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On High-Frequency Oscillations Spectra of Superfluid ^{3}He-A and ^{3}He-B

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ABSTRACT. In strong magnetic field the fundamental differences between the properties of superfluid ^{3}He-A and ^{3}He-B phases are revealed. The spin dynamics of the phases is investigated. It is demonstrated that high frequency part of dipole-dipole potencials has essentially different dependence on rapid angular variable $\alpha(t)$. In particular, it is shown that in ^{3}He-A phase this dependence even in different cases is always of integer multiple type, when in ^{3}He-B phase, except integer, there exist fraction multiples. *© 2019 Bull. Georg. Natl. Acad. Sci.*

Key words: superfluid ^{3}He-A and ^{3}He-B phases

In the present paper the fundamental difference between the properties of superfluid ^{3}He-A and ^{3}He-B will be discussed. Our main goal is to describe the cause of the mentioned dependence. To analize this situation we address first the order parameters of ^{3}He-A and ^{3}He-B, which are based on the orts $\left(\hat{d},\hat{e},\hat{s}\right)$ and $\left(\hat{m},\hat{n},\hat{l}\right)$ in the spin and orbital spaces.

The superfluid ^{3}He-A in the "antiferromagnetic" configuration is described by the order parameter

$$A_{\mu i}=\frac{1}{2}\left\{\Delta_{\uparrow\uparrow}\left(\hat{d}+i\hat{e}\right)_{\mu}+\Delta_{\downarrow\downarrow}\left(\hat{d}-i\hat{e}\right)_{\mu}\right\}\left(\hat{m}+i\hat{n}\right)_{i}e^{i\chi}.$$

In the zero magnetic field case $\Delta_{\uparrow\uparrow}=\Delta_{\downarrow\downarrow}=\Delta_A\left(T\right)$, so that

$$A_{\mu i}=\Delta_A\left(T\right)d_{\mu}\left(\hat{m}+i\hat{n}\right)_{i}e^{i\chi}.$$

The superfluid ^{3}He-B is described by the order parameter

$$A_{\mu i}=\left\{\frac{1}{2}\left[\Delta_{\uparrow\uparrow}\left(\hat{d}+i\hat{e}\right)_{\mu}\left(\hat{m}-i\hat{n}\right)_{i}+\Delta_{\downarrow\downarrow}\left(\hat{d}-i\hat{e}\right)_{\mu}\left(\hat{m}+i\hat{n}\right)_{i}\right]+\Delta_{\uparrow\downarrow}\hat{s}_{\mu}\hat{l}_{i}\right\}e^{i\chi}.$$

In the zero magnetic field case $\Delta_{\uparrow\uparrow}=\Delta_{\downarrow\downarrow}=\Delta_{\uparrow\downarrow}=\Delta_B\left(T\right)$ and the order parameter of ^{3}He-B is

$$A_{\mu i}=\Delta_B\left(T\right)\left(d_{\mu}m_i+e_{\mu}n_i+s_{\mu}l_i\right)e^{i\chi}.$$

In using the relative spin-orbital 3D rotation matrix $R(\hat{v},\theta)$ we have:

$$d_\mu = R_{\mu j} m_j, \; e_\mu = R_{\mu j} n_j, \; s_\mu = R_{\mu j} l_j,$$

so that

$$A_{\mu i} = \Delta_B(T) R_{\mu j} \left(m_i m_j + n_i n_j + l_i l_j \right),$$

because of the completeness of the orts $\left(\hat{m},\hat{n},\hat{l}\right)$

$$m_i m_j + n_i n_j + l_i l_j = \delta_{ij}$$

and for ^{3}He-B

$$A_{\mu i} = \Delta_B(T) R_{\mu i}(\hat{v},\theta) e^{i\chi}.$$

Alternatively instead of the variables $(\hat{v},\theta)$, the combination of the Euler triple angles $(\alpha_s,\beta_s,\gamma_s)$ and $(\alpha_L,\beta_L,\gamma_L)$ will be preferred.

After having described the main aspects of the order parameters, one has to introduce, as an important content of our consideration, the dipole-dipole potentials of ^{3}He-A and ^{3}He-B.

