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Cybernetics

On Network Maintenance Problem. Mixed-Type Semi-Markov Queuing System with Bifurcation of Arrivals

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ABSTRACT. In the present paper a multi-unit redundant system with unreliable, repairable units is considered. Two types of maintenance operations are performed in the system: 1) the replacement of the failed main unit by the redundant one; 2) the repair of the failed unit. For this system mixed-type semi-Markov queuing model with bifurcation of arrivals is constructed. It represents a non-classical boundary value problem of mathematical physics with non-local boundary conditions. At present the problem is still being investigated. *© 2018 Bull. Georg. Natl. Acad. Sci.*

Key words: replacement, repair, semi-Markov queuing model, bifurcation

Our research topic belongs to one branch of the applied Mathematics – Operations Research (and /or Management Science), namely to the Mathematical Theory of Reliability (MTR). The letter applies methods and models of Queuing Theory (QT). That is why we could formulate our problem in terms of Queuing Theory. But we prefer to formulate it in terms of the practical problems, for solution of which, the necessity of our research has been arisen. This is necessary to underline the significance of the research topic as well as its place within the network maintenance problem [1-6].

The research subject of the paper is multi-unit redundant system with repairable units, such as info-communication, computer and transportation networks, power and defense systems, etc.

We should mention here the high importance of open and mix type queuing models for dependability and performability analysis of above mentioned territorially distributed networks. The fact is that for a long period of time in MTR and QT there was generally accepted the idea, that in problems of reliability and maintenance of redundant complex systems only finite-source (closed) queuing models were applicable.

This idea, however, is valid for classical machine maintenance problem, but for modern network maintenance problem open queuing models or mixed type models are mainly applied. This is convincingly verified by experts from the Georgian Technical University in their publications for the last years [7-11].

For illustration we can consider a specific example – the number of Radio Base Stations (RBS) in modern mobile communication networks may be hundreds, thousands and more. That means that in mathematical models (as it is accepted in QT) we can consider the set of RBS as infinite ***(m*** $= \infty$***)*** source of failures. Due to the same factor we can consider the total failure rate to be constant. Consequently, we will have a Poisson stream of requests to maintenance facilities. As it is known, this is very important for construction and investigation of suitable mathematical models.

Our research subject consists of ***m (m*** $= \infty$***)*** identical main (operative) and ***n*** redundant units. It is supposed that for normal operation of the system it's desirable that in the set of main units all m units be operative.). But if their number is less, the system continuous functioning with reduced economical effectiveness. The total failure rate of the set of all main units is $\boldsymbol{\alpha}$ and the failure rate of individual redundant units' is $\boldsymbol{\beta}$.

The failed main unit must be replaced with operative redundant one. Thus, if at failure moment there is a free redundant unit in the system, its replacement operation will commence. The failed units, both main and redundant ones, must be repaired. It is supposed that repaired unit becomes identical to the new one. In general, there are ***l*** and ***k*** maintenance servers for replacement and repair, respectively. Replacement and repair times are random variables with ***H*** and ***G*** distribution functions. In case when servers are busy, request queues for replacement or repair are formed. The service discipline is FCFS (First Come, First Served). Thus, we have a mixed type queuing system (queuing system with infinite and finite sources simultaneously) with two types of services – replacement and repair. The request for replacement arises when the main unit fails. The same event, as well as the failure of redundant unit, generates request for the repair (see Fig.).

Such semi-Markov queuing models are not considered in scientific publications up to now. As for Markov models, they are only discussed in earlier publications of the authors of the present paper [7-12].

The diagram in Figure will be helpful to explain queuing models of network maintenance problem [7].

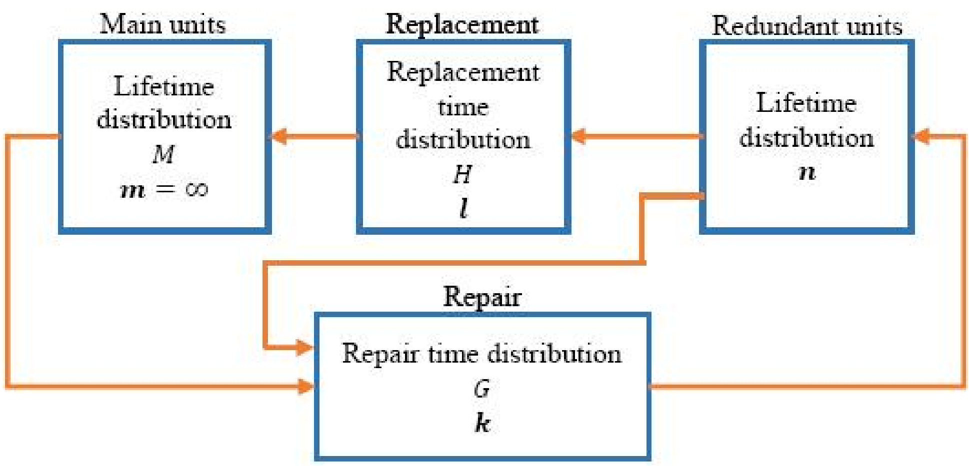

Fig. Diagram ilustrating the network maintenance problem.

