during the earthquake.

This problem has been pointed out in recent publication [6] of Kouertzis et al., where is assessed the effect of secondary P-waves resulting from reflection/refraction of S-waves on the final tunnel lining. Analytical expressions are used for a circular tunnel internal forces under geostatic stress field, transforming these to calculate the seismic forces in the liner at the secondary seismic *P*-wave propagation with a plane front. This is done by setting maximum free-field normal stress in the *P*-waves - ($\sigma_{\max}$) instead of the in situ vertical component of gravitational stresses (γH). The lateral pressure ($K_0\ \gamma H$) in the P-wave is ignored as its ratio $K_0 = 0$. That can be justified in only theoretically possible exceptional cases of site conditions, for example, if the Poisson's ratio of the rock mass is zero. The geostatic loads on the liner from rock mass relaxation are also ignored. This assumption is appropriate for an unreinforced tunnel section constructed with the NATM method in competent rock mass, where the final lining is installed after the primary lining has reached equilibrium. It experiences only self-weight load and must withstand the possible additional influences, the seismic loads [6], initial supports provide a seemingly stable opening but it is known that additional support is required for long term stability then that support must be provided by the final lining [2: 6-53].

The issues of calculation of the tunnels under the influence of both longitudinal P and transverse S seismic waves were discussed by Napetvaridze [7], Fotieva [8,9] and others. This group applied a method of the theory of elasticity [10] for determination of the extreme stresses in the final tunnel support of any cross-section form under “overpressure” loading. In this case a temporary lining has not been taken into account and it is considered as a reserve of bearing capacity of the final support.

In both of the above-mentioned cases of the ignoring of primary or secondary lining the value of the reserve of bearing capacity of the final support may be significant and must be evaluated if it is possible. Besides it is important the level and character of the existing stress-strain condition of a structure from geostatic forces at the moment of seismic impact.

Some of the results of early analytical studies of these problems, published monographs in Russian [11-14], are proposed below in a finalized form of the “excavation loading” scheme, taking in to account the sequence of the excavation and reinforcing of the opening. The rock mass is assumed as a homogenous, isotropic linearly deformable medium, simulated by apparatus of theory of elasticity. The tunnel liner is considered by methods of strength of materials. Both cases of full-slip and no-slip contact conditions for rock-support interface are considered. The action of arbitrarily directed tension/compression (P) and shear (S) long waves on a system "rock-support", being under geostatic loads are considered in this work. Therefore, the formulae are set out below: at the beginning due to geostatic forces, then from the seismic P and S waves and finally is made their superposition.

Analysis of Circular Tunnel in the Geostatic Stress Field of Weak Rock Mass

If in situ main geostatic stresses are designated P_{st} and λP_{st} , the radial and tangential stress components by the circular line of the prospective tunnel excavation can be represented in polar coordinates as

$$\sigma_r = 0.5P_{st}[1+\lambda+(1-\lambda)\cos 2\theta]; \qquad (1)$$
$$\tau_{r\theta} = 0.5P_{st}(1-\lambda)\sin 2\theta;$$

where P_{st} is maximum normal geostatic stress; λ - in situ stress ratio; θ is polar angle, measured from axis of maximum normal stress. If in situ stresses are only gravitational $P_{st} = \gamma H,\ \lambda = \nu/(1-\nu)$.

This means that the excavation of the tunnel leads to the removal of the radial and tangential, (1) stresses. Corresponding equations of radial u_r and tangential u_θ displacements of points of the tunnel contour of radius R, derived [12] by solution of a plane problem of elasticity theory [10], are given by:

$$u_r = \frac{P_{st}R(1+\nu)}{2E}[1+\lambda+(3-4\nu)(1-\lambda)\cos 2\theta]; \qquad (2)$$

$$u_\theta = \frac{P_{st}R(1+\nu)}{2E}(3-4\nu)(1-\lambda)\sin 2\theta, \qquad (3)$$

where E is the modulus of elasticity and ν is Poisson's ratio of the rock mass.

