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Mathematics

On the Investigation of One-Dimensional Models of Thermo-Electro-Magneto-Elastic Bars

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In this paper a linear dynamic model of thermo-electro-magneto-elastic bars made of anisotropic inhomogeneous material is considered. An algorithm of approximation of the corresponding three-dimensional initial-boundary value problem by one-dimensional problems for bar with variable cross-section is constructed, when density of surface force, and components of the electric displacement and magnetic induction along the outward normal vector of the boundary are given on the lateral face surfaces of the bar. The obtained one-dimensional initial-boundary value problems are investigated in suitable function spaces. Moreover, it is proved that the sequence of vector-functions of three space variables, restored from the solutions of the constructed one-dimensional problems, converges, in the corresponding function space, to the exact solution of the original three-dimensional initial-boundary value problem and, under additional conditions, the rate of convergence is estimated. *© 2019 Bull. Georg. Natl. Acad. Sci.*

Thermo-electro-magneto-elastic solids, initial-boundary value problems, bars with variable cross-section, dimensional reduction algorithms, error estimates

Piezoelectric materials are the most popular materials currently being used in smart structures, which often contain bars, and hence, it is important to construct and investigate algorithms of approximation of three-dimensional models of bars by one-dimensional problems. After discovery of the direct and converse piezoelectric effects, one of the first theoretical models of piezoelectricity was constructed by Voigt [1] and further various models of piezoelectric solids were constructed and investigated by various authors (see [2-4] and the references given therein). In order to construct one-dimensional models of thermo-electro-magneto-elastic bars we use the generalization of the approximation method that was suggested by Vekua [5] for plates with variable thickness in the classical theory of elasticity. Various two-dimensional models constructed by Vekua were collected in his monograph [6]. The estimates of the order of approximation of the static three-dimensional problem by two-dimensional ones, constructed in [5], first were obtained in the spaces of classical regular functions [7], and the reduced two-dimensional models for thin shallow shells, constructed in [6], were investigated in Sobolev spaces in [8]. Later on, various static and dynamic hierarchical models were constructed and investigated applying Vekua's dimensional reduction methods and their generalizations in [9-15].

In the present paper, we construct and investigate an algorithm of approximation of three-dimensional dynamic model for thermo-electro-magneto-elastic bar with variable rectangular cross-section, which may vanish on one of the butt ends, by one-dimensional problems. We consider the variational formulation of the initial-boundary value problem corresponding to the three-dimensional model of thermo-electro-magneto-elastic bar made of inhomogeneous anisotropic material, when temperature vanishes on the entire boundary. Applying the variational formulation we construct a hierarchy of one-dimensional problems approximating the three-dimensional one, when the mechanical displacement, electric and magnetic potentials vanish on the butt end with positive area, and densities of surface force and normal components of the electric displacement and magnetic induction are given on the remaining part of the boundary. We investigate the constructed one-dimensional initial-boundary value problems in suitable spaces of vector-valued distributions with respect to the time variable with values in weighted Sobolev spaces. Moreover, we prove that the sequence of vector-functions of three space variables, restored from the solutions of the constructed one-dimensional problems, converges to the solution of the original three-dimensional problem and, under additional regularity conditions, we estimate the rate of convergence.

We denote by $W^{r,2}(D)=H^r(D)$ and $H^r(\hat{\Gamma})$, $r\in\mathbf{R}$, the Sobolev spaces of order r based on the spaces $H^0(D)=L^2(D)$ and $H^0(\hat{\Gamma})=L^2(\hat{\Gamma})$ of square-integrable functions, respectively, where $D\subset\mathbf{R}^p$, $p\in\mathbf{N}$, is a bounded Lipschitz domain [16] and $\hat{\Gamma}\subset\partial D$ is a Lipschitz surface. We denote by $\mathbf{H}^r(D)=[H^r(D)]^3$, $\mathbf{H}^r(\hat{\Gamma})=[H^r(\hat{\Gamma})]^3$, $\mathbf{L}^2(D)=[L^2(D)]^3$, $\mathbf{L}^s(\hat{\Gamma})=[L^s(\hat{\Gamma})]^3$, $s\geq 1$, $r,s\in\mathbf{R}$, the corresponding spaces of vector-valued functions. The trace operators are denoted by $tr_{\hat{\Gamma}}:H^1(D)\to H^{1/2}(\hat{\Gamma})$ and $\mathbf{tr}_{\hat{\Gamma}}:\mathbf{H}^1(D)\to\mathbf{H}^{1/2}(\hat{\Gamma})$. For a Banach space X, we denote by $C([0,T];X)$ the space of continuous functions on $[0,T]$ with values in X. $L^q(0,T;X)$, $1\leq q\leq\infty$, is the space of such measurable functions $g:(0,T)\to X$ so that $\|g(t)\|_X\in L^q(0,T)$ and the generalized derivative of g is denoted by $g'=dg/dt$ [17].

Let us consider a thermo-electro-magneto-elastic bar $\overline{\Omega}$ with variable rectangular cross-section with thickness or width that may vanish on the lower butt end, i.e. the initial configuration $\overline{\Omega}$ of the bar is a closure of Lipschitz domain of the following form

$$\Omega=\left\{(x_1,x_2,x_3)\in\mathbf{R}^3;\, h_1^-(x_3)<x_1<h_1^+(x_3),\, h_2^-(x_3)<x_2<h_2^+(x_3),\quad x_3\in I=(h_3^-,h_3^+)\right\},$$

where $h_1^{\pm},h_2^{\pm}\in C^0(\bar{I})\cap C^{0,1}_{loc}(I)$ are continuous on $\bar{I}$ and Lipschitz continuous in I, $h_1^+(x_3)>h_1^-(x_3)$ and $h_2^+(x_3)>h_2^-(x_3)$, for $x_3\in(h_3^-,h_3^+]$, $h_1^{\pm}(x_3)$ and $h_2^{\pm}(x_3)$ are equal or different for $x_3=h_3^-$. The upper butt end of the bar $\overline{\Omega}$, defined by the equation $x_3=h_3^+$ we denote by $\Gamma_0=\{x\in\mathbf{R}^3;\ h_\alpha^-(x_3)\leq x_\alpha\leq h_\alpha^+(x_3),\ \alpha=1,2, x_3=h_3^+\}$ and the remaining part of the boundary we denote by Γ_1.

