

## Fuzzy Aggregation Operators Approach in Location/Transportation Problem

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**ABSTRACT.** Two-stage Fuzzy Multi-Objective Emergency Location-Transportation Problem (FMOELTP) is presented. Some independent attributes act on humanitarian aid distribution centers' (HADCs) selection process. At the first stage, based on experts' evaluations on attributes, HADC's selection index is defined by the constructed associated fuzzy probabilistic OWA (AsFPOWA) operator. At the second stage, four objective functions are constructed for the FMOELTP. The constraints of the model provide deep detail of the problem. An epsilon-constraint approach is proposed for the exact solution of the FMOELTP. © 2018 Bull. Georg. Natl. Acad. Sci.

**Key words:** Aggregation operators, multi-objective location-transportation problem, fuzzy sets and possibility theory

Our goal is to create an expert knowledge-based intelligent support system, which will serve as assistant tool to provide optimal solutions for the FMOELTP. The input to the mathematical model of the system will be objective data, as well as expert evaluations. The outputs of the system will solve Fuzzy Multi-Objective Emergency Location-Transportation Problem (FMOELTP), which is an extension of general Facility Location/transportation Problem (FLTP) [1]. The FMOELTP modeling is very actual, because objective data on the model parameters does not always exist in the disaster process. Therefore, our approach must be more feasible and reliable in the research of emergency management. In our models we primarily concentrate on detailed modeling of the location-transportation problem with different fuzzy objective functions and constraints.

**Definition 1** [2]:  $\tilde{c}(t): R^1 \rightarrow [0;1]$  is called the Fuzzy Number (FN) which can be considered as a generalization of the interval number:

$$\tilde{c}(t) = \begin{cases} 1 & \text{if } t \in [c'_2, c''_2] \\ \frac{t - c_1}{c'_2 - c_1} & \text{if } t \in [c_1, c'_2] \\ \frac{c_3 - t}{c_3 - c''_2} & \text{if } t \in [c''_2, c_3] \\ 0 & \text{otherwise} \end{cases},$$

where  $c_1 \leq c'_2 \leq c''_2 \leq c_3 \in R^1$  ( $\tilde{c} \equiv (c_1, c'_2, c''_2, c_3)$ ).

In the following we use triangular fuzzy numbers (TFN), which means that  $c'_2 = c''_2$  in  $\tilde{c}(t)$ . Here we present a brief review of TFN arithmetic operations: Let  $\tilde{c}$  and  $\tilde{b}$  be two TFNs, where  $\tilde{c} = (c_1, c_2, c_3)$  and  $\tilde{b} = (b_1, b_2, b_3)$ . Then [2]: 1.  $\tilde{c} + \tilde{b} = (c_1 + b_1, c_2 + b_2, c_3 + b_3)$ ; 2.  $\tilde{c} - \tilde{b} = (c_1 - b_3, c_2 - b_2, c_3 - b_1)$ ; 3.  $\tilde{c} \times k = (kc_1, kc_2, kc_3)$ ,  $k > 0$ ; 4.  $\tilde{c}^k = (c_1^k, c_2^k, c_3^k)$ ,  $k > 0, c_i > 0$ ; 5.  $\tilde{c} \cdot \tilde{b} = (c_1 b_1, c_2 b_2, c_3 b_3)$ ,  $c_i > 0, b_i > 0$ ; 6.  $1/\tilde{b} = \{1/b_3, 1/b_2, 1/b_1\}$ ,  $b_i > 0$ ; 7.  $\tilde{c} > \tilde{b}$  if  $c_2 > b_2$  and if  $c_2 = b_2$  then  $\tilde{c} > \tilde{b}$  if  $c_1 + c_3 > b_1 + b_3$ , otherwise  $\tilde{c} = \tilde{b}$ .