Having elastic displacements it is possible to receive expressions for viscous-elastic displacements of a contour of tunnel buried in weak rock mass. For this, the elastic parameters in these functions must be replaced by time operators:

$$E(t) = E/(1+\Phi);$$
$$G(t) = G/\left[1+3\Phi/2(1+\nu)\right]; \tag{4}$$

where: $\Phi = \delta/(1-\alpha)t^{1-\alpha}$ is a creep function, t (days counted from the excavation of tunnel) ; α and δ are creep parameters of Abel's kernel [15] what for rock mass vary in limits: $\alpha = 0.6-0.9$; $\delta = 0.5-1.5$. As a result of the interpretation of obtained operator expressions on the Volterra-Rabotnov principle, equations such as expressions (2), take into account the creep of the rock mass. That gives a possibility to develop the equations of internal forces in the support depending on the contact conditions, time factor and location of the lining loading. However, the practical application of these equations is somewhat difficult in cases when the rheological parameters of the rock mass are not known in advance. Therefore, in this work in order to simplify the calculations, the effect of decreasing the elastic modulus over time is realized by using the rock mass modulus of deformation, including all elastic as well as non elastic linear deformations.

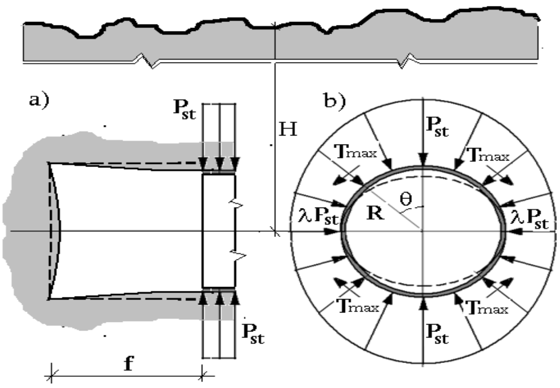

Fig. 1. Scheme of erection of support on a distance f from excavation face (a) and distribution of normal and shear stresses on the rock-support contact interface (b).

After this simplification, the loads developed per unit length of the support, set at a distance *f* from the tunnel face, can be schematically represented in Fig. 1 and written as:

$$P_\theta^{'} = P_0 + P_2^{'} \cos 2\theta, \tag{5}$$

$$T_\theta = T_2 \sin 2\theta; \tag{6}$$

where the uniform part of contact pressure, i.e. half the sum of the maximum and minimum contact pressures, forming in the result of rock-support interaction, is given by:

$$P_0 = \frac{P_{st}\,(1+\lambda)}{2}\,\frac{E_l F(1-A)}{R^2 K_0 + E_l F}. \tag{7}$$

The half-difference of the extreme values of radial contact pressures $P_2^{'}$ and maximum tangential contact stress T_2 at a no-slip conditions on their contact surface:

$$P_2^{'} = \frac{P_{st}\left(1-\lambda\right)}{2} \frac{\left[\frac{18E_l I}{R^4} - C\left(1-\frac{h}{R}\right)\left(1-\omega\right)\right]\left(1-A\right)}{\frac{18E_l I}{R^4} + C\left(1-\frac{h}{R}\right)\omega + 2K_2^{'}}; \quad (8)$$

$$T_2 = \frac{P_{st}\left(1-\lambda\right)}{2} \frac{\left[\frac{18E_l I}{R^4} + 2K_2^{'}\left(1-\omega\right)\right]\left(1-A\right)}{\left[\frac{18E_l I}{R^4 C} + \frac{2K_2^{'}}{C} + \left(1-\frac{h}{R}\right)\omega\right]K_2^{'}}; \quad (9)$$

where:

E_l , *h*, *F* and I are Young's modulus, thickness, cross-sectional area and moment of inertia per unit length of liner respectively; *A* is a dimensionless quantity taking into account the support erecting lag f from the tunnel face. Its value approximately [12] may be defined as $A = 1 - \exp(-0.6 f \,/\, R)$;