Thus, we have a queuing system, which is open with respect to the main units' failures stream (source of requests is infinite) and, at the same time, it is a closed queuing system with respect to the redundant units' failures stream (source of requests is finite).

From above description it is obvious that the failure of main units generates two requests for service: 1) the replacement of failed unit by redundant one; 2) the repair of failed unit. That is called the bifurcation of arrivals (failures) [7]. Therefore, both of maintenance operations should be performed in parallel mode.

Mathematical Model

Suppose we have one repair server and one replacement server (k= 1 =1). Replacement time has an exponential distribution with parameter μ . Repair time distribution function is arbitrary G function. Denote repair rate by $\eta(u)$, so that $\eta(u)=\dfrac{G'(u)}{1-G(u)}$.

We introduce the random processes, which define the states of the considered system at the time instant *t*:

$i(t)$ is the number of units missing in the group of main units;

$j(t)$ is the number of non-operative (failed) units in the system;

$\xi(t)$ is the time interval length from the beginning of the repair operation to the time instant t .

To describe system state, we define probability characteristics:

$P(i,t)=P\{i(t)=i,\ j(t)=0\},\ i=\overline{0,\infty}$

$q(i,j,t,u)=\lim\left(\frac{1}{h}P\{i(i)=i, j(t)=j, u<\xi(t)<u+h\}\right)\ i=\overline{0,\infty},\ \ j=\overline{1,n+i}.$

We assume, that $P(0,0)=1, P(i,0)=0, i>0$ and $q(i,j,0,u)=0$ for any i and j .

Suppose,

A) Functions $P(i,t)$ have continuous derivatives when $t>0$;

B) Functions $q(i,j,t,u)$ have continuous partial derivatives when $t>0$, $u\ge 0$.

According to the usual probabilistic considerations, the following theorems are proved.

Theorem 1. *Suppose, the conditions A) and B) are valid. Then, functions* $P(i,t)$ *satisfy the following system of integro-differential equations:*

$$\frac{dP(0,t)}{dt}=-(\alpha+n\beta)P(0,t)+\mu P(1,t)+\int_0^t q(0,1,t,u)\eta(u)du,$$
$$\frac{dP(i,t)}{dt}=-(\alpha+(n+i-1)\beta+\mu)P(i,t)+\mu P(i+1,t)+\int_0^t q(i,1,t,u)\eta(u)du,\quad i\ge 1. \tag{1}$$

Theorem 2. *Suppose, the condition B) is valid. The Functions* $q(i,j,t,u)$ *satisfy the following infinite system of partial differential equations:*

$$\frac{\partial q(0,1,t,u)}{\partial t}+\frac{\partial q(0,1,t,u)}{\partial u}=-(\alpha+(n-1)\beta+\eta(u))q(0,1,t,u)+\mu q(1,1,t,u),$$
$$\frac{\partial q(0,j,t,u)}{\partial t}+\frac{\partial q(0,j,t,u)}{\partial u}=-(\alpha+(n-j)\beta+\eta(u))q(0,j,t,u)+\mu q(1,j,t,u)+$$
$$+(n-(j-1))\beta q(0,j-1,t,u),\quad 1<j\le n,$$
$$\frac{\partial q(i,1,t,u)}{\partial t}+\frac{\partial q(i,1,t,u)}{\partial u}=-(\alpha+(n+i-2)\beta+\mu+\eta(u))q(i,1,t,u)+\mu q(i+1,1,t,u),\ i\ge 1, \tag{2}$$

$$\frac{\partial q(i,j,t,u)}{\partial t}+\frac{\partial q(i,j,t,u)}{\partial u}=-\left(\alpha+(n+i-j-1)\beta+\mu+\eta(u)\right)q(i,j,t,u)+\mu q(i+1,j,t,u)+$$
$$+\alpha q(i-1,j-1,t,u)+(n+i-j)\beta q(i,j-1,t,u),\quad i\geq 1,\quad 1<j\leq n+i-1,$$
$$\frac{\partial q(i,j,t,u)}{\partial t}+\frac{\partial q(i,j,t,u)}{\partial u}=-\left(\alpha+(n+i-j)\mu+\eta(u)\right)q(i,j,t,u)+(n+i-j+1)\mu q(i+1,j,t,u)+$$
$$+\alpha q(i-1,j-1,t,u),\quad i\geq 1,\quad n+i-1<j\leq n+i.$$

Theorem 3. *The functions* $q(i,j,t,0)$ *satisfy the infinite system of recursive equations:*

$$\begin{aligned}
&q(0,1,t,0)=n\beta P(0,t)+\int_0^t q(0,2,t,u)\eta(u)\,du,\\
&q(i,1,t,0)=\alpha P(i-1,t)+(n+i-1)\beta P(i,t)+\int_0^t q(i,2,t,u)\eta(u)\,du,\quad i\geq 1,\\
&q(i,j,t,0)=\int_0^t q(i,j+1,t,u)\eta(u)\,du,\quad i\geq 0,\quad 1<j<n+i,\\
&q(i,n+i,t,0)=0,\quad i\geq 0.
\end{aligned}\tag{3}$$

The systems of equations (1), (2), (3) represent non-classical boundary value problem of mathematical physics with non-local boundary conditions (3).