The set of all TFNs is denoted by  $\Psi$  and positive TFNs ( $c_i > 0$ ) by  $\Psi^+$ . Suppose, that the expert evaluation of time for moving from customer  $i$  to center  $j$  is represented by the positive triangular fuzzy number -  $\tilde{t}_{ij} = (t_{ij}^{(1)}, t_{ij}^{(2)}, t_{ij}^{(3)}) \in \Psi^+$  ("approximately time  $t_{ij}^{(2)}$ "). By definition of possibility measure on the  $R^1$  we construct possibility distribution [2]:  $\forall t > 0, Pos(\tilde{t}_{ij} \leq t) = \sup_{\tau \leq t} \tilde{t}_{ij}(\tau)$  or

$$Pos(\tilde{t}_{ij} \leq t) = \begin{cases} 1, & t > t_{ij}^{(2)}; \\ \frac{t - t_{ij}^{(1)}}{t_{ij}^{(2)} - t_{ij}^{(1)}}, & t_{ij}^{(1)} \leq t \leq t_{ij}^{(2)}; \\ 0, & t < t_{ij}^{(1)}. \end{cases} \tag{1}$$

**OWA – Type New Fuzzy Probability Aggregations**

As it is well-known, many fuzzy probabilistic aggregations were researched in the OWA and other operators ([3,4] and others). We present here one of them:

Let the set of all attributes which act on the selection process of candidate sites be  $\Omega = \{\omega_1, \omega_2, \dots, \omega_l\}$ . Let  $\tilde{r}$  be a fuzzy probabilistic variable of experts' evaluations on  $\Omega$ :  $\tilde{r}(\omega_i) = \tilde{r}_i, i = 1, \dots, l$ .

**Definition 2** [3]: Let  $\Psi$  be the set of TFNs. A fuzzy probabilistic OWA operator - FPOWA of dimension  $l$  is a mapping  $FPOWA: \Psi^l \Rightarrow \Psi$  as a weighted sum of the fuzzy OWA (FOWA) operator and Mathematical Expectation values of arguments of  $\tilde{r}$ :

$$FPOWA(\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_l) = \beta \cdot FOWA(\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_l) + (1 - \beta) \cdot E_p(\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_l), \tag{2}$$

where  $E_p(\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_l) \equiv E_p(\tilde{r}) = \sum_{i=1}^l p_i \tilde{r}_i$ ;  $p_i \equiv P(\tilde{r} = \tilde{r}_i)$  is a probability of  $\tilde{r}_i$  with  $\sum_{j=1}^l p_j = 1, 0 \leq p_j \leq 1$ ;

$\beta \in [0, 1]$  is a weight;  $FOWA(\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_l) = \sum_{j=1}^l w_j \tilde{r}_{i(j)}$ ,  $w_j$  is a weight of  $\tilde{r}_i$  and  $\tilde{r}_{i(j)}$  is the  $j$ -th largest of the  $\{\tilde{r}_i\}, i = 1, \dots, l$ .

**Definition 3** [4]: Let  $S_l$  be the set of all permutations of the set  $\{1, 2, \dots, l\}$ . Let  $\{\pi(\omega)\}_{\omega \in \Omega}$  be the possibility distribution of the measure -  $Pos: 2^\Omega \rightarrow [0, 1]$ . The associated probability class  $\{P_\sigma\}_{\sigma \in S_l}$  of a possibility measure -  $Pos$  is called:  $\forall \sigma \in S_l,$

$$\begin{aligned}
P_{\sigma}(\omega_{\sigma(i)}) &= \text{Pos}(\omega_{\sigma(1)}, \omega_{\sigma(2)}, \dots, \omega_{\sigma(v)}) - \text{Pos}(\omega_{\sigma(1)}, \omega_{\sigma(2)}, \dots, \omega_{\sigma(v-1)}) = \\
&= \max_{v=1, i} \pi(\omega_{\sigma(v)}) - \max_{v=1, i-1} \pi(\omega_{\sigma(v)}), \quad (3)
\end{aligned}$$

$i = 1, \dots, l$ ; for each  $\sigma = (\sigma(1), \sigma(2), \dots, \sigma(l)) \in S_l$   $\pi(s_{\sigma(0)}) \equiv 0$ .