ω is a dimensionless quantity that depends on the shear modulus of the rock, *G*, shear modulus of the liner, G_l and liner thickness, *h*. For approximate calculations ω can be taken equal to -0.5. A more precise value of ω can be determined by the expressions obtained from the closed form solution of the contact problem of the plane [16] with a reinforced circular hole:

$$\omega = \left(a-b\right)/\left(a+b\right),$$

where: $a = \left(\kappa_l + 1\right)\left(n^6 - n^4\right)$; $n = \left(R+h\right)R$; $\kappa = \left(3-4\nu\right)$; $\kappa_l = \left(3-4\nu_l\right)$;

$$b = \left(\frac{G}{G_l} - 1\right)\left(3n^6 - 6n^4 + 4n^2\right) + \kappa\left(1 + \kappa_l \frac{G}{G_l}\right)n^6 + \left(1 + \kappa_l \frac{G}{G_l}\right)n^8 + \kappa\left(\frac{G}{G_l} - 1\right).$$

In the expressions (7)-(9) K_0, $K_2^{'}$ and C are functions having physical meaning and dimension of Winkler's coefficient. The first of them is the widely used "spring stiffness" [2] for the relationship of axisymmetric radial displacement of the excavation circular contour with uniform part of contact stresses,

$$K_0 = \frac{E}{R\left(1+\nu\right)}; \quad (10)$$

The functions $K_2^{'}$ and *C* are "spring stiffness" for uneven radial and tangential displacements for the no-slip contact conditions on rock-support interface respectively:

$$K_2^{'} = \frac{E\left[5 - 6\nu - \left(4-6\nu\right)\omega\right]}{R\left(1+\nu\right)\left(3-4\nu\right)}, \quad (11)$$

$$C = \frac{E\left[5 - 6\nu - \left(4-6\nu\right)\frac{1}{\omega}\right]}{R\left(1+\nu\right)\left(3-4\nu\right)}. \quad (12)$$

The normal force, $N_\theta^{'}$, bending moment, $M_\theta^{'}$ and radial displacement, $U_\theta^{'}$ for a unit length of the support in the no slip contact conditions are given by

$$N_\theta^{'} = R\left[P_0 - 0.333\left(P_2^{'} + 2T_2\right)\cos 2\theta\right], \quad (13)$$

$$M_\theta^{'} = \frac{IP_0}{F} - \frac{R^2}{6}\left(2P_2^{'} + T_2\right)\cos 2\theta, \quad (14)$$

$$U_\theta^{'} = \frac{P_0 R^2}{E_l F} + \frac{R^4}{18E_l I}\left(2P_2^{'} + T_2\right)\cos 2\theta. \quad (15)$$

If full slip on the rock-support interface is possible, only the normal component of the contact stresses (5) will develop on the support. In this case, the evenly distributed part of the load P_0 will be expressed by the same formula (7). The half-difference of the extreme values of radial contact pressures $P_2^{''}$ at a full-slip contact condition will be given by

$$P_2^{''} = \frac{P_{st}(1-\lambda)}{2}\frac{(1-A)}{\frac{R^4 K_2^{''}}{9E_l I}+1}, \tag{16}$$

where the function of "spring stiffness" for uneven radial displacements of rock-support interface at a full-slip conditions will be given by

$$K_2^{''} = \frac{3E}{R\left[(1+\nu)(5-6\nu)\right]}. \tag{17}$$

The corresponding equations of normal force, bending moment and radial displacement of the unit length support for full-slip conditions will be respectively:

$$N_\theta^{''} = P_0 R - \frac{P_2^{''} R}{3}\cos 2\theta, \tag{18}$$

$$M_\theta^{''} = \frac{IP_0}{F} - \frac{P_2^{''} R^2}{3}\cos 2\theta, \tag{19}$$

$$U_\theta^{''} = \frac{P_0 R^2}{E_l F} + \frac{P_2^{''} R^4}{9E_l I}\cos 2\theta. \tag{20}$$

Circular Tunnel Under the Influence of Seismic *P* and *S* Waves

The following is based on the generally accepted provision that seismic stresses can usually be considered as pseudo-static superposition on the existing stresses, because the seismic wave is almost always much longer than the cross-sectional dimension of the typical underground structure.