Let us consider, in the first place, the dipole-dipole potential of ^{3}He-A.

$$u_A = -\frac{1}{2}\chi_N \left(\frac{\Omega_A}{g}\right)^2 f_A \text{ , where } f_A = \left(\hat{d}\cdot\hat{l}\right)^2, \; \vec{H}_0 = H_0 \hat{z} \tag{1}$$

$$\begin{aligned} \hat{d} &= \vec{R}(\alpha,\beta,\gamma)\hat{x} \\ \hat{l} &= l_\perp \hat{x} + l_z \hat{z} \end{aligned} \tag{2}$$

$$\begin{aligned} l_z &= \left(\hat{z}\cdot\hat{l}\right) = \cos\varphi \\ l_\perp &= \sqrt{1-l_z^2} = \sin\varphi \\ 0 &\le \varphi \le \pi. \end{aligned} \tag{3}$$

The spin-orbital function is:

$$f_A = \left(d_x l_\perp + d_z l_z\right)^2 = d_x^2 \sin^2\varphi + d_z^2 \cos^2\varphi + d_x d_z \sin 2\varphi. \tag{4}$$

In the Euler angles representation with (α,β,γ):

$$d_x = R_{xx} = \frac{1}{2}\left\{(1+\cos\beta)\cos(\alpha+\gamma) - (1-\cos\beta)\cos(\alpha-\gamma)\right\}, \tag{5a}$$

$$d_z = R_{zx} = -\sin\beta\cos\gamma. \tag{5b}$$

In the strong magnetic field case $\left(\omega_0 = gH_0 \gg \Omega_A\right)$ the spin-orbital function can be decomposed as

$$f_A = \overline{f}_A + \tilde{f}_A(t), \tag{6}$$

where the time-averaged (van der Pol) contribution is:

$$\overline{f}_A(\beta,\Phi) = \frac{1}{4}\left\{\left[1+\cos^2\beta + \frac{1}{2}(1+\cos\beta)^2\cos 2\Phi\right] l_\perp^2 + 2 l_z^2 \sin^2\beta\right\}. \tag{7}$$

With $\Phi = \alpha + \gamma$ being a slow phase and rapidly time-oscillating part is given as:

$$\tilde{f}_A(t) = \frac{1}{4}\left\{2l_z^2 \sin^2\beta\cos 2\gamma - \left[\sin^2\beta\left(\cos 2\alpha + \cos 2\gamma\right) - \frac{1}{2}(1-\cos\beta)^2\cos(2\alpha - 2\gamma)\right]l_\perp^2 - \right.$$

$$\left. -2\sin\beta\left[2\cos\beta\cos\alpha + (1+\cos\beta)\cos(\alpha+2\gamma) - (1-\cos\beta)\cos(\alpha-2\gamma)\right]l_\perp l_z\right\}. \quad (8)$$

After having used the substitution $\gamma = -\alpha + \Phi$, from Eq. (8) it is concluded that in the (α, Φ) representation

$$\tilde{f}_A(t) = \tilde{f}_A(\alpha,\Phi) = \frac{1}{4}\left\{2l_z^2 \sin^2\beta\cos(2\alpha - 2\Phi)\right.$$

$$-\left[\sin^2\beta\left(\cos 2\alpha + \cos(2\alpha - 2\Phi)\right) - \frac{1}{2}(1-\cos\beta)^2\cos(4\alpha - 2\Phi)\right]l_\perp^2 -$$

$$\left. -2\sin\beta\left[2\cos\beta\cos\alpha + (1+\cos\beta)\cos(\alpha - 2\Phi) - (1-\cos\beta)\cos(3\alpha - 2\Phi)\right]l_\perp l_z\right\}. \quad (9)$$

In Eq. (9) two angular variables appear: rapid $\alpha(t)$ and slow Φ. To exclude slow variable, it is necessary to know the meaning of its stationary value. This value is established as a result of minimization of van der Pol part of spin orbital function. In the case $\Phi = \alpha + \gamma$, the stationary quantity $\Phi_{st} = (0, \pi)$. After substitution of this quantity, we have:

$$\tilde{f}_A(t) = \tilde{f}_A(\alpha, \Phi_{st}) = \tilde{f}_A(\alpha, 2\alpha, 3\alpha, 4\alpha). \quad (10)$$

