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Informatics

Time-Cost Trade-off Method in Project Management

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ABSTRACT. The process of creating, researching and managing the projects is possible to be conducted by using network flows. The network theory and appropriate mathematical models provide such projects with great help. The following paper discusses one of the most significant current approaches in the method of managing construction and/or research projects, which is Time-Cost Trade-off method. In order to minimize costs in such projects the network model is built and programing realization is shown. *© 2018 Bull. Georg. Natl. Acad. Sci.*

Key words: network model, mathematical model

Networks are one of the best ways to graphically represent the flow of activities in the main project, such as construction, research and development projects. Therefore, network theories and their applications are stimulating and simplifying the process of managing such projects [1, 2].

During the second half of 1950s, two operation research methods (OR) were established, the PERT and CPM. The methods were being developed for during years and finally were formed into the method called PERT/CPM method [1, 3]. Nowadays it is widely spread and used in project management [2, 4].

PERT/CPM method has many qualities, but a fundamental property of PERT/CPM is CPM time-cost trade-off method, for two reasons. Firstly, this is a network optimization model; secondly, it clearly illustrates how this method is applicable in real life.

Before discussing the example of using this method we need to know some concepts:

Project network is the network that is used to represent the project. It consists of nodes (they usually have shapes of square or circle) and directed arcs, which connect two nodes with each other.

Critical Path is the longest path in the project network.

Crashing activity means taking measures connected to the costs, in order to consume time needed for work. These measures can be hiring temporary help, using special equipment etc.

Crashing the project means crashing particular activities in order to reduce time.

Immediate ancestor is the activity that must be finished before starting current activity.

Immediate successor is the activity that starts only after its previous activity is finished.

Prototype example.

Consider, the company is ready to start a new project. The customer gave 92 weeks time to finish the project. In case of delay the company will be fined.

In order to meet the deadline, the manager has to create some groups, that will work on different parts of the project.

Table 1. Initial project plan

Activity	Immediate predecessor(s)	Estimated duration (Weeks)
A	-	32
B	-	28
C	A	36
D	B	16
E	B	32
F	B	54
G	D	17
H	E, G	20
I	E, G	34
J	C ,F	18

In order to create the timetable of activities, the manager must discuss the time needed for every activity in the usual speed with the workers (Table 1).

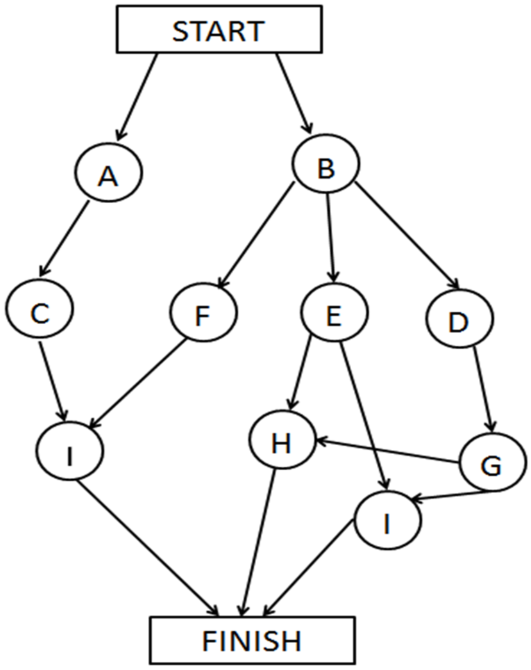

Fig. 1. Project Network.

Now we need to calculate probable paths and their lengths. These calculations are given in Table 2. According to them the length must be between 80 and 100.

Time needed is 287 weeks, which is a lot more than given deadline. Fortunately, some activities may be accomplished simultaneously, which reduces the time.

In Fig. 1. we see the project network. Due to A and B not having immediate predecessor, we implement new node "START", which is connected to both A and B.

In the same way, for J, I and H, which have no immediate successor we add new node "FINISH". This scheme clearly represents the project and its activities.

Table 2. Possible paths in the project and lengths

Path	*Length*
START→A→C→J→FINISH	86
START→B→D→G→H→FINISH	81
START→B→D→G→I→FINISH	95
START→B→E→H→FINISH	80
START→B→E→I→FINISH	94
START→B→F→J→FINISH	100

Time needed to reach the "FINISH" is the path length. Furthermore, the shortest path length will reach the "FINISH" at last. Therefore, the critical path is: START→B→F→J→FINISH. According to this, the approximate time for the project is 100 weeks, but as we already know the deadline is 92 weeks, so now the manager has to find the way to fit in given time with least expenses.

CPM method for time-cost trade-off decisions is for discovering opportunities of crashing in order to save time. The data, necessary for this, is given in the Table. The manager makes decisions according to it.

For example: Activity J.

Normal point: time=18 weeks, cost = 80 million dollars.

Crash point: time = 16 weeks, cost = 84 million dollars.

Maximum reduction of time =18-16= 2 weeks

Crash cost per week: $= \frac{84-80}{2} = 2$ million dollars.