Seismic stresses in the rock mass, according to law of energy conservation and wave theory [17] can be determined by

$$\sigma_{\substack{\max \\ \min}} = \pm\frac{1}{g}\gamma C_1 V_1; \quad \tau_{\substack{\max \\ \min}} = \pm\frac{1}{g}\gamma C_2 V_2 \tag{21}$$

where γ - unit weight of the rock mass; V_1 and V_2 - maximum velocities of particles from compressive and shear waves, respectively; $C_1 = \sqrt{G/\rho(1-2\nu)}$ and $C_2 = \sqrt{G/\rho}$ - compressive and shear waves propagation velocities respectively; g - acceleration of gravity.

Consequently, taking into account the condition for the absence of transverse deformations in a plane wave [17], the components of normal P_{din}, λP_{din} and shear Q_{din} stresses in longitudinal, *P* and transverse, *S* seismic waves are determined by:

$$P_{din} = \pm\frac{1}{2\pi}k\gamma C_1 T_0; \quad \lambda P_{din} = \frac{\nu}{1-\nu}P_{din}; \quad Q_{din} = \pm\frac{1}{2\pi}k\gamma C_2 T_0. \tag{22}$$

where *k=a/g* - coefficient of seismicity; *a* is peak ground acceleration as the most common index of the intensity of strong ground motion at a site [1,2]; T_0 - dominant rock particles oscillation period (T_0 = 0.5 s if data is missing).

This formulation is more general and somewhat different from the concept of authors [3,6,18], taking the assumption λ = 0, which reflects the uniaxial loading conditions and can correspond to the propagation of a plane P wave in the theoretically possible particular case when Poisson's ratio of rock mass, ν = 0.

At the long wave seismic impact, concentrated stresses and displacements around the tunnel that have the main significance for design, are defined using so called quasi static method. Such an approach for a circular tunnel was analyzed in the works [13.14].

Two contact problems of a circular tunnel were solved for both longitudinal and transverse waves. In this case the problems are considered in the "overpressure loading" scheme, because, in contrast to the gravitational field of stresses, seismic waves propagate in the mass where the tunnel already exists.

To obtain expressions of internal forces in a circular liner under the influence of the long seismic waves, the solution of the problem of action of compressive and shear stresses on "infinite" boundaries of the plane was used.

The normal, $P^{'}_{\alpha+\theta}(P)$ and shear, $T^{'}_{\alpha+\theta}(P)$ contact stresses on the support in no slip conditions, caused by only longitudinal wave (Fig.2,a) propagating at an angle α from the vertical, can be written as

$$P^{'}_{\alpha+\theta}(P) = P^{'}_{0(din)} + P^{'}_{2(din)}\cos 2(\alpha+\theta); \tag{23}$$

$$T^{'}_{\alpha+\theta}(P) = T_{2(din)}\sin 2(\alpha+\theta), \tag{24}$$

where P_0, $P_2^{'}$, T_2 will be expressed by formulae (7)- (9), replacing P_{st} by P_{din} and substituting A= 0, because of the assumption that tunnel face is now far from the site under consideration and it cannot reduce the effect of seismic stresses.

At the full slip conditions on the rock-support interface, shear contact stresses $T_{2(din)} = 0,$ the evenly distributed part of the radial contact pressure P_0 will be expressed by (7) and the half-difference of the extreme values of radial contact pressures $P^{''}_{2(din)}$ will be given by (16) when A= 0.

Similarly, the axial forces, bending moments and radial displacements of the support caused by longitudinal seismic *P* wave at the no slip and full slip contact conditions can be determined by equations (13)-(15) and (17)-(19) respectively, replacing P_0, $P_2^{'}$, $P_2^{''}$ by $P_{0(din)}$, $P^{'}_{2(din)}$, $P^{''}_{2(din)}$ to take the $\cos 2(\theta+\alpha)$ instead of $\cos 2\theta$ and A= 0.