Properties of associated probabilities of the possibility measure see in [4]. Therefore, we have  $l!$  mathematical expectations

$$E_{P_{\sigma}}(\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_l) = \sum_{i=1}^l P_{\sigma}(\omega_{\sigma(i)}) \tilde{r}_{\sigma(i)}, \quad \sigma \in S_l$$

instead of one value, as in FPOWA operator. We use all  $l!$  mathematical expectations in our aggregations for the possibilistic uncertainty:

Let  $M: \Psi^k \Rightarrow \Psi$  ( $k = l!$ ) be some mean aggregation function with the following properties - monotonicity, boundedness, idempotency and symmetricity [5].

**Definition 4:** An associated fuzzy probabilistic OWA operator AsFPOWA of dimension  $l$  is mapping  $AsFPOWA: \Psi^l \Rightarrow \Psi$ , that has an associated objective weighted vector  $W$  of dimension  $l$  such that

$w_j \in (0, 1)$ ,  $\sum_{j=1}^l w_j = 1$  and a possibility measure  $Pos: 2^{\Omega} \Rightarrow [0, 1]$ , is defined according the following

formula:

$$AsFPOWA(\tilde{r}_1, \tilde{r}_2, \dots, \tilde{r}_l) = \beta \sum_{j=1}^l w_j \tilde{r}_{i(j)} + (1 - \beta) M(E_{P_{\sigma_1}}(\tilde{r}), E_{P_{\sigma_2}}(\tilde{r}), \dots, E_{P_{\sigma_k}}(\tilde{r})), \quad (4)$$

where  $\tilde{r}_{i(j)}$  is the  $j$ -th largest of the  $\{\tilde{r}_i\}, i = 1, \dots, l$ ;  $E_{P_{\sigma_i}}(\tilde{r}), \sigma_i \in S_l$  is a Mathematical Expectation of  $\tilde{r}$  with respect to associated probability  $P_{\sigma_i}$ ; ( $k = l!$ ) of the possibility measure  $Pos$ .

The selection of  $M$  depends on the decision maker's preferences with respect to decision risks and other factors of extreme situations.

#### Fuzzy Multi-Objective Emergency Location/Transportation Problem for Extreme Environment

In disaster situations, it makes sense to think that each particular house or building within the affected region could require relief or humanitarian aid, thus becoming a potential demand point. In a severe crisis that affects a large area, the number of demand points and the number of types of products and services required may be very high. The amount of information to be managed is huge, and it would be impractical (if not impossible) to consider the fine details in designing and deploying humanitarian aid distribution networks. To cope with these difficulties, demand points are aggregated into demand zones and needed products are grouped into generic humanitarian functions.

Based on this particular context, our work deals with a Fuzzy Multi-objective Emergency Location–Transportation Problem (FMOELTP). We assume fuzziness in the model because the experts (emergency managers) knowledge and evaluations on the parameters of disaster response are the main and important data.

At the stage  $I_2$  we are focusing on a multi-criteria decision making approach for location selection problem for HADCs under uncertainty and extreme environment. We will use new fuzzy aggregation operator (AsFPOWA) to model decision making parameters. The problem of location selection for HADCs can be classified as a special case of the more general facility location problems. The facility location problem usually involves a set of location candidate HADCs (candidate sites) which are evaluated against

a set of weighted attributes independent from each other. The alternative that performs best with respect to all attributes is chosen for implementation. The distinct feature in location planning for HADCs is the consideration of interests of other stakeholders like geographical point residents, municipal administrators etc.

As the work shows, our approach in mentioned tasks is different from all and means to use the second pole of representation of the expert information - measure of the information uncertainty – using fuzzy measure with the first pole – imprecision of data ([2, 4] and others). It is giving us more convincing aggregation tool, both to evaluate parameters and construct new objective function in the FMOELTP for selection of HADCs. As a fuzzy measure, there will be used possibilistic measure and *possibility measure theory* [2] in decision making aggregation operators [5].