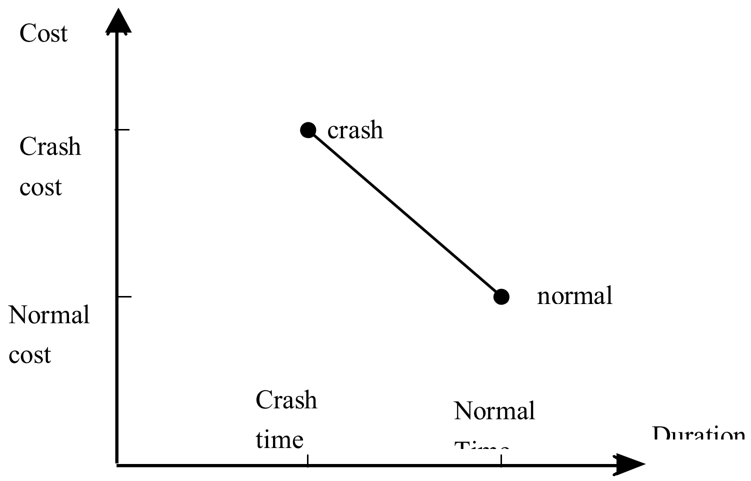

Fig.2. Relation between cost and duration of the project.

We study all activities in the same way (Table 3).

Table 3. Project network matrix

Activity	Time		Cost		Maximum reduction in time	Crash cost per week saved
	Normal	Crash	Normal	Crash		
A	32	28	160	180	4	5
B	28	25	125	146	3	7
C	36	31	170	210	5	8
D	16	13	60	72	3	4
E	32	27	135	160	5	5
F	54	47	215	257	7	6
G	17	15	90	96	2	3
H	20	17	120	132	3	4
I	34	30	190	226	4	9
J	18	16	80	84	2	2

According to Table 3 we have:

Sum of the Normal costs = 1345 million dollars and sum of crash costs = 1563 million dollars.

Let us state the program in the following way: Z is the Sum of expenses needed for crashing. Our goal is to minimize Z. The variables will be:

x_i = Reduction of the duration of activity j by crashing this activity, for $j = A, B, ..., J$, each 14 decisions on the right-hand side need to be restricted to non-negative values.

y_{FINISH} = Duration of the project (time needed to reach the "FINISH" node, which shows that the project ended) $y_{FINISH} \leq 92$

We must introduce the following additional variables:

$y_j = start\, time\, of\, the\, activity\, j \quad j = C, D, \ldots, J\; and\; given\, values\, of\;\; x_A, x_B, \ldots, x_J$.

For activity A and B there are no such variables, as the project starts with these activities. So we assign 0 to them from the beginning of the project. Because we consider node FINISH as one of activities, we assign a value to it.

For every activity (C, D, … , J, FINISH) and every immediate predecessor we have:

Start time of activity ≥ (start time + duration)

Duration of activity j = normal time of activity$-x_j$

Finally, we can obtain the LP model:

$$Z = 180x_A + 146x_B + \cdots + 84x_j$$

Constraints are:

1. Maximum Reduction: $x_A \leq 4, \quad x_B \leq 3, ..., \quad x_j \leq 2.$
2. Non negativity:

$$x_A \geq 0, \quad x_B \geq 0, ..., \quad x_N \geq 0$$
$$y_C \geq 0, \ldots, y_N \geq 0, y_{FINISH} \geq 0.$$

3. Start-time: One immediate predecessor:

$$y_C \geq 0 + 32 - x_A$$
$$y_F \geq 0 + 28 - x_B$$
$$y_E \geq 0 + 28 - x_B$$
$$\dots$$
$$y_H \geq y_E + 32 - x_E.$$

Two immediate predecessors:

$$y_H \geq y_G + 17 - x_G$$
$$y_H \geq y_E + 20 - x_E.$$
$$\dots$$
$$y_{FINISH} \geq y_J + 18 - x_J$$
$$y_{FINISH} \geq y_H + 20 - x_H$$
$$y_{FINISH} \geq y_I + 34 - x_I.$$

4. Project duration:

$$y_{FINISH} \leq 92$$

According to the data given in Table 3 and described algorithm, we get:

Table 4. Solution Table

	Start time	Time reduction	Finish time
A	0	0	32
B	0	2	26
C	32	0	68
D	26	0	42
E	26	0	58
F	0	0	54
G	42	1	58
H	58	0	78
I	58	0	92
J	74	0	92
FINISH	92	<=	92

Z	17		

Where “time reduction” and “start time” columns are x_i and y_i variables respectively. Z is the function, which must be minimized.

According to the results the manager must reduce time for activities: B and G by 2 and 1 weeks respectively. The minimum additional cost will be 17 million dollars and project will finish in 92 weeks as it was desired.

ინფორმატიკა

პროექტების მართვა შესრულების დროისა და დანახარჯების ოპტიმიზაციის მიხედვით

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