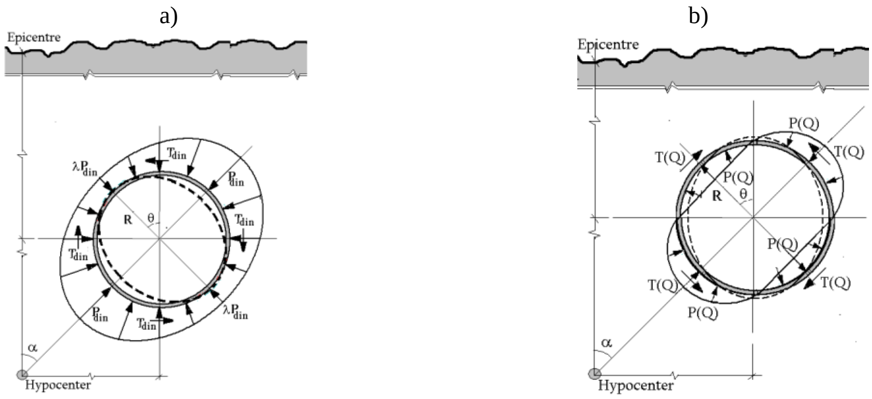

Fig. 2. Schema of circular tunnel under long, arbitrarily directed *P*, longitudinal, (a) and *S*, transverse, (b) seismic waves.

In order to obtain a solution to the problem of pure shear at infinity, it is necessary in the formulae for *P* wave to replace: P_{din} by Q_{din}, $(\theta+\alpha)$ by $(\theta+\alpha+\pi/4)$, and substitute the value λ =-1. As a result of simple transformations: $\cos 2(\alpha+\pi/4) = -\sin 2\alpha;$ $\sin 2(\alpha+\pi/4) = \cos 2\alpha$, the following expressions are obtained.

The normal $P^{'}_{\alpha+\theta}(Q)$ and shear $T^{'}_{\alpha+\theta}(Q)$ contact stresses, caused by only transverse seismic wave propagation at an angle α from the vertical (Fig.2,b), in no slip conditions can be written as

$$P^{'}_{\alpha+\theta}(Q) = -P^{'}_{2(din)}\sin 2(\alpha+\theta); \tag{25}$$

$$T_{\alpha+\theta}(Q) = T_{2(din)}\cos 2(\alpha+\theta), \tag{26}$$

where:

$$P'_{2(din)} = \frac{Q_{дин}\left[\dfrac{18E_l I}{bR^4} - C\left(1-\omega\right)\right]}{\dfrac{18E_l I}{bR^4} - C\dfrac{V_2'}{U_2'} + 2K_2^{'}}; \tag{27}$$

$$T_{2(din)} = \frac{Q_{дин}\left[\dfrac{18E_l I}{bR^4} + 2K_2^{'}\left(1-\omega\right)\right]}{\dfrac{18E_l I}{bR^4} + \left(\dfrac{2K_2^{'}}{C} + \dfrac{V_2'}{U_2'}\right)K_2^{'}}; \tag{28}$$

The axial forces, bending moments and radial displacements in the support caused by transverse seismic Q wave at the no slip conditions are respectively:

$$N'_{\theta+\alpha}\left(Q\right) = \frac{bR}{3}\left(P'_{2(din)} + 2T'_{2(din)}\right)\sin 2\left(\theta+\alpha\right); \tag{29}$$

$$M'_{\theta+\alpha}\left(Q\right) = \frac{bR^2}{6}\left(2P'_{2(din)} + T'_{2(din)}\right)\sin 2\left(\theta+\alpha\right); \tag{30}$$

$$U^{'}_{\theta+\alpha}\left(Q\right) = \frac{bR^4}{18E_l I}\left(2P'_{2(din)} + T'_{2(din)}\right)\sin 2\left(\theta+\alpha\right). \tag{31}$$

The pressures on the rock-support interface in full slip conditions will be represented by only radial contact stresses

$$P''_{\theta+\alpha}\left(Q\right) = -\frac{9E_l I Q_{din}}{bR^4 K_2^{''} + 9E_l I}\sin 2\left(\theta+\alpha\right). \tag{32}$$