At present the problem is still being investigated.

Conclusion

The bifurcation of arrivals mean that the failure of main units generates two requests for service: 1) the replacement of failed unit by redundant one; 2) the repair of failed unit. Therefore, both maintenance operations should be performed in parallel mode. In our previous papers [7-11] we discussed the exponential models of our research subject with bifurcation of arrivals. For them we constructed suitable mathematical models as infinite systems of ordinary differential equations. These equations in steady state conditions are reduced to infinite system of linear algebraic equations.

The main difficulty in building and analyzing the mathematical model of the discussed system is the bifurcation of arrivals. Overcoming this difficulty is the main challenge to the model obtained.

In this paper we present the semi-Markov queuing model, considering the bifurcation of failures. Unlike exponential systems we obtain here much more complex mathematical description; namely, the integro-differential equations (1), infinite system of partial differential equations (2) and the infinite system of recursive functional equations (3). Overall, they form the non-classical boundary value problem of mathematical physics. At present the problem is still being investigated. Investigation of this task (the existence and uniqueness of the solution) is associated with significant difficulties. However, note that constructing a particular solution of the problem (1), (2), (3) is expected using the 2-variable generating functions' method.

From the results of this research we can easily derive many important probabilistic characteristics of the system. Direction for future research can be the investigation of system's behavior for different distribution functions of repair time.

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§ ივანე ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტი, ზუსტ და საბუნებისმეტყველო მეცნიერებათა ფაკულტეტი, თბილისი, საქართველო
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წარმოდგენილ ნაშრომში განხილულია მრავალელემენტიანი დარეზერვებული სისტემა არასაიმედო აღდგენადი ელემენტებით. ამ სისტემაში მიმდინარეობს ორი ტიპის მომსახურების ოპერაცია: 1) მტყუნებული ძირითადი ელემენტების ჩანაცვლება სარეზერვოთი; 2) მტყუნებული ელემენტის აღდგენა. ამ სისტემისათვის აგებულია შერეული ტიპის ნახევრად მარკოვული რიგების მოდელი შემოსვლათა ბიფურკაციით. ის წარმოადგენს მათემატიკური ფიზიკის არაკლასიკურ სასაზღვრო ამოცანას არალოკალური სასაზღვრო პირობებით. ამჟამად ეს მოდელი გამოკვლევის პროცესშია.

REFERENCES

1. Ushakov I. (2006) Reliability: Past, Present, Future. *RTA Journal*, **1**, 1:10-16.
2. Ushakov I. (2012) Reliability theory: history & current state in bibliographies. *RTA Journal*, **1**, 1: 8-35.
3. Limnios N., Oprisan G. (2001) Semi-Markov Processes and Reliability. Boston, Birkhauser.
4. Korolyuk V.S., Limnios N. (2005) Stochastic Systems in Merging Phase Space. World Scientific, Imperial Coledge Press.
5. Gertsbakh E. (2000) Reliability Theory: with Applications to Preventive Maintenance. Springer-Verlag.
6. Epstein B., Weissman I. (2008) Mathematical Models for Systems Reliability. Taylor and Francis Group LLC.
7. Prangishvili A., Meladze H., Kakubava R., Svanidze N. (2017) On network maintenance problem. Open markovian queuing system with bifurcation of arrivals. *Bull. Georg. Natl. Acad. Sci.*; **11**, 3: 34-42.
8. Prangishvili A., Meladze H., Kakubava R. (2016) Open queuing system for two parallel maintenance operations as mathematical model for dependability and performance analysis. *Bull. Georg. Natl. Acad. Sci.*; **10**, 3: 69-74.
9. Prangishvili A., Meladze H., Kakubava R. (2015) Queuing models for large scale technical systems' structural control. *Proc. Intern. Sci. Conf. "Information and Computer Technologies, Modeling-Control"*, 131-135. Tbilisi, Georgia.
10. Kakubava R. (2010) Multi-line Markov closed queuing system for two maintenance operations. *RTA Journal,* **1**: 15-22.
11. Kakubava R. (2013) Reliability model for stand by system with replacement delays of failed elements. *Autom.Contr. and Com. Sci.*, **47**, 2: 94- 98

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