Therefore, the formation of expert input data for the construction of attributes is an important task of the problem. To decide on the location of HADCs, it is assumed that a set of candidate sites already exists. This set is denoted by  $L = \{1, \dots, u\}$  in the following, where we can locate HADCs and  $\Omega = \{\omega_1, \omega_2, \dots, \omega_l\}$  is the set of all attributes (transformed in benefit attributes) which define HADCs selection. For example: 1. Access by public and special transport modes to the candidate site post disaster; 2. Security of the candidate site from accidents, theft and vandalism post disaster; 3. Connectivity of the location with other modes of transport (highways, railways, seaport, airport etc.) post disaster; 4. Costs in vehicle resources, required products and etc. for the location of HADC in candidate center; 5. Impact of the candidate site on the environment, such as important objects of Critical Infrastructure and others; 6. Distances of the candidate site to the central locations; 7. Distances of the candidate site from demand points; 8. Availability of raw material and labor resources in the candidate site; 9. Ability to conform to sustainable freight regulations imposed by emergency managers post disaster for e.g. restricted delivery hours, special delivery zones; 10. Ability to increase size to accommodate growing demands post disaster, and others.

Let  $W = \{w_1, w_2, \dots, w_l\}$  be the weights of attributes and  $\Pi = \{\pi_1, \pi_2, \dots, \pi_l\}$  be the vector of degrees of possibilities of attributes. These degrees indicate on their possibility of influence on the HADCs' selection process within disaster region. For each emergency manager (expert)  $e_k$  from the group of experts  $E = \{e_1, e_2, \dots, e_t\}$ , let  $r_{ij}^k$  be the rating (utility, degree of membership, etc.) of his evaluation for each candidate point  $i, i \in L$ , with respect to each attribute  $\omega_j, (j = 1, \dots, l)$ . For the expert  $e_k$  we construct binary relations  $\tilde{R}_k = \{r_{ij}^k, i = 1, \dots, u; j = 1, \dots, l\}, k = 1, \dots, t$ , elements of which will be represented in triangular fuzzy numbers. Based on recent results of this work's authors in the decision making aggregation operators ([4] and others) we construct (Def. 4) new fuzzy aggregation operator –AsFPOWA for candidate site selection problem. The method for each candidate site will aggregate presented objective and subjective data into scalar value – *candidate site selection index*  $\delta(i), i = 1, \dots, L$ . This aggregation formally can be represented as:

$$\tilde{\delta}_i \equiv \tilde{\delta}(i) = AsFPOWA(\Omega, W, \Pi, [\tilde{R}_1]_i, \dots, [\tilde{R}_t]_i), i \in L. \tag{5}$$

At the stage II, we consider a new mathematical model – fuzzy multi-objective Mixed Integer Programming Problem of emergency location-transportation planning within disaster.

In the following, the set of demand points and required products are denoted, respectively, by  $I$  and  $J$ . The request for each demand point for each product is denoted by  $d_{ij}$ . As was defined, a set of candidate sites is denoted by  $L$  in the following. In disaster situations, one wants to ensure that every demand point is accessible (can be covered) from at least one HADC in a time less than or equal to a maximum covering time, denoted by  $\pi$ . But the difficulty of movement in extreme conditions between the candidate sites and

their demand points cause the uncertainty of time. We denote  $\tilde{t}_{il}$  the approximate time, evaluated by experts in positive triangular fuzzy number -  $\tilde{t}_{il} = (t_{il}^{(1)}, t_{il}^{(2)}, t_{il}^{(3)})$  needed to travel from site  $l \in L$  to demand point  $i \in I$  which takes into account the state of roads (e.g., broken, damaged, intact). The possibility distribution on the  $R_0^+$  induced by the fuzzy time  $\tilde{t}_{il}$  is defined as:  $\forall t \geq 0, Pos(\tilde{t}_{ij} \leq t) = \sup_{\tau \leq t} \tilde{t}_{ij}(\tau)$  [2]. The possibilistic