The corresponding equations of axial forces, bending moments and radial displacements in the support caused by transverse seismic Q wave at the no slip conditions are respectively:

$$N''_{\theta+\alpha}\left(Q\right) = \frac{1}{3}RP''_{\theta+\alpha}\left(Q\right)\sin 2\left(\theta+\alpha\right), \tag{33}$$

$$M''_{\theta+\alpha}\left(Q\right) = -\frac{1}{3}R^2 P''_{\theta-\alpha}\left(Q\right)\sin 2\left(\theta+\alpha\right), \tag{34}$$

$$U^{''}_{\theta+\alpha}\left(Q\right) = \frac{bR^4 P''_{\theta-\alpha}\left(Q\right)}{9E_l I}\sin 2\left(\theta+\alpha\right). \tag{35}$$

Shear waves can cause stretching radial stresses on skew-symmetric $0 \le \left(\theta+\alpha\right) \le \pi/2$ and $\pi \le \left(\theta+\alpha\right) \le 3\pi/2$ parts of the contact surface (Fig.2,b). As noted in the works [6,18] there will take place a separation on the rock mass-structure interface. This is possible if there are no special bonds between the rocks and support. Otherwise the axial forces, bending moments and radial displacements (29) - (35) should be reduced approximately twice. But when the loads of the static field act on the structure, they could be reduced by tensile contact stresses generated by the transverse waves. This will depend on the intensity of the static and seismic main stresses and their respective orientation, which will be determined by the angle α . If the superposed contact stresses are not less than zero, then estimates of equations (29)

- (35) are valid. Such forces generated from compressive, *P* and shear, *S* waves will superpose to forces under geostatic field of stresses. Further, the extreme meanings of internal stresses and displacements will be determined by confirmatory analysis of structural mechanics method.

The sequence of calculations by the proposed closed-form solutions and analysis of the obtained results for specific numerical examples are given in appendix.

CONCLUSIONS

Analytical solutions were developed for determination of influence of both longitudinal tension/compression (*P*) as well as the lateral, shear (*S*) seismic waves on the circular tunnel support being under the loading of geostatic gravitational forces.

Closed-form solutions are given for underground structures of circular section at significant depth in a weak rock mass. Problems are considered in the “excavation loading” scheme, taking into account the sequence of excavation and reinforcing of the opening.

Four functions are proposed for establishing the relationship between uniform and non-uniform radial and tangential displacements with the corresponding contact stresses forming at the rock-support interaction. These functions have the physical meaning of "spring stiffness" and dimension of Winkler's coefficient and take into account the deformation parameters of the rock mass, conditions on contact interface, and the geometry of the liner profile.

The rock-support interaction problems for underground structure under the gravitational forces are determined in the “excavation loading” scheme. Assumed is that the lining is installed at some distance from the tunnel face. Both cases of full-slip and no-slip contact conditions at the rock-support interface are considered.

Same problems for *P* and *S* waves are considered in the "overpressure loading" scheme, because, in contrast to the geostatic field of stresses, seismic waves propagate when the tunnel already exists.

The examples of numerical modeling, performed by proposed analytical apparatus lead to the conclusion, that: maximum hoop stress generated by an S-wave in the liner can be greater than that by a P-wave at low values of the mass modulus, but for greater values of the modulus the maximum internal stress due to S-waves is smaller than that from P-waves. The difference between them increases with increasing mass modulus;

Maximum total stresses, that depend on directions of main geostatic stresses and the propagation of seismic waves, can be formed by superposing geostatic stresses more with P-wave than with S-wave.

All this puts under doubt the widespread current opinion that dominant and critical impact on the tunnel should be only the transverse waves. The numerical examples show: the maximum stresses in the underground structure imposed by longitudinal waves may exceed those due to transverse waves; summed stresses after superposition of static and seismic fields of stress differ greatly from same by shear seismic waves. Therefore ignoring any of them will decrease the accuracy of calculation**.**

Appendix

Numerical Verification and Comparative Analyses of Suggested Expressions

For a comparative analysis of the obtained results, the initial data are taken to be the same as used in the US Technical Manual [2: E-6] for calculating a tunnel of circular section. Input data of the examples are converted into a metric system and are listed in Table 1.