On the other hand, at $\hat{\Phi} = \alpha + 2\gamma$ being a slow phase, in using the substitution $2\gamma = -\alpha + \tilde{\Phi}$ instead of Eq. (8) it is concluded that

$$\tilde{f}_A\left(\alpha,\tilde{\Phi}\right) = \frac{1}{4}\left\{\left[\frac{1}{2}(1+\cos\beta)^2\cos\left(\alpha+\tilde{\Phi}\right) - \sin^2\beta\cos\left(\alpha - \tilde{\Phi}\right)\right]l_\perp^2 - \right.$$

$$-4l_\perp l_z \sin\beta\cos\beta\cos\alpha - \left[l_\perp^2\sin^2\beta\cos 2\alpha - 2l_\perp l_z\sin\beta(1-\cos\beta)\cos\left(2\alpha - \tilde{\Phi}\right)\right] +$$

$$\left. +2l_z^2\sin^2\beta\cos\left(\alpha - \tilde{\Phi}\right) + \frac{1}{2}l_\perp^2(1-\cos\beta)^2\cos\left(3\alpha - \tilde{\Phi}\right)\right\}. \quad (11)$$

In this case the stationary value depends on the signon of l_z and $\tilde{\Phi}_{st}\begin{cases}\pi & l_z > 0\\ 0 & l_z < 0\end{cases}$. Substituting this value in Eq. (11) we concluded that

$$\tilde{f}_A\left(\alpha, \tilde{\Phi}_{st}\right) = \tilde{f}_A(\alpha, 2\alpha, 3\alpha). \quad (12)$$

In comparing Eq.(10) and Eq.(12), it is noticed that at $\tilde{\Phi} = \alpha + 2\gamma$ as a slow phase, the rapidly time-oscillating part of $\tilde{f}_A(t)$ lost the 4α component.

Now we pass to consider the structure of the dipole-dipole potential of ^{3}He-B phase

$$u_B = \frac{2}{15}\chi_B\left(\frac{\Omega_B}{g}\right)^2 f_B, \quad \text{with} \quad f_B = \left(T_r R - \frac{1}{2}\right)^2. \quad (13)$$

The 3D matrix $\vec{R}$ describes relative rotation of the spin and orbital spaces: $\vec{R} = \vec{R}_s \vec{R}_L^{-1}$. In what follows, the triples of the Euler angles $(\alpha_s, \beta_s, \gamma_s)$ and $(\alpha_L, \beta_L, \gamma_L)$ will be used. The crucial role will be played by the variables:

$$\alpha = \alpha_s - \alpha_L, \quad \gamma = \gamma_s - \gamma_L, \quad s_z = \cos\beta_z, \quad l_z = \cos\beta_L, \quad s_\perp = \sqrt{1-s_z^2}\,. \tag{14}$$

The spin-orbital function f_B in the strong magnetic field case can be decomposed as usually:

$$f_B = \bar{f}_B + \tilde{f}_B(t)\,. \tag{15}$$

Of special interest is the case of $\alpha + 2\gamma = \tilde{\Phi}$, where

$$\begin{aligned}\tilde{f}_B(t) = {} & S_\perp^2 l_\perp^2 \left\{\frac{3}{4}(\cos 2\alpha + \cos 2\gamma) + \cos(\alpha+\gamma) + \cos(\alpha-\gamma)\right\} + \\ & + \left(S_z l_z - \frac{1}{2}\right)\left\{2S_\perp l_\perp(\cos\alpha + \cos\gamma) + (1+S_z)(1+l_z)\cos(\alpha+\gamma) + (1-S_z)(1-l_z)\cos(\alpha+\gamma)\right\} + \\ & + \frac{1}{8}\left\{(1+S_z)^2(1+l_z)^2\cos(2\alpha+2\gamma) + (1-S_z)^2(1-l_z)^2\cos(2\alpha-2\gamma)\right\} + \\ & + \frac{1}{2}S_\perp l_\perp\left\{(1+S_z)(1+l_z)\left[\cos\alpha + \cos\gamma + \cos(2\alpha+\gamma)\right] + (1-S_z)(1-l_z)\left[\cos\alpha + \cos\gamma + \right.\right. \\ & \left.\left. + \cos(2\alpha-\gamma) + \cos(\alpha-2\gamma)\right]\right\}.\end{aligned} \tag{16}$$