We assume that thermoelastic piezoelectric bar consists of general inhomogeneous anisotropic material with the mass density ρ in the initial configuration, the elasticity tensor $(c_{ijpq})_{i,j,p,q=1}^3$, the piezoelectric $(\varepsilon_{pij})_{i,j,p=1}^3$ and piezomagnetic $(b_{pij})_{i,j,p=1}^3$ coefficients, the stress-temperature tensor $(\lambda_{ij})_{i,j=1}^3$, the permittivity $(d_{ij})_{i,j=1}^3$ and permeability $(\zeta_{ij})_{i,j=1}^3$ tensors, the coupling coefficients connecting electric and magnetic fields $(a_{ij})_{i,j=1}^3$, the thermal conductivity tensor $(\eta_{ij})_{i,j=1}^3$ and the thermal capacity χ. We neglect the influence of thermal field on electric and magnetic fields and assume that pyroelectric and pyromagnetic coefficients vanish. The applied body force density is denoted by $\mathbf{f}=(f_i)_{i=1}^3:\Omega\times(0,T)\to\mathbf{R}^3$, the density of electric charges is denoted by $f^\varphi:\Omega\times(0,T)\to\mathbf{R}$, and the density of heat sources is denoted by

$f^{\theta}:\Omega\times(0,T)\to\mathbf{R}$. The temperature $\theta:\overline{\Omega}\times[0,T]\to\mathbf{R}$ vanishes along the boundary $\Gamma=\partial\Omega$ of the domain Ω. The bar is clamped along the upper butt end Γ_0 and, on the remaining part Γ_1 of the boundary, surface force with density $\mathbf{g}=(g_i):\Gamma_1\times(0,T)\to\mathbf{R}^3$ is given. The electric field $\mathbf{E}$ is conservative, i.e. $\mathbf{E}=-(\partial\varphi/\partial x_i)_{i=1}^3$, and the electric potential $\varphi:\overline{\Omega}\times[0,T]\to\mathbf{R}$ vanishes along Γ_0 and, on the remaining part Γ_1 of the boundary, the normal component of the electric displacement with density $g^{\varphi}:\Gamma_1^{\varphi}\times(0,T)\to\mathbf{R}$ is given. The magnetic field $\mathbf{H}$ is conservative, i.e. $\mathbf{H}=-(\partial\psi/\partial x_i)_{i=1}^3$, and the magnetic potential $\psi:\overline{\Omega}\times[0,T]\to\mathbf{R}$ vanishes along Γ_0 and, on the remaining part Γ_1 of the boundary, the normal component of the magnetic induction with density $g^{\psi}:\Gamma_1^{\psi}\times(0,T)\to\mathbf{R}$ is given. The linear dynamic three-dimensional model of the thermo-electro-magneto-elastic bar $\overline{\Omega}$ in differential form, with quasi-static equations for electro-magnetic fields, where the rate of change of magnetic field is small, i.e. the electric field is curl free, and there is no electric current, i.e. the magnetic field is curl free, is given by the following initial-boundary value problem [2, 3]:

$$\rho\frac{\partial^2 u_i}{\partial t^2}-\sum_{j=1}^{3}\frac{\partial\sigma_{ij}}{\partial x_j}=f_i \quad \text{in } \Omega\times(0,T),\ i=1,2,3, \tag{1}$$

$$\sum_{j=1}^{3}\frac{\partial D_j}{\partial x_j}=f^{\varphi} \quad \text{in } \Omega\times(0,T), \tag{2}$$

$$\sum_{j=1}^{3}\frac{\partial B_j}{\partial x_j}=0 \quad \text{in } \Omega\times(0,T), \tag{3}$$

$$\chi\frac{\partial\theta}{\partial t}-\sum_{j=1}^{3}\frac{\partial}{\partial x_i}\left(\eta_{ij}\frac{\partial\theta}{\partial x_j}\right)+\Theta_0\frac{\partial}{\partial t}\sum_{i,j=1}^{3}\lambda_{ij}e_{ij}(\mathbf{u})=f^{\theta} \quad \text{in } \Omega\times(0,T), \tag{4}$$

$$\mathbf{u}=\mathbf{0} \quad \text{on } \Gamma_0\times(0,T), \qquad \sum_{j=1}^{3}\sigma_{ij}n_j=g_i \quad \text{on } \Gamma_1\times(0,T),\ i=1,2,3, \tag{5}$$

$$\varphi=0 \quad \text{on } \Gamma_0\times(0,T), \qquad \sum_{i=1}^{3}D_i n_i=g^{\varphi} \quad \text{on } \Gamma_1\times(0,T), \tag{6}$$

$$\psi=0 \quad \text{on } \Gamma_0\times(0,T), \qquad \sum_{i=1}^{3}B_i n_i=g^{\psi} \quad \text{on } \Gamma_1\times(0,T), \tag{7}$$

$$\theta=0 \quad \text{on } \Gamma\times(0,T), \tag{8}$$

$$u_i(x,0)=u_{0i}(x),\quad \frac{\partial u_i}{\partial t}(x,0)=u_{1i}(x),\quad \theta(x,0)=\theta_0(x) \quad \text{in } \Omega,\ i=1,2,3, \tag{9}$$

where $\mathbf{n}=(n_i)_{i=1}^3$ is the unit outward normal vector to Γ, $\mathbf{u}=(u_i)_{i=1}^3:\overline{\Omega}\times[0,T]\to\mathbf{R}^3$ is the mechanical displacement vector-function, $\Theta_0>0$ is the temperature of the thermo-electro-magneto-elastic body in the natural state of no deformation and electromagnetic fields, which is considered as a reference temperature, $\mathbf{u}_0=(u_{0i})_{i=1}^3$ and $\mathbf{u}_1=(u_{1i})_{i=1}^3$ are the initial displacement and velocity vector-functions, θ_0 is the initial distribution of temperature, $e_{ij}(\mathbf{v})=1/2(\partial v_i/\partial x_j+\partial v_j/\partial x_i)$, $i,j=1,2,3$, $\mathbf{v}=(v_i)_{i=1}^3$, is the strain tensor. $(\sigma_{ij})_{i,j=1}^3$ is the mechanical stress tensor, $\mathbf{D}=(D_j)_{j=1}^3$ is the electric displacement vector, and $\mathbf{B}=(B_j)_{j=1}^3$ is the magnetic induction vector, which are given by the following constitutive equations:

$$\sigma_{ij}=\sum_{p,q=1}^{3}c_{ijpq}e_{pq}(\mathbf{u})+\sum_{p=1}^{3}\varepsilon_{pij}\frac{\partial\varphi}{\partial x_p}+\sum_{p=1}^{3}b_{pij}\frac{\partial\psi}{\partial x_p}-\lambda_{ij}\theta,\quad i,j=1,2,3,$$

$$D_i=\sum_{p,q=1}^{3}\varepsilon_{ipq}e_{pq}(\mathbf{u})-\sum_{j=1}^{3}d_{ij}\frac{\partial\varphi}{\partial x_j}-\sum_{j=1}^{3}a_{ij}\frac{\partial\psi}{\partial x_j},\qquad i=1,2,3,$$

$$B_i = \sum_{p,q=1}^{3} b_{ipq} e_{pq}(\mathbf{u}) - \sum_{j=1}^{3} a_{ij} \frac{\partial \varphi}{\partial x_j} - \sum_{j=1}^{3} \zeta_{ij} \frac{\partial \psi}{\partial x_j}, \qquad i = 1,2,3.$$