Table 1. Input data of the example case study

Rock mass properties		*Liner propertie*	
Modulus of deformation , MPa E	101.5	Depth of a tunnel location, m H	31.5
Poisson's ratio ν	0.41	Modulus of elasticity, MPa E_l	29000
Shear modulus, MPa G	385	Poisson's ratio ν_l	0.25
Soil unit weight, MN/m^3 γ	0.0217	Shear modulus, MPa G_l	7900
Coefficient of lateral pressure λ	0.7	Radius of liner , m R	3.3
Seismicity coefficient k	0.6	Thickness of liner, m h	0.45
P-wave stress, MPa. (eq.3.2) P_{din}	0.34	Lining erecting lag, m f , A	0
S-wave stress, MPa. (eq.3.2) Q_{din}	0.14	Angle of wave propagation, ^{0} α	0

The maximum internal forces of a quasi-static gravitational nature in the liner springline $(\theta = \pi / 2)$ section, computed by the proposed expressions and , from various solutions [2] are given in Table 2. The comparison for the special case when the support is erected directly at the tunnel face (f=0) shows a slight divergence between them.

Table 2. The maximum internal forces at springline $(\theta = \pi / 2)$ of the lining of quasi static gravitational nature calculated from various analytical solutions

Analytical Solutions	Trast, kN/m			Moment, kN m/m		
Contact conditions	Full slip	No slip	Differ. %	Full slip	No slip	Differ. %
Einstein and Schwartz [19]	2092	1948	6.9	244	225	7.8
Ranken et al. [20]	1948	1778	8.7	216	234	-8.3
Curtis [21]	1802	1884	-4.5	115	115	0
Proposed expressions	1923	2011	-4.6	158	173	-9.5

The extreme contact pressures, $P_{\max}$, $P_{\min}$, thrusts, $N_{\max}$, bending moments, $M_{\max}$, and radial displacements $U_{\max}$; extreme tangential normal stresses in the springline $(\theta = \pi / 2)$ section of the lining, $\sigma_{\max}$, $\sigma_{\min}$ due to: quasi static gravitational forces, *P* and *S* - waves, for both contact conditions, calculated using proposed analytical expressions are given in Table 3.

Table 3. Loads, thrusts and bending moments, stresses and displacements in the lining due to: geostatic, *P* and *S* - waves, for no-slip and full-slip contact condition

Natur	Contact condition	$P_{\max}$ KPa	$P_{\min}$ KPa	$T_{\max}$ KPa	$M_{\max}$ KN	$N_{\max}$ KN	$\sigma_{\max}$ MPa	$\sigma_{\min}$ MPa	$U_{\max}$ cm
Geo-static	No-slip	296	273	50	173	2011	10	-1.2	0.29
P-wave	Full-slip	305	264	0.0	158	1923	9.0	-0.4	0.25
	No-slip	294	271	25	91	998	5.0	-0.5	0.17
S-wave	Full-slip	151	131	0.0	78	951	4.4	-0.2	0.14
	No-slip	14.5	-14.5	63.5	238	177	7.0	-6.2	0.40
	Full-slip	28.2	-28.2	0.0	188	57	6.3	-5.8	0.34

The summed stress fields arising after superposition of qeostatic and seismic stresses are formed at different times because of the different propagation velocities of the longitudinal and lateral waves. The maximum hoop stresses of different natures act also in different sections of the lining structure. Therefore, the determination of the location and magnitude of the total compressive and tensile stresses in the support is possible using the graphic devices constructed by analytical expressions. Such graphic's for maximum hoop stresses in $0 \le \theta \le \pi/4$ sections of the support, being in the no slip contact condition, calculated for two values of seismic waves propagation angles: $\alpha = 0$ and $\pi/4$, are given in Figure 3.

a) b)

