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Informatics

To the Analogy between Information Theory and Physics

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ABSTRACT. The work deals with the analogy between information theory and physics, namely mechanics, atomic physics and quantum mechanics. Some of the issues are more simplified in the theory of information that demonstrates the primacy of this theory. Generally such analogues are noticed by scientists. In mechanics the analogy between the exact differential of mechanical energy and the exact differential of information, analogy between the energy conservation law and information conservation law are considered. The analogy of proofs is almost the same in these two great subjects. In atomic physics, the analogy between the energy of light quants and the energy of information quants is treated. The difference lies in the fact that the Planck constant equals one for the case of information. This simplifies the issue under study. The analogy between Heisenberg's Uncertainty Principle and the same principle from information theory is treated as well. It is demonstrated that in the similar formulas the Plank constant is substituted by the value of one that finally simplifies the issue under study. In quantum mechanics it is asserted that coordinates and speed of elementary particles cannot be measured simultaneously. Similar analogy takes place during transmission of information when the transmission is performed by errors. At this point, the location and speed of transmission of the information quants are uncertain, so we are not able to talk about coordinates and speeds at the given moment of time. Here the coordinates and speed of information quants may be characterized by so called Heming distance. An example is discussed when in case of transmission of information, the external fields are acting on the information sequence of length n, thus distorting the coding word. This example can be compared with ejection of electrons from the atoms under the action of external forces. *© 2019 Bull. Georg. Natl. Acad. Sci.*

Key words: information, quantum, uncertainty principles, conservation laws

As a result of our studies, the analogy between information theory and physics, namely mechanics, atomic physics and quantum mechanics was revealed [1].

In our opinion, many problems are formulated more simply in the theory of information that testifies the primacy of this theory.

Total Differential and Conservation Laws

As an example, let us consider several issues from the theory, namely the total differential of energy and information and the laws of conservation.

For the case of energy

$$du = Tds - Pdv ,$$

where T is absolute temperature, P – pressure, s – entropy and v – volume.

The partial derivatives in this formula are:

$$\frac{\partial u}{\partial s} = T\ , \quad \frac{\partial u}{\partial v} = -P\ .$$

In the case of information

$$dI = d(X:Y) = \frac{\partial I}{\partial S(x)} dS(x) - \frac{\partial I}{\partial S_y(x)} dS_y(x)\ ,$$

where $d(X:Y)$ is a mutual information of information source (entrance of the channel) and the entry of the recipient (channel outlet), $S(x)$ – entropy of source output $S_y(x)$ – the conditional entropy of entrance of the channel, when channel output Y is known.

The partial derivatives in this formula are $\frac{\partial I}{\partial S(x)} = 1,\ \frac{\partial I}{\partial S_y(x)} = -1$ as entropic and information variables are the same for it.

As for the conservation laws, all the conservation laws may be approved in the same way as energy conservation law [2]. For this purpose it is necessary to determine the Lagrangian function for this system and to obtain complete derivative of it. For information, the derivative of Lagrangian function is written in the following way:

$$\frac{dL}{dt} = \sum_i \frac{\partial L}{\partial d_i} R_i + \sum_i \frac{\partial L}{\partial R_i} R_i^{'}\ ,$$

where d is Hemming's distance and R – information transmission speed.

Lagrangian for information is given by $I_y(x)$ that represents loss of information caused by interference in the channel. This is an analog to the work performed with the change in energy. This can be proved similarly to the proof given in [2].

Similarity with Atomic Physics

According to Planck [3] the energy of light quantum emitted or absorbed by a body is given by the formula

$$\varepsilon = h\nu,$$

where h is Planck constant, ν – frequency of emission.

The same is in the formula for determining energy of information quantum:

$$\varepsilon = h\nu.$$

Here the coefficient h is equal to 1, ν – frequency of dropping out bits from transmitted information that is caused by interference.

Let us consider briefly the analogy with the Heisenberg uncertainty principle given in [1].

Minimum uncertainty that can be added to the information, is equal to one bit. This allows to detect one error in information transmitted.