We assume that the coefficients characterizing elastic, thermal, electric and magnetic properties satisfy the following symmetry conditions

$$\begin{aligned} &c_{ijpq}(x) = c_{ijqp}(x) = c_{jipq}(x), \quad \varepsilon_{pij}(x) = \varepsilon_{pji}(x), \quad b_{pij}(x) = b_{pji}(x), \\ &d_{ij}(x) = d_{ji}(x), \quad a_{ij}(x) = a_{ji}(x), \quad \zeta_{ij}(x) = \zeta_{ji}(x), \quad \lambda_{ij}(x) = \lambda_{ji}(x), \quad x \in \Omega, \ i,j,p,q = 1,2,3. \end{aligned} \tag{10}$$

Let us consider the variational formulation of the three-dimensional initial-boundary value problem (1)-(9), which is equivalent to the differential formulation in spaces of smooth enough functions: Find the unknown vector-function $\mathbf{u} \in C([0,T];\mathbf{V}(\Omega))$, $\mathbf{u}' \in C([0,T];\mathbf{L}^2(\Omega))$, and functions $\varphi \in C([0,T];V^{\varphi}(\Omega))$, $\psi \in C([0,T];V^{\psi}(\Omega))$, $\theta \in L^2(0,T;H_0^1(\Omega))$, $\theta' \in L^2(0,T;H^{-1}(\Omega))$, which satisfy the following equations in the sense of distributions on $(0,T)$,

$$\frac{d}{dt}(\rho \mathbf{u}', \mathbf{v})_{\mathbf{L}^2(\Omega)} + c(\mathbf{u},\mathbf{v}) + \varepsilon(\varphi,\mathbf{v}) + b(\psi,\mathbf{v}) - \lambda(\theta,\mathbf{v}) = L^{\mathbf{u}}(\mathbf{v}), \qquad \forall \mathbf{v} \in \mathbf{V}(\Omega), \tag{11}$$

$$-\varepsilon(\overline{\varphi},\mathbf{u}) + d(\varphi,\overline{\varphi}) + a(\psi,\overline{\varphi}) = L^{\varphi}(\overline{\varphi}), \qquad \forall \overline{\varphi} \in V^{\varphi}(\Omega), \tag{12}$$

$$-b(\overline{\psi},\mathbf{u}) + a(\varphi,\overline{\psi}) + \zeta(\psi,\overline{\psi}) = L^{\psi}(\overline{\psi}), \qquad \forall \overline{\psi} \in V^{\psi}(\Omega), \tag{13}$$

$$\frac{d}{dt}(\chi\theta,\overline{\theta})_{L^2(\Omega)} + \eta(\theta,\overline{\theta}) + \Theta_0 \lambda(\overline{\theta},\mathbf{u}') = L^{\theta}(\overline{\theta}), \qquad \forall \overline{\theta} \in H_0^1(\Omega), \tag{14}$$

together with the initial conditions

$$\mathbf{u}(0) = \mathbf{u}_0, \quad \mathbf{u}'(0) = \mathbf{u}_1, \quad \theta(0) = \theta_0, \tag{15}$$

where $\mathbf{V}(\Omega) = \{\mathbf{v} \in \mathbf{H}^1(\Omega);\ \mathbf{tr}_{\Gamma}(\mathbf{v}) = \mathbf{0} \text{ on } \Gamma_0\}$, $V^{\varphi}(\Omega) = \{\overline{\varphi} \in H^1(\Omega);\ tr_{\Gamma}(\overline{\varphi}) = 0 \text{ on } \Gamma_0\}$, $V^{\psi}(\Omega) = \{\overline{\psi} \in H^1(\Omega);\ tr_{\Gamma}(\overline{\psi}) = 0 \text{ on } \Gamma_0\}$, $H_0^1(\Omega) = \{\overline{\theta} \in H^1(\Omega);\ tr_{\Gamma}(\overline{\theta}) = 0 \text{ on } \Gamma\}$, $H^{-1}(\Omega)$ is the dual space of $H_0^1(\Omega)$,

$$c(\mathbf{u},\mathbf{v}) = \int_{\Omega} \sum_{i,j,p,q=1}^{3} c_{ijpq} e_{pq}(\mathbf{u}) e_{ij}(\mathbf{v}) dx, \quad \varepsilon(\varphi,\mathbf{v}) = \int_{\Omega} \sum_{i,j,p=1}^{3} \varepsilon_{pij} \frac{\partial \varphi}{\partial x_p} e_{ij}(\mathbf{v}) dx,$$

$$b(\psi,\mathbf{v}) = \int_{\Omega} \sum_{i,j,p=1}^{3} b_{pij} \frac{\partial \psi}{\partial x_p} e_{ij}(\mathbf{v}) dx, \quad \lambda(\theta,\mathbf{v}) = -\int_{\Omega} \sum_{i,j=1}^{3} v_i \frac{\partial}{\partial x_j}(\lambda_{ij}\theta) dx, \quad d(\varphi,\overline{\varphi}) = \int_{\Omega} \sum_{i,j=1}^{3} d_{ij} \frac{\partial \varphi}{\partial x_j} \frac{\partial \overline{\varphi}}{\partial x_i} dx,$$

$$a(\psi,\overline{\varphi}) = \int_{\Omega} \sum_{i,j=1}^{3} a_{ij} \frac{\partial \psi}{\partial x_j} \frac{\partial \overline{\varphi}}{\partial x_i} dx, \quad \zeta(\psi,\overline{\psi}) = \int_{\Omega} \sum_{i,j=1}^{3} \zeta_{ij} \frac{\partial \psi}{\partial x_j} \frac{\partial \overline{\psi}}{\partial x_i} dx, \quad \eta(\theta,\overline{\theta}) = \int_{\Omega} \sum_{i,j=1}^{3} \eta_{ij} \frac{\partial \theta}{\partial x_j} \frac{\partial \overline{\theta}}{\partial x_i} dx,$$