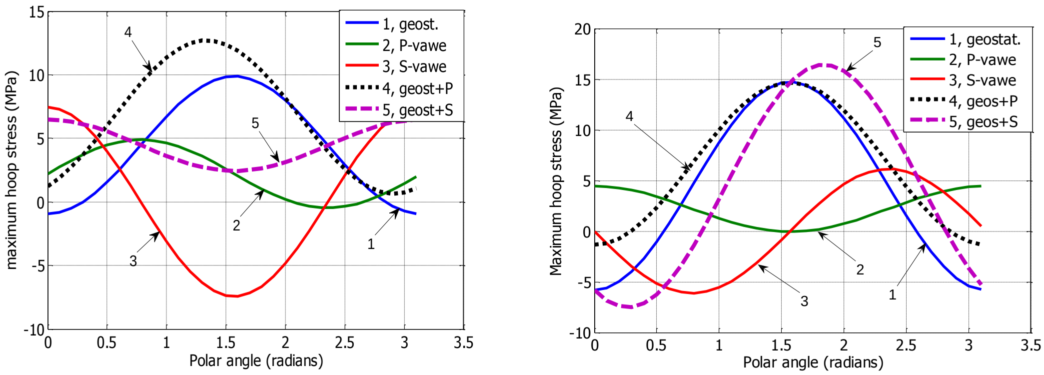

Fig. 3. Maximum hoop stresses, formed in the lining by: geostatic (1), *P*-wave (2) and *S*-wave (3) seismic forces in the superposition of geostatic to the: *P*-wave (4), and *S*-wave (5) stresses, for the seismic waves propagation angles: α=0 , (a) and α= $\pi/4$ **(b).**

The above-mentioned dependencies are valid also for free slip contact conditions and for other initial data typical for practice.

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მექანიკა

სეისმური P და S ტალღებით და გეოსტატიკური ძალებით გამოწვეული შიგა ძალვების გაანგარიშება წრიული კვეთის გვირაბის კონსტრუქციაში

ლ. ჯაფარიძე

აკადემიის წევრი, გ. წულუკიძის სამთო ინსტიტუტი, თბილისი, საქართველო

სტატიაში მოცემულია გეოსტატიკური და სეისმური ძალების მოქმედების ქვეშ მყოფი ქანების მასივის და წრიული კვეთის გვირაბის კონსტრუქციის ერთობლივი დეფორმაციის რეჟიმში წარმოშობილი შიგა ძალვების გაანგარიშების ანალიზური აპარატი. ქანების მასივი განხილულია როგორც ერთგვაროვანი, იზოტროპული, წრფივად დეფორმირებადი ტანი და მოდელირებულია დრეკადობის თეორიის მეთოდებით. გვირაბის სამაგრი კონსტრუქციის დაძაბულ-დეფორმირებული მდგომარეობა შეფასებულია მასალათა გამძლეობის საშუალე-ბებით და ითვალისწინებს გვირაბის გაყვანის და გამაგრების ოპერაციების ტექნოლოგიურ ფაქტორებს და „სამაგრი-ქანის“ მექანიკური სისტემის საკონტაქტო პირობებს.

ანალიზური აპარატი მოწოდებულია გვირაბის საექსპლოატაციო პირობების ზღვრულ მდგომარეობებზე გაანგარიშებისათვის მასზე გრავიტაციული სტატიკური და სეისმური, ***P*** და *შ* გრძივი და განივი ტალღებით გამოწვეული დატვირთვების ზემოქმედებისას.

ჩატარებული რიცხვითი მაგალითები აჩვენებს, რომ გრძივი ტალღებით გამოწვეული მაქსიმალური ძაბვები გვირაბის კონსტრუქციაში შეიძლება აჭარბებდეს განივი ტალღებით გამოწვეულ ძაბვებს, ხოლო გეოსტატიკური და სეისმური წარმოშობის ძაბვების სუპერპო-ზიციით მიღებული ჯამური ძაბვები შეიძლება ბევრად აღემატებოდეს მხოლოდ განივი ტალღებისას. შესაბამისად, მათი ზემოქმედების ცალ-ცალკე შეფასება, ანდა გრძივი ტალღების საერთოდ იგნორირება, რასაც ადგილი აქვს თანამედროვე სპეციალური ლიტერატურის დიდ უმრავლესობაში, მნიშვნელოვნად ამცირებს გაანგარიშების სიზუსტეს და, საბოლოო ჯამში, ამცირებს ნაგებობის საიმედოობას.

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