To keep the rate of transfer unchanged it should be increased by 1bit/sec. At this moment information pulse increment will be equal to

$$\Delta P = \Delta m \Delta R\ ,$$

where m is mass of information, R – rate of transfer, and increment of distance in Hemming metric would be $\Delta x = 1$. This leads to

$$\Delta P \Delta x = 1 \cdot 1 = 1.$$

If t – fold error is expected during transmission, the distance that background information will pass by Hemming metric would be equal $\Delta x = t$. For t -fold error to be recovered the background information should be increased by at least $t \cdot k$ bits, where k is an integer number. Here, also, the rate of transfer to be kept unchanged, it should be increased in the same amount. In this case, in general, it can be written as:

$$\Delta P \Delta x \geq 1.$$

The Heisenberg uncertainty principle, as we know, looks as follows:

$$\Delta P \Delta x \geq h.$$

This result once again confirms that for the case of information the Plank's constant is equal to 1.

Analogy with Quantum Mechanics

As for the similarity among the Theory of Information and Quantum Mechanics [4], it should be mentioned that mathematical foundation of Quantum Mechanics is based on the statement that the description of the state of a system can be achieved by introducing a certain function of the coordinates $\psi(q)$. The square of this function $|\psi(q)|^2\, dq$ is the probability for the coordinate to be found in some dq-size environment of configuration space, as a result of measurement.

It is known that in quantum mechanics the coordinate and speed of elementary particle could not be measured simultaneously, which means that we cannot obtain certain values at the same time.

Similar situation is in the case of transmission of information when the transmission is with error i.e. the code bits (quanta) in codewords has been changed. In this case the state of the bits, particularly their coordinate and the transmission rate are not defined. Therefore, we are not able to talk definitely about coordinate and speeds at the given time. The movement of information bits may be characterized by the so-called Hemming's distance.

In the Theory of Information this means that transmitted information sequence of the length n, in case of linear coding may occur in one of the neighboring classes.

And now let us consider an important analogy between Quantum Mechanics and Information Theory.

In Quantum Mechanics we consider physical quantity f as the characteristic of the state of quantum system. The values that may be given to that physical quantity are called the eigenvalues and their aggregate the range of values of physical quantity.

Suppose, we are given two such values f_n and f_m. According to relation given in [4]

$$(f_n - f_m)\int \psi_n \psi_m^* dq = 0,$$

where ψ_m^* complex conjugate of ψ_m. Those values, themselves, are the eigenvalues of the wave function and thus different eigenvalues are mutually orthogonal:

$$\int \psi_n \psi_m^* dq = 0$$

for $f_n \neq f_m$.

For normalized functions, the result obtained is written as follows:

$$\int \psi_n \psi_m^* dq = \delta_{nm}$$

where $\delta_{nm} = 1$ whenever $m = n$. Otherwise $\delta_{nm} = 0$.

As for the arbitrary state function $\psi = \sum a_n \psi_n$ the following relation is hold:

$$a_n = \int \psi_n \psi_m^* dq$$

for any $n \neq m$.

In the Theory of Information the analogy considered above will be written as follows [3]:

$$\int \varphi_i(t)\varphi_j^*(t)dt = \delta_{ij}$$

where φ_i and φ_j are orthogonal and normalized functions. $\delta_{ij} = 1$ whenever $i = j$. Otherwise $\delta_{ij} = 0$.

The same is for code functions:

$$x(t) = \sum_i x_i \varphi_i(t),$$

where the coefficient

$$x_i = \int x(t)\varphi_i^*(t)dt.$$

There are also other analogies. For example, let us give analogy between Perturbation theory applied in Quantum Mechanics [4] and the Theory of Coding [5].

In general, when these theories are compared with each other, there is a difference between the mathematical apparatus of research but the content of the study does not differ as much.

Consider an example, when in the case of transmitting information, the informative sequence is under effect of external fields causing the distortion of the coded word (binary sequence of length n). This example can be aligned with the case of knock out of electrons from the atoms of matter with the assistance of external forces.

The code that corrects the erroneous bits in case of transmission of information can be considered in the form of two mutually orthogonal subspaces [5], one of which is called a coding subsurface, another checking subspace. Let us denote their base matrices by G and H, respectively. Here G is a coding matrix and H decoding (checking) matrix.

If we multiply coded vector received during transmission without error by H matrix, we receive zero. In case of errors, the result would be a linear combination of its columns. In the Theory of Coding it is called a syndrome. The code should be built in a way that the error were corrected by means of the syndrome (syndromes). (Syndrome is an address or identifier of class and error).

For example, consider (7,4) binary cyclical code [5]. The length of this code is 7, and the length of the information sequence is 4. It corrects any particular bug in the code sequence.