$$L^{\mathbf{u}}(\mathbf{v}) = \int_{\Omega} \sum_{i=1}^{3} f_i v_i dx + \int_{\Gamma_1} \sum_{i=1}^{3} g_i tr_{\Gamma_1}(v_i) d\Gamma, \quad L^{\varphi}(\overline{\varphi}) = \int_{\Omega} f^{\varphi} \overline{\varphi} dx - \int_{\Gamma_1^{\varphi}} g^{\varphi} tr_{\Gamma_1^{\varphi}}(\overline{\varphi}) d\Gamma,$$

$$L^{\psi}(\overline{\psi}) = -\int_{\Gamma_1^{\psi}} g^{\psi} tr_{\Gamma_1^{\psi}}(\overline{\psi}) d\Gamma, \quad L^{\theta}(\overline{\theta}) = \int_{\Omega} f^{\theta} \overline{\theta} dx,$$

$(.,.)_{\mathbf{L}^2(\Omega)}$ and $(.,.)_{L^2(\Omega)}$ are the scalar products in the spaces $\mathbf{L}^2(\Omega)$ and $L^2(\Omega)$, respectively.

In order to construct the hierarchy of one-dimensional models, let us consider the subspaces $\mathbf{V}_{\mathbf{N}^1\mathbf{N}^2}(\Omega)$ and $\mathbf{H}_{\mathbf{N}^1\mathbf{N}^2}(\Omega)$ of $\mathbf{V}(\Omega)$ and $\mathbf{L}^2(\Omega)$, respectively, $\mathbf{N}^1 = (N_1^1, N_2^1, N_3^1)$, $\mathbf{N}^2 = (N_1^2, N_2^2, N_3^2)$, consisting of vector-functions with components $v_{\mathbf{N}^1\mathbf{N}^2 i}$ $(i = 1,2,3)$, which are polynomials with respect to the variables x_1 and x_2,

$$\mathbf{v}_{\mathbf{N}^1\mathbf{N}^2} = (v_{\mathbf{N}^1\mathbf{N}^2 i})_{i=1}^3, \quad v_{\mathbf{N}^1\mathbf{N}^2 i} = \sum_{r_i^1=0}^{N_i^1} \sum_{r_i^2=0}^{N_i^2} \frac{1}{h_1 h_2} \left(r_i^1 + \frac{1}{2}\right)\left(r_i^2 + \frac{1}{2}\right) \overset{r_i^1 r_i^2}{v}_{\mathbf{N}^1\mathbf{N}^2 i} P_{r_i^1}(z_1) P_{r_i^2}(z_2),$$

$$(h_1 h_2)^{-1/2} \overset{r_i^1 r_i^2}{v}_{\mathbf{N}^1\mathbf{N}^2 i} \in L^2(I), \ 0 \le r_i^{\alpha} \le N_i^{\alpha}, \quad z_{\alpha} = \frac{x_{\alpha} - \overline{h}_{\alpha}}{h_{\alpha}}, \ h_{\alpha} = \frac{h_{\alpha}^+ - h_{\alpha}^-}{2}, \ \overline{h}_{\alpha} = \frac{h_{\alpha}^+ + h_{\alpha}^-}{2}, \ \alpha = 1,2, \ i = 1,2,3.$$

We also consider the subspaces $V^{\varphi}_{N^1_\varphi N^2_\varphi}(\Omega)$ and $V^{\psi}_{N^1_\psi N^2_\psi}(\Omega)$ of $V^{\varphi}(\Omega)$ and $V^{\psi}(\Omega)$, respectively, which consist of the following functions

$$\overline{\varphi}_{N^1_\varphi N^2_\varphi}=\sum_{r^1_\varphi=0}^{N^1_\varphi}\sum_{r^2_\varphi=0}^{N^2_\varphi}\frac{1}{h_1h_2}\left(r^1_\varphi+\frac{1}{2}\right)\left(r^2_\varphi+\frac{1}{2}\right)\overline{\varphi}^{r^1_\varphi r^2_\varphi}_{N^1_\varphi N^2_\varphi}P_{r^1_\varphi}(z_1)P_{r^2_\varphi}(z_2),\quad (h_1h_2)^{-1/2}\overline{\varphi}^{r^1_\varphi r^2_\varphi}_{N^1_\varphi N^2_\varphi}\in L^2(I),\ r^\alpha_\varphi=0,...,N^\alpha_\varphi,\alpha=1,2,$$

$$\overline{\psi}_{N^1_\psi N^2_\psi}=\sum_{r^1_\psi=0}^{N^1_\psi}\sum_{r^2_\psi=0}^{N^2_\psi}\frac{1}{h_1h_2}\left(r^1_\psi+\frac{1}{2}\right)\left(r^2_\psi+\frac{1}{2}\right)\overline{\psi}^{r^1_\psi r^2_\psi}_{N^1_\psi N^2_\psi}P_{r^1_\psi}(z_1)P_{r^2_\psi}(z_2),\ (h_1h_2)^{-1/2}\overline{\psi}^{r^1_\psi r^2_\psi}_{N^1_\psi N^2_\psi}\in L^2(I),r^\alpha_\psi=0,...,N^\alpha_\psi,\alpha=1,2,$$

and subspaces $V^{\theta}_{N^1_\theta N^2_\theta}(\Omega)$ and $H^{\theta}_{N^1_\theta N^2_\theta}(\Omega)$ of $H^1_0(\Omega)$ and $L^2(\Omega)$, respectively, consisting of the functions

$$\overline{\theta}_{N^1_\theta N^2_\theta}=\sum_{r^1_\theta=0}^{N^1_\theta}\sum_{r^2_\theta=0}^{N^2_\theta}\frac{1}{h_1h_2}\left(r^1_\theta+\frac{1}{2}\right)\left(r^2_\theta+\frac{1}{2}\right)\overline{\theta}^{r^1_\theta r^2_\theta}_{N^1_\theta N^2_\theta}P_{r^1_\theta}(z_1)P_{r^2_\theta}(z_2)$$