Excluding γ according to the substitution $\gamma = \dfrac{-\alpha + \tilde{\Phi}}{2}$ it is found that in the $(\alpha, \tilde{\Phi})$ representation

$$\begin{aligned}\tilde{f}_B(\alpha,\tilde{\Phi}) = {} & \cos\alpha\left\{3S_z l_z\sqrt{(1-S_z^2)(1-l_z^2)}\right\} + \cos(\alpha+\tilde{\Phi})\left\{\frac{1}{8}(1+S_z)^2(1+l_z)^2\right\} + \\ & + \cos(\alpha-\tilde{\Phi})\left\{\frac{3}{4}(1-S_z^2)(1-l_z^2)\right\} + \cos 2\alpha\left\{\frac{3}{4}(1-S_z^2)(1-l_z^2)\right\} + \\ & + \cos(2\alpha-\tilde{\Phi})\left\{\frac{1}{2}\sqrt{(1-S_z^2)(1-l_z^2)}(1-S_z)(1-l_z)\right\} + \cos(3\alpha-\tilde{\Phi})\left\{\frac{1}{8}(1-S_z)^2(1-l_z)^2\right\} + \\ & + \cos\frac{\alpha+\tilde{\Phi}}{2}\left\{(1-S_z^2)(1-l_z^2) + \left(S_z l_z - \frac{1}{2}\right)(1+S_z)(1+l_z)\right\} + \cos\frac{\alpha-\tilde{\Phi}}{2}\left\{3S_z l_z\sqrt{(1-S_z^2)(1-l_z^2)}\right\} + \\ & + \cos\frac{3\alpha-\tilde{\Phi}}{2}\left\{(1-S_z^2)(1-l_z^2) + \left(S_z l_z - \frac{1}{2}\right)(1-S_z)(1-l_z)\right\} + \\ & + \cos\frac{3\alpha+\tilde{\Phi}}{2}\left\{\frac{1}{2}\sqrt{(1-S_z^2)(1-l_z^2)}(1+S_z)(1+l_z)\right\} + \\ & + \cos\frac{5\alpha+\tilde{\Phi}}{2}\left\{\frac{1}{2}\sqrt{(1-S_z^2)(1-l_z^2)}(1-S_z)(1-l_z)\right\}.\end{aligned} \tag{17}$$

In this case $\tilde{\Phi}_{st} = \pi$ and consequently

$$\tilde{f}_B(t) = \tilde{f}_B(\alpha, \tilde{\Phi}_{st}) = \tilde{f}_B\left(\alpha, 2\alpha, 3\alpha, \frac{1}{2}\alpha, \frac{3}{2}\alpha, \frac{5}{2}\alpha\right). \tag{18}$$

It is seen that in contrast to $\tilde{f}_A(t)$ in $\tilde{f}_B(t)$ the fractional components appear, which are absent in ^{3}He-A case.

This fundamental difference between the behavior of ^{3}He-A and ^{3}He–B at $\hat{\Phi} = \alpha + 2\gamma$ can be found in the publication by G.Kharadze, N. Suramlishvili and G. Vachnadze [1,2].

ფიზიკა

^{3}He-A და ^{3}He-B ზედენადი ფაზების მაღალი სიხშირული ოსცილაციების სპექტრის შესახებ

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გამოვლენილია ^{3}He-A და ^{3}He-B ზედენადი ფაზების თვისებათა ფუნდამენტური ხასიათის განსხვავება. განხილულია ამ ფაზებში სპინური დინამიკა ძლიერ მაგნიტურ ველში. ნაჩვენებია, რომ დიპოლ-დიპოლური პოტენციალის მაღალი სიხშირული (ოსცილაციური) ნაწილი მნიშვნელოვნად განსხვავებულადაა დამოკიდებული სწრაფ კუთხურ ცვლადზე. ნაჩვენებია, რომ ^{3}He-A ფაზაში ეს დამოკიდებულება სხვადასხვა შემთხვევაშიც კი ყოველთვის მთელი რიცხვის ჯერადია, მაშინ როდესაც ^{3}He-B ფაზაში მთელი რიცხვოვანი წევრების გარდა არსებობს წილადი რიცხვების ჯერადი წევრებიც.

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