The coding matrix would be written as

$$G = \begin{vmatrix} 1101000 \\ 0110100 \\ 0011010 \\ 0001101 \end{vmatrix}$$

and decoding (checking) matrix would be

$$H = \begin{vmatrix} 0010111 \\ 0101110 \\ 1011100 \end{vmatrix}.$$

If during transmission one error occurs in any digit of code word than the syndrome multiplied by H matrix will take the form of one of the columns, which indicates a single bug and its location in codeword.

As we can see, in case of damaging information atom the procedure to be applied for recovery is simpler than in the case of atom of matter.

If we consider continuous codes, its theory is directly related to the theory of motion of particle studied in Quantum Mechanics. Above, we talked about primacy of information and connected it to supernatural creations [1], where everything is absolute and no subject to change. Therefore, there is no time there (Time is characteristic of variability).

Due to this, there is a lot of conservation laws. They are all absolute and are not considered directly in time. Therefore, our aspiration should be directed towards the discovery of these laws. For example, we found two - the law of Conservation of Information and the law of Conservation of its momentum [1].

If the change of an event is not reduced directly to time, as in the case of speed of change of coordinate in Quantum Mechanics, this already brings us closer to Almighty.

When the time tends to zero, and the speed and the distance to the infinity, we are not able to speak about time in the literal sense. Here there is no reason to talk about the distance between two points. In our point of view, in this case we should introduce the notion of separation. Thus, it is necessary to determine its unit. For example, our points of view are strongly distant from each other. Introduction of meters and kilometers here is no use. As the separation is general notion and it contains some amount of uncertainty, it could be measured in entropic units, i.e. in bits. Thus, we would be able to mention directly that the separation between these two different points of view is equal to one or another number of bits.

ინფორმატიკა

ინფორმაციის თეორიასა და ფიზიკას შორის არსებული ანალოგიების შესახებ

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(წარმოდგენილია აკადემიის წევრის ა. ფრანგიშვილის მიერ)

ნაშრომში განხილულია ანალოგია ინფორმაციის თეორიასა და ფიზიკას შორის, კერძოდ, მექანიკასთან, ატომურ ფიზიკასა და კვანტურ მექანიკასთან. ზოგიერთი საკითხი ინფორმაციის თეორიაში უფრო გამარტივებულია, რაც მეტყველებს ამ თეორიის პირველადობაზე. საერთოდ ასეთი ანალოგიები მეცნიერების მიერ შემჩნეულია. მექანიკიდან განხილულია ანალოგია ენერგიისა და ინფორმაციის სრულ დიფერენციალს შორის, ენერგიისა და ინფორმაციის შენახვის კანონს შორის. მტკიცებების ანალოგია თითქმის ერთნაირია ამ ორ დიდ საგანს შორის. ატომური ფიზიკიდან მოყვანილია ანალოგია სინათლისა და ინფორმაციული კვანტების ენერგიებს შორის. მოყვანილია ჰაიზენბერგის განუზღვრელობის პრინციპის ანალოგია ინფორმაციის თეორიის იმავე პრინციპთან. ნაჩვენებია, რომ ერთსა და იმავე ფორმულებში პლანკის მუდმივას ცვლის ერთი, რაც, ჩვენი აზრით, ამარტივებს განხილულ საკითხს. კვანტურ მექანიკაში დამტკიცებულია, რომ ელემენტარული ნაწილაკის კოორდინატი და სიჩქარე არ შეიძლება ერთდროულად ზუსტად იყოს გაზომილი. არსებობს ასეთივე ანალოგია ინფორმაციის გადატანის დროსაც, როცა გადაცემა წარმოებს შეცდომით. ამ დროს განუზღვრელობას წარმოადგენს ინფორმაციის კვანტების მდებარეობა და გადაცემის სიჩქარე, ამიტომ არ შეიძლება განვსაზღვროთ, დროის აღებულ მომენტში, კოორდინატი და სიჩქარე. ამ დროს ინფორმაციის კვანტების კოორდინატი და სიჩქარე შეიძლება დახასიათდეს ე. წ. ხემინგის მანძილით. განხილულია მაგალითი, როდესაც ინფორმაციის გადაცემის შემთხვევაში, **n** სიგრძის ინფორმაციულ თანმიმდევრობაზე მოქმედებს გარე ველები, რომლებიც იწვევენ კოდური სიტყვის დამახინჯებას. ეს მაგალითი შეიძლება შევუსაბამოთ ნივთიერების ატომიდან, გარე ველების ზემოქმედებით, ელექტრონების ამოყრას.

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