$$-\sum_{r^1_\theta=0}^{N^1_\theta}\sum_{r^2_\theta=0}^{N^2_\theta}\frac{1}{2h_1h_2}\left(r^1_\theta+\frac{1}{2}\right)\left(r^2_\theta+\frac{1}{2}\right)\overline{\theta}^{r^1_\theta r^2_\theta}_{N^1_\theta N^2_\theta}\sum_{\alpha,\beta=1}^{2}(1+(-1)^{r^{3-\alpha}_\theta+N^{3-\alpha}_\theta+\beta})P_{r^\alpha_\theta}(z_\alpha)P_{N^{3-\alpha}_\theta+\beta}(z_{3-\alpha})$$

$$+\sum_{r^1_\theta=0}^{N^1_\theta}\sum_{r^2_\theta=0}^{N^2_\theta}\frac{1}{4h_1h_2}\left(r^1_\theta+\frac{1}{2}\right)\left(r^2_\theta+\frac{1}{2}\right)\overline{\theta}^{r^1_\theta r^2_\theta}_{N^1_\theta N^2_\theta}\sum_{\alpha,\beta=1}^{2}(1+(-1)^{r^1_\theta+N^1_\theta+\alpha})(1+(-1)^{r^2_\theta+N^2_\theta+\beta})P_{N^1_\theta+\alpha}(z_1)P_{N^2_\theta+\beta}(z_2),$$

where $(h_1h_2)^{-1/2}\overline{\theta}^{r^1_\theta r^2_\theta}_{N^1_\theta N^2_\theta}\in L^2(I),\ 0\le r^1_\theta\le N^1_\theta,\ 0\le r^2_\theta\le N^2_\theta$.

Since the functions $h^\pm_1$ and $h^\pm_2$ are Lipschitz continuous in I, from Rademacher's theorem [18] we have that $h^\pm_1$ and $h^\pm_2$ are differentiable almost everywhere in I and $\partial_3 h^\pm_1$, $\partial_3 h^\pm_2\in L^\infty(I^*)$, for all $I^*=(h^{-,*}_3,h^{+,*}_3)$, $h^-_3<h^{-,*}_3<h^{+,*}_3<h^+_3$. Therefore, the positiveness of $h^\pm_1$ and $h^\pm_2$ in I implies that, for any vector-function $\mathbf{v}_{\mathbf{N}^1\mathbf{N}^2}=(v_{\mathbf{N}^1\mathbf{N}^2 i})^3_{i=1}\in\mathbf{V}_{\mathbf{N}^1\mathbf{N}^2}(\Omega)$, the corresponding functions $\overset{r^1_i r^2_i}{v}_{\mathbf{N}^1\mathbf{N}^2 i}\in H^1(I^*)$, for all I^*, $\overline{I^*}\subset I$, i.e. $\overset{r^1_i r^2_i}{v}_{\mathbf{N}^1\mathbf{N}^2 i}\in H^1_{loc}(I)$, $0\le r^1_i\le N^1_i$, $0\le r^2_i\le N^2_i$, $i=1,2,3$. Similarly, for all functions $\overline{\varphi}_{N^1_\varphi N^2_\varphi}\in V^{\varphi}_{N^1_\varphi N^2_\varphi}(\Omega)$, $\overline{\psi}_{N^1_\psi N^2_\psi}\in V^{\psi}_{N^1_\psi N^2_\psi}(\Omega)$, $\overline{\theta}_{N^1_\theta N^2_\theta}\in V^{\theta}_{N^1_\theta N^2_\theta}(\Omega)$, the functions $\overline{\varphi}^{r^1_\varphi r^2_\varphi}_{N^1_\varphi N^2_\varphi}$, $\overline{\psi}^{r^1_\psi r^2_\psi}_{N^1_\psi N^2_\psi}$, $\overline{\theta}^{r^1_\theta r^2_\theta}_{N^1_\theta N^2_\theta}$ of one space variable in the expressions of $\overline{\varphi}_{N^1_\varphi N^2_\varphi}$, $\overline{\psi}_{N^1_\psi N^2_\psi}$, $\overline{\theta}_{N^1_\theta N^2_\theta}$ belong to $H^1(I^*)$, $\overline{I^*}\subset I$, i.e. $\overline{\varphi}^{r^1_\varphi r^2_\varphi}_{N^1_\varphi N^2_\varphi}$, $\overline{\psi}^{r^1_\psi r^2_\psi}_{N^1_\psi N^2_\psi}$, $\overline{\theta}^{r^1_\theta r^2_\theta}_{N^1_\theta N^2_\theta}\in H^1_{loc}(I)$, $0\le r^1_\varphi\le N^1_\varphi$, $0\le r^2_\varphi\le N^2_\varphi$, $0\le r^1_\psi\le N^1_\psi$, $0\le r^2_\psi\le N^2_\psi$, $0\le r^1_\theta\le N^1_\theta$, $0\le r^2_\theta\le N^2_\theta$. Moreover, the norms $\|.\|_{\mathbf{H}^1(\Omega)}$ and $\|.\|_{H^1(\Omega)}$ in the spaces $\mathbf{H}^1(\Omega)$ and $H^1(\Omega)$ define the weighted norms $\|.\|_*$ and $\|.\|_{\varphi*}$, $\|.\|_{\psi*}$, $\|.\|_{\theta*}$ for vector-functions $\vec{v}_{\mathbf{N}^1\mathbf{N}^2}=(\overset{r^1_i r^2_i}{v}_{\mathbf{N}^1\mathbf{N}^2 i})\in[H^1_{loc}(I)]^{N^{1,2}_{1,2,3}}$, $N^{1,2}_{1,2,3}=\sum_{i=1}^{3}(N^1_i+1)(N^2_i+1)$, and $\vec{\overline{\varphi}}_{N^1_\varphi N^2_\varphi}=(\overline{\varphi}^{r^1_\varphi r^2_\varphi}_{N^1_\varphi N^2_\varphi})\in[H^1_{loc}(I)]^{N^{1,2}_\varphi}$, $N^{1,2}_\varphi=(N^1_\varphi+1)(N^2_\varphi+1)$, $\vec{\overline{\psi}}_{N^1_\psi N^2_\psi}=(\overline{\psi}^{r^1_\psi r^2_\psi}_{N^1_\psi N^2_\psi})\in[H^1_{loc}(I)]^{N^{1,2}_\psi}$, $N^{1,2}_\psi=(N^1_\psi+1)(N^2_\psi+1)$, $\vec{\overline{\theta}}_{N^1_\theta N^2_\theta}=(\overline{\theta}^{r^1_\theta r^2_\theta}_{N^1_\theta N^2_\theta})\in[H^1_{loc}(I)]^{N^{1,2}_\theta}$, $N^{1,2}_\theta=(N^1_\theta+1)(N^2_\theta+1)$, such that $\|\vec{v}_{\mathbf{N}^1\mathbf{N}^2}\|_*=\|\mathbf{v}_{\mathbf{N}^1\mathbf{N}^2}\|_{\mathbf{H}^1(\Omega)}$, $\|\vec{\overline{\varphi}}_{N^1_\varphi N^2_\varphi}\|_{\varphi*}=\|\overline{\varphi}_{N^1_\varphi N^2_\varphi}\|_{H^1(\Omega)}$, $\|\vec{\overline{\psi}}_{N^1_\psi N^2_\psi}\|_{\psi*}=\|\overline{\psi}_{N^1_\psi N^2_\psi}\|_{H^1(\Omega)}$ and $\|\vec{\overline{\theta}}_{N^1_\theta N^2_\theta}\|_{\theta*}=\|\overline{\theta}_{N^1_\theta N^2_\theta}\|_{H^1(\Omega)}$. Using the properties of the Legendre polynomials [19], we can obtain explicit expressions of the norms $\|\cdot\|_*$, $\|\cdot\|_{\varphi*}$, $\|\cdot\|_{\psi*}$, and $\|\cdot\|_{\theta*}$. In particular, $\|\cdot\|_*$ is given by the following expression:

$$\|\vec{v}_{\mathbf{N}^1\mathbf{N}^2}\|_*^2=\sum_{i=1}^{3}\sum_{r_i^1=0}^{N_i^1}\sum_{r_i^2=0}^{N_i^2}\left(r_i^1+\frac{1}{2}\right)\left(r_i^2+\frac{1}{2}\right)\left[\left\|\frac{\overset{r_i^1r_i^2}{v}_{\mathbf{N}^1\mathbf{N}^2i}}{\sqrt{h_1h_2}}\right\|_{L^2(I)}^2+\sum_{\alpha=1}^{2}\left\|\sum_{k_i^\alpha=r_i^\alpha}^{N_i^\alpha}\left(k_i^\alpha+\frac{1}{2}\right)\frac{1-(-1)^{k_i^\alpha+r_i^\alpha}}{h_1h_2\sqrt{h_\alpha}}\right.\right.$$

$$\left.\times\left((2-\alpha)\overset{k_i^1r_i^2}{v}_{\mathbf{N}^1\mathbf{N}^2i}+(\alpha-1)\overset{r_i^1k_i^2}{v}_{\mathbf{N}^1\mathbf{N}^2i}\right)\right\|_{L^2(I)}^2+\left\|\frac{1}{\sqrt{h_1h_2}}\left((\overset{r_i^1r_i^2}{v}_{\mathbf{N}^1\mathbf{N}^2i})'-\sum_{\alpha=1}^{2}\frac{(h_\alpha)'}{h_\alpha}(r_i^\alpha+1)\overset{r_i^1r_i^2}{v}_{\mathbf{N}^1\mathbf{N}^2i}\right.\right.$$

$$\left.\left.\left.-\sum_{\alpha=1}^{2}\sum_{k_i^\alpha=r_i^\alpha+1}^{N_i^\alpha}\left(k_i^\alpha+\frac{1}{2}\right)\frac{(h_\alpha^+)'-(-1)^{k_i^\alpha-r_i^\alpha}(h_\alpha^-)'}{h_\alpha}\left((2-\alpha)\overset{k_i^1r_i^2}{v}_{\mathbf{N}^1\mathbf{N}^2i}+(\alpha-1)\overset{r_i^1k_i^2}{v}_{\mathbf{N}^1\mathbf{N}^2i}\right)\right)\right\|_{L^2(I)}^2\right],$$

where we assume that the sums with the lower limit greater than the upper one equal to zero.

For components $\overset{r_i^1r_i^2}{v}_{\mathbf{N}^1\mathbf{N}^2i}$ and $\overset{r_\varphi^1r_\varphi^2}{\varphi}_{N_\varphi^1N_\varphi^2}$, $\overset{r_\psi^1r_\psi^2}{\psi}_{N_\psi^1N_\psi^2}$, $\overset{r_1r_2}{\theta}_{N_\theta^1N_\theta^2}$ of $\vec{v}_{\mathbf{N}^1\mathbf{N}^2}$ and $\vec{\vec{\varphi}}_{N_\varphi^1N_\varphi^2}$, $\vec{\vec{\psi}}_{N_\psi^1N_\psi^2}$, $\vec{\vec{\theta}}_{N_\theta^1N_\theta^2}$, which possess the properties $\|\vec{v}_{\mathbf{N}^1\mathbf{N}^2}\|_*<\infty$ and $\|\vec{\vec{\varphi}}_{N_\varphi^1N_\varphi^2}\|_{\varphi*}<\infty$, $\|\vec{\vec{\psi}}_{N_\psi^1N_\psi^2}\|_{\psi*}<\infty$, $\|\vec{\vec{\theta}}_{N_\theta^1N_\theta^2}\|_{\theta*}<\infty$, we can define the traces on $x_3=h_3^+$. Indeed, the corresponding vector-function of three space variables $\mathbf{v}_{\mathbf{N}^1\mathbf{N}^2}=(v_{\mathbf{N}^1\mathbf{N}^2i})_{i=1}^3$ and functions $\varphi_{N_\varphi^1N_\varphi^2}$, $\psi_{N_\psi^1N_\psi^2}$, $\theta_{N_\theta^1N_\theta^2}$ belong to the spaces $\mathbf{V}_{\mathbf{N}^1\mathbf{N}^2}(\Omega)\subset\mathbf{H}^1(\Omega)$ and $V^\varphi_{N_\varphi^1N_\varphi^2}(\Omega)$, $V^\psi_{N_\psi^1N_\psi^2}(\Omega)$, $V^\theta_{N_\theta^1N_\theta^2}(\Omega)\subset H^1(\Omega)$, respectively, and using the trace operator tr_{Γ_0} we define the traces of $\overset{r_i^1r_i^2}{v}_{\mathbf{N}^1\mathbf{N}^2i}$ and $\overset{r_\varphi^1r_\varphi^2}{\varphi}_{N_\varphi^1N_\varphi^2}$, $\overset{r_\psi^1r_\psi^2}{\psi}_{N_\psi^1N_\psi^2}$, $\overset{r_\theta^1r_\theta^2}{\theta}_{N_\theta^1N_\theta^2}$ on $x_3=h_3^+$, in particular,

$$tr_{h_3^+}(\overset{r_i^1r_i^2}{v}_{\mathbf{N}^1\mathbf{N}^2i})=\int_{h_2^-}^{h_2^+}\int_{h_1^-}^{h_1^+}tr_{\Gamma_0}(v_{\mathbf{N}^1\mathbf{N}^2i})P_{r_i^1}(z_1)P_{r_i^2}(z_2)dx_1dx_2,\quad r_i^1=0,..,N_i^1,\ r_i^2=0,..,N_i^2,\ i=1,2,3.$$