expectation of a triangular fuzzy number  $\tilde{t}_{il}$  with respect to a possibility distribution is defined by the formula  $E^{Pos}(\tilde{t}_{il}) = (t_{il}^{(2)} + t_{il}^{(3)} - 2t_{il}^{(2)} + t_{il}^{(1)})/6$ . We also define for each demand point  $i \in I$ , a subset  $L_i$  of candidate sites which cover the demand point  $i$  within the maximum covering time, i.e.,  $L_i = \{l \in L : E^{(Pos)}(\tilde{t}_{il}) \leq \pi\}$  (another way of formation set  $L_i$  is also considered:  $L_i = \{l, l \in L / Pos(\tilde{t}_{il} \leq \pi) \geq \alpha\}$ , where  $\alpha$  is some minimal allowable possibility level). Number of required agents which operate at opened HADC  $l \in L$  is denoted  $N_l, \forall l \in L$ . Moreover, each potential site has a global and a per product capacity that fixes the maximum quantity that can be stored within the site. The global capacity of a site  $l$  is denoted by  $S_l$  and its capacity for product  $j$  is denoted by  $s_{jl}$ . In addition, it is assumed that at each potential site  $l$ , there are  $m_l$  vehicle types,  $h = 1, \dots, m_l$ , and  $u_{hl}$  vehicles of each type  $h$ . Since all potential sites may not be equally equipped for receiving a particular vehicle type, different approximate docking times,  $\tilde{\tau}_{lh}$ , are considered, one for each vehicle type  $h$  and the corresponding site  $l$ . The vehicles available at candidate sites have different characteristics. Indeed, some vehicles may have certain handling equipment that makes them more efficient at manipulating some products. The time needed for loading and unloading one unit (for example, a pallet) of product  $j$  into a vehicle of type  $h$  is defined as  $\alpha_{jh}$ , where  $\alpha_{jh} = \infty$  if product  $j$  cannot be loaded into a type- $h$  vehicle. There are also some restrictions on the total weight and the total volume associated with vehicles. These restrictions depend on the vehicle type used. Formally, a loaded vehicle of type  $h$  must not weigh more than  $Q_h$  weight units nor have a volume over  $V_h$  volume units. To determine the total weight (the total volume) corresponding to a given vehicle's load, the weight  $w_j$  in weight units (the volume  $v_j$  in volume units) of each product  $j$  is assumed to be known with certainty. Finally, a maximum daily work time,  $D_h$  (in time units) for each vehicle type  $h$  is imposed. A given vehicle can perform as many trips as needed during a day as long as the corresponding work time limit is respected. As requested quantities are generally large in terms of vehicle capacity (in weight and/or volume), each vehicle trip is assumed to visit only one demand point at a time. In other words, only back and forth trips are considered. Obviously, a demand point may be visited many times. However, because of the maximum daily work time, the number of trips performed to delivery point  $i$  by a specific vehicle will be limited to a maximum value  $r$ . We describe decision variables:  $y_l$  - equal to 1, if a HADC is open at site  $l$ , 0 otherwise;  $x_{ilhkv}$  - equal to 1 if demand point  $i$  is visited from HADC  $l$  with the  $k$ -th vehicle of type  $h$  on its  $v$ -th trip to  $i$ ;  $Q_{ijlhkv}$  - quantity of product  $j$  delivered to point  $i$  from HADC  $l$  with the  $k$ -th vehicle of type  $h$  on its  $v$ -th trip to  $i$ ;  $p_{jl}$  - quantity of product  $j$  provided at site  $l$ .

The FMOELTP as a Multi-objective Mixed Integer Programming Problem [7] can be formulated as an Expected Value Programming Problem with Possibilistic Constraints:

$$\text{Min } f_1 = \sum_{i=1}^n \sum_{l=1}^u \sum_{h=1}^{m_l} \sum_{k=1}^{u_{hl}} \sum_{v=1}^r ((2E^{(Pos)}(\tilde{t}_{il}) + E^{(Pos)}(\tilde{\tau}_{lh}))x_{ilhkv} + \sum_{j=1}^p \alpha_{jh}Q_{ijlhkv}) \quad (6)$$

$$Max f_2 = \sum_{l=1}^u E^{(Pos)}(\tilde{\delta}_l)y_l, \tag{7}$$

$$Min f_3 = \sum_{l=1}^u N_l y_l, \tag{8}$$

$$Min f_4 = \sum_{i=1}^n \sum_{j=1}^p (d_{ij} - \sum_{l=1}^u \sum_{h=1}^{m_l} \sum_{k=1}^{u_{hl}} \sum_{v=1}^r Q_{ijlhkv}), \tag{9}$$

Subject to

$$\sum_{l \in L_i} \sum_{h=1}^{m_l} \sum_{k=1}^{u_{hl}} \sum_{v=1}^r Q_{ijlhkv} \leq d_{ij}, \quad i = 1, \dots, n; \quad j = 1, \dots, p \tag{10}$$

$$\sum_{i \in N_l} \sum_{h=1}^{m_l} \sum_{k=1}^{u_{hl}} \sum_{v=1}^r Q_{ijlhkv} \leq p_{jl}, \quad j = 1, \dots, n; \quad l = 1, \dots, u \tag{11}$$

$$Pos(\sum_{i \in N_l} \sum_{v=1}^r ((2\tilde{t}_{il} + \tilde{\tau}_{lh})x_{ilhk} + \sum_{j=1}^p \alpha_{jh} Q_{ijlhkv}) \leq D_h y_l) \geq \beta, \quad l = 1, \dots, u; \quad h = 1, \dots, m_l; \quad k = 1, \dots, u_{hl} \tag{12}$$

$$\sum_{j=1}^p w_j Q_{ijlhkv} \leq Q_h x_{ilhk}, \quad i = 1, \dots, n; \quad l \in L_i; \quad h = 1, \dots, m_l; \quad k = 1, \dots, u_{hl}; \quad v = 1, \dots, r \tag{13}$$

$$\sum_{j=1}^p v_j Q_{ijlhkv} \leq V_h x_{ilhk}, \quad i = 1, \dots, n; \quad l \in L_i; \quad h = 1, \dots, m_l; \quad k = 1, \dots, u_{hl}; \quad v = 1, \dots, r \tag{14}$$

$$\sum_{j=1}^p p_{jl} \leq S_l y_l, \quad l = 1, \dots, u, \tag{15}$$

$$p_{jl} \leq s_{lj}, \quad j = 1, \dots, p; \quad l = 1, \dots, u, \tag{16}$$

$$y_l \in \{0,1\}, \quad l = 1, \dots, u, \tag{17}$$

$$x_{ilhk} \in \{0,1\}, \quad i = 1, \dots, n; \quad l \in L_i; \quad h = 1, \dots, m_l; \quad k = 1, \dots, u_{hl}; \quad v = 1, \dots, r \tag{18}$$

$$Q_{ijlhkv} \geq 0, \quad i = 1, \dots, n; \quad j = 1, \dots, p; \quad l \in L_i; \quad h = 1, \dots, m_l; \quad k = 1, \dots, u_{hl}; \quad v = 1, \dots, r, \tag{19}$$

$$p_{jl} \geq 0, \quad j = 1, \dots, p; \quad l = 1, \dots, u. \tag{20}$$

The four objectives are given by Eqs. (6)-(9). The objective function (6) minimizes an expectation of the total transportation durations. In fact, the possibilistic expected duration of the  $v$ -th trip of the  $k$ -th vehicle of type  $h$  from site  $l$  to demand point  $i$  is given by  $(2E^{(Pos)}(\tilde{t}_{il})x_{ilhk} + E^{(Pos)}(\tilde{\tau}_{lh})x_{ilhk} + \sum_{j=1}^p \alpha_{jh} Q_{ijlhkv})$ , where the first part  $(2E^{(Pos)}(\tilde{t}_{il}))$  represents the back and forth expectations of travel times, the second part  $(E^{(Pos)}(\tilde{\tau}_{lh}))$  is the expected docking time, and the last part  $(\sum_{j=1}^p \alpha_{jh} Q_{ijlhkv})$  is the loading and unloading time of all the products delivered from site  $l$  to point  $i$ . Objective function (7) maximizes the total selection index of opened HADCs. Based on the results of stage I this objective function minimizes HADCs opening risks in disaster region. The third objective function (8) minimizes the number of agents needed to operate the opened HADCs. The fourth objective function (9) minimizes the non-covered demand for all demand points. A multi-objective problem with two or three above mentioned objective functions may be considered depending on specific situations and objectives. Constraints (10) ensure that the quantity of product  $j$  delivered for each demand point  $i$