Since the vector-functions $\mathbf{v}_{\mathbf{N}^1\mathbf{N}^2}$, from the subspaces $\mathbf{V}_{\mathbf{N}^1\mathbf{N}^2}(\Omega)$ and $\mathbf{H}_{\mathbf{N}^1\mathbf{N}^2}(\Omega)$, and the functions $\varphi_{N_\varphi^1N_\varphi^2}\in V^\varphi_{N_\varphi^1N_\varphi^2}(\Omega)$, $\psi_{N_\psi^1N_\psi^2}\in V^\psi_{N_\psi^1N_\psi^2}(\Omega)$, and $\theta_{N_\theta^1N_\theta^2}$, from $V^\theta_{N_\theta^1N_\theta^2}(\Omega)$ and $H^\theta_{N_\theta^1N_\theta^2}(\Omega)$, are uniquely defined by the functions $\overset{r_i^1r_i^2}{v}_{\mathbf{N}^1\mathbf{N}^2i}$, $\overset{r_\varphi^1r_\varphi^2}{\varphi}_{N_\varphi^1N_\varphi^2}$, $\overset{r_\psi^1r_\psi^2}{\psi}_{N_\psi^1N_\psi^2}$, $\overset{r_\theta^1r_\theta^2}{\theta}_{N_\theta^1N_\theta^2}$ of one space variable, and considering the original three-dimensional problem (11)-(15) on these subspaces, we obtain the following hierarchy of one-dimensional initial-boundary value problems: Find $\vec{u}_{\mathbf{N}^1\mathbf{N}^2}\in C([0,T];\vec{V}_{\mathbf{N}^1\mathbf{N}^2}(I))$, $\vec{u}'_{\mathbf{N}^1\mathbf{N}^2}\in C([0,T];\vec{H}_{\mathbf{N}^1\mathbf{N}^2}(I))$, $\vec{\varphi}_{N_\varphi^1N_\varphi^2}\in C([0,T];\vec{V}^\varphi_{N_\varphi^1N_\varphi^2}(I))$, $\vec{\psi}_{N_\psi^1N_\psi^2}\in C([0,T];\vec{V}^\psi_{N_\psi^1N_\psi^2}(I))$, $\vec{\theta}_{N_\theta^1N_\theta^2}\in L^2(0,T;\vec{V}^\theta_{N_\theta^1N_\theta^2}(I))$, $\vec{\theta}'_{N_\theta^1N_\theta^2}\in L^2(0,T;(\vec{V}^\theta_{N_\theta^1N_\theta^2}(I))')$, which satisfy the following equations in the sense of distributions on $(0,T)$,

$$\frac{d}{dt}R_{\mathbf{N}^{1,2}}(\vec{u}'_{\mathbf{N}^1\mathbf{N}^2},\vec{v}_{\mathbf{N}^1\mathbf{N}^2})+c_{\mathbf{N}^{1,2}}(\vec{u}_{\mathbf{N}^1\mathbf{N}^2},\vec{v}_{\mathbf{N}^1\mathbf{N}^2})+\varepsilon_{N_\varphi^{1,2}\mathbf{N}^{1,2}}(\vec{\varphi}_{N_\varphi^1N_\varphi^2},\vec{v}_{\mathbf{N}^1\mathbf{N}^2})+b_{N_\psi^{1,2}\mathbf{N}^{1,2}}(\vec{\psi}_{N_\psi^1N_\psi^2},\vec{v}_{\mathbf{N}^1\mathbf{N}^2})$$
$$-\lambda_{N_\theta^{1,2}\mathbf{N}^{1,2}}(\vec{\theta}_{N_\theta^1N_\theta^2},\vec{v}_{\mathbf{N}^1\mathbf{N}^2})=L^{\mathbf{u}}_{\mathbf{N}^{1,2}}(\vec{v}_{\mathbf{N}^1\mathbf{N}^2}),\tag{16}$$

$$-\varepsilon_{N_\varphi^{1,2}\mathbf{N}^{1,2}}(\vec{\vec{\varphi}}_{N_\varphi^1N_\varphi^2},\vec{u}_{\mathbf{N}^1\mathbf{N}^2})+d_{N_\varphi^{1,2}}(\vec{\varphi}_{N_\varphi^1N_\varphi^2},\vec{\vec{\varphi}}_{N_\varphi^1N_\varphi^2})+a_{N_\psi^{1,2}N_\varphi^{1,2}}(\vec{\psi}_{N_\psi^1N_\psi^2},\vec{\vec{\varphi}}_{N_\varphi^1N_\varphi^2})=L^\varphi_{N_\varphi^{1,2}}(\vec{\vec{\varphi}}_{N_\varphi^1N_\varphi^2}),\tag{17}$$

$$-b_{N_\psi^{1,2}\mathbf{N}^{1,2}}(\vec{\vec{\psi}}_{N_\psi^1N_\psi^2},\vec{u}_{\mathbf{N}^1\mathbf{N}^2})+a_{N_\varphi^{1,2}N_\psi^{1,2}}(\vec{\varphi}_{N_\varphi^1N_\varphi^2},\vec{\vec{\psi}}_{N_\psi^1N_\psi^2})+\zeta_{N_\psi^{1,2}}(\vec{\psi}_{N_\psi^1N_\psi^2},\vec{\vec{\psi}}_{N_\psi^1N_\psi^2})=L^\psi_{N_\psi^{1,2}}(\vec{\vec{\psi}}_{N_\psi^1N_\psi^2}),\tag{18}$$

$$\frac{d}{dt}R^\theta_{N_\theta^{1,2}}(\vec{\theta}_{N_\theta^1N_\theta^2},\vec{\vec{\theta}}_{N_\theta^1N_\theta^2})+\eta_{N_\theta^{1,2}}(\vec{\theta}_{N_\theta^1N_\theta^2},\vec{\vec{\theta}}_{N_\theta^1N_\theta^2})+\Theta_0\lambda_{N_\theta^{1,2}\mathbf{N}^{1,2}}(\vec{\vec{\theta}}_{N_\theta^1N_\theta^2},\vec{u}'_{\mathbf{N}^1\mathbf{N}^2})=L^\theta_{N_\theta^{1,2}}(\vec{\vec{\theta}}_{N_\theta^1N_\theta^2}),\tag{19}$$