does not exceed its demand. Constraints (11) ensure that the total quantity of a given product  $j$  delivered from a HADC  $l$  does not exceed the quantity of product  $j$  available in this HADC. Possibilistic type chance constraints (12) express the maximum daily work time restrictions associated with each vehicle  $k$  of type  $h$  located at a HADC  $l$  with some minimal level  $\beta$ ,  $0 < \beta < 1$ . These constraints also prohibit trips from unopened sites. Constraints (13) and (14) impose the vehicle capacity constraints for each trip, in terms of weight ( $Q_h$ ) and volume ( $V_h$ ). Constraints (15) and (16), respectively, ensure that the global, respectively, the per product, capacity of each HADC is satisfied. Constraints (17)-(20) express the nature of decision variables used in the model.

New models of FMOELTP (6)-(18) can be constructed for the cases when some input parameters will be evaluated in triangular fuzzy numbers. For example, demands -  $\tilde{d}_{ij}$ , capacities -  $\tilde{s}_{ij}$ , time parameters -  $\tilde{\alpha}_{jh}$  and others.

As the results show, our approach in mentioned tasks is different from all existing ones and means to use the other pole of representation of the expert information - measure of the information uncertainty – using possibility measure. It is giving us more convincing aggregation tool, both to evaluate parameters and construct new objective functions (6) and (7) in the FMOELTP for selection location and transportation. A possibility measure is also used in the aggregation and formation of expert input data for the construction of HADC's selection index.

We proposed an epsilon-constraint ( $\varepsilon$ -constraint) method (general  $\varepsilon$ -constraint approach in multi-objective optimization problems see in [6]). It was proved that this approach generates the Pareto front of the multi-objective location–transportation problem addressed (omitted here).

### Conclusion

The new constructed FMOELTP model (6)-(20) represents Multi-objective Mixed Integer Optimization Problem. Sometimes for large dimensions of a computational experiment that this exact method requires large computing times. For large dimension of the FMOELTP the Estimation of Distribution Algorithm's (EDA) approach will be developed in our future works.

Based on the optimal solution of the FMOELTP the Intelligent Support System for Emergency Location and Transportation Planning in Disaster Regions will be developed. The output of the system will be the solution of Fuzzy Multi-Objective Emergency Location-Transportation Problem (FMOELTP). Programming, testing and implementation of the system will be made on the example of an experimental disaster region (for some geographical zone of Georgia) which will be generated by the Monte-Carlo simulation modeling.

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სტატიაში წარმოდგენილია ორეტაპიანი ფაზი-მრავალკრიტერიუმისანი გადაუდებელი დახმარების ობიექტების განთავსება/ტრანსპორტირების ამოცანა (FMOELTP). ზოგიერთი ატრიბუტი ზემოქმედებს ჰუმანიტარული დახმარების სადისტრიბუციო ცენტრების (HADC) შერჩევის პროცესზე. პირველ ეტაპზე, ატრიბუტების ექსპერტულ შეფასებებზე დაფუძნებული, HADC-ების სელექციის ინდექსია განმარტებული ასოცირებული ფაზი-ალბათური OWA (AsFPOWA) ოპერატორით. მეორე ეტაპზე, ოთხი მიზნობრივი ფუნქციაა განმარტებული FMOELTP-მოდელისთვის. მოდელის შეზღუდვები უზრუნველყოფს პრობლემის ღრმა დეტალიზაციას. FMOELTP-ის ზუსტი გადაწყვეტისთვის შექმნილია ექსილონი-შეზღუდვების მიდგომა.

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