for all $\vec{v}_{\mathbf{N}^1\mathbf{N}^2}\in\vec{V}_{\mathbf{N}^1\mathbf{N}^2}(I)$, $\vec{\vec{\varphi}}_{N_\varphi^1N_\varphi^2}\in\vec{V}^\varphi_{N_\varphi^1N_\varphi^2}(I)$, $\vec{\vec{\psi}}_{N_\psi^1N_\psi^2}\in\vec{V}^\psi_{N_\psi^1N_\psi^2}(I)$, $\vec{\vec{\theta}}_{N_\theta^1N_\theta^2}\in\vec{V}^\theta_{N_\theta^1N_\theta^2}(I)$, and the initial conditions

$$\vec{u}_{\mathbf{N}^1\mathbf{N}^2}(0)=\vec{u}_{\mathbf{N}^1\mathbf{N}^20},\quad \vec{u}'_{\mathbf{N}^1\mathbf{N}^2}(0)=\vec{u}_{\mathbf{N}^1\mathbf{N}^21},\quad \vec{\theta}_{N_\theta^1N_\theta^2}(0)=\vec{\theta}_{N_\theta^1N_\theta^20},\tag{20}$$

where $\vec{V}_{\mathbf{N}^1\mathbf{N}^2}(I)=\{\vec{v}_{\mathbf{N}^1\mathbf{N}^2}=(\overset{r_i^1 r_i^2}{v_{\mathbf{N}^1\mathbf{N}^2 i}})\in[H^1_{loc}(I)]^{N^{1,2}_{1,2,3}};\left\|\vec{v}_{\mathbf{N}^1\mathbf{N}^2}\right\|_*<\infty, tr_{h_3^+}(\overset{r_i^1 r_i^2}{v_{\mathbf{N}^1\mathbf{N}^2 i}})=0,\ 0\le r_i^\alpha\le N_i^\alpha,\ \alpha=1,2,\ i=1,2,3\}$, $\vec{H}_{\mathbf{N}^1\mathbf{N}^2}(I)=\{\vec{v}_{\mathbf{N}^1\mathbf{N}^2}\in[L^2(I)]^{N^{1,2}_{1,2,3}};(h_1h_2)^{-1/2}\overset{r_i^1 r_i^2}{v_{\mathbf{N}^1\mathbf{N}^2 i}}\in L^2(I),\ 0\le r_i^\alpha\le N_i^\alpha,\ \alpha=1,2,\ i=1,2,3\}$, $\vec{V}^{\varphi}_{N_\varphi^1 N_\varphi^2}(I)=\{\vec{\overline{\varphi}}_{N_\varphi^1 N_\varphi^2}=(\overset{r_\varphi^1 r_\varphi^2}{\overline{\varphi}_{N_\varphi^1 N_\varphi^2}})\in[H^1_{loc}(I)]^{N_\varphi^{1,2}};\left\|\vec{\overline{\varphi}}_{N_\varphi^1 N_\varphi^2}\right\|_{\varphi *}<\infty, tr_{h_3^+}(\overset{r_\varphi^1 r_\varphi^2}{\overline{\varphi}_{N_\varphi^1 N_\varphi^2}})=0, 0\le r_\varphi^\alpha\le N_\varphi^\alpha, \alpha=1,2\}$, $\vec{V}^{\psi}_{N_\psi^1 N_\psi^2}(I)=\{\vec{\overline{\psi}}_{N_\psi^1 N_\psi^2}=(\overset{r_\psi^1 r_\psi^2}{\overline{\psi}_{N_\psi^1 N_\psi^2}})\in[H^1_{loc}(I)]^{N_\psi^{1,2}};\left\|\vec{\overline{\psi}}_{N_\psi^1 N_\psi^2}\right\|_{\psi *}<\infty, tr_{h_3^+}(\overset{r_\psi^1 r_\psi^2}{\overline{\psi}_{N_\psi^1 N_\psi^2}})=0, 0\le r_\psi^\alpha\le N_\psi^\alpha, \alpha=1,2\}$, $\vec{V}^{\theta}_{N_\theta^1 N_\theta^2}(I)=\{\vec{\overline{\theta}}_{N_\theta^1 N_\theta^2}=(\overset{r_\theta^1 r_\theta^2}{\overline{\theta}_{N_\theta^1 N_\theta^2}})\in[H^1_{loc}(I)]^{N_\theta^{1,2}};$ $\left\|\vec{\overline{\theta}}_{N_\theta^1 N_\theta^2}\right\|_{\theta *}<\infty, tr_{h_3^+}(\overset{r_\theta^1 r_\theta^2}{\overline{\theta}_{N_\theta^1 N_\theta^2}})=0, 0\le r_\theta^\alpha\le N_\theta^\alpha, \alpha=1,2\}$, $\vec{H}^{\theta}_{N_\theta^1 N_\theta^2}(I)=\{\vec{\overline{\theta}}_{N_\theta^1 N_\theta^2}=(\overset{r_\theta^1 r_\theta^2}{\overline{\theta}_{N_\theta^1 N_\theta^2}})\in[L^2(I)]^{N_\theta^{1,2}};(h_1h_2)^{-1/2}\times$ $\overset{r_\theta^1 r_\theta^2}{\overline{\theta}_{N_\theta^1 N_\theta^2}}\in L^2(I), 0\le r_\theta^\alpha\le N_\theta^\alpha, \alpha=1,2\}$, the bilinear forms $R_{\mathbf{N}^{1,2}}$, $c_{\mathbf{N}^{1,2}}$, $\varepsilon_{N_\varphi^{1,2}\mathbf{N}^{1,2}}$, $b_{N_\psi^{1,2}\mathbf{N}^{1,2}}$, $\lambda_{N_\theta^{1,2}\mathbf{N}^{1,2}}$, $d_{N_\varphi^{1,2}}$, $a_{N_\varphi^{1,2}N_\psi^{1,2}}$, $a_{N_\psi^{1,2}N_\varphi^{1,2}}$, $\zeta_{N_\psi^{1,2}}$, $R^\theta_{N_\theta^{1,2}}$, $\eta_{N_\theta^{1,2}}$ are defined as follows $R_{\mathbf{N}^{1,2}}(\vec{\tilde{v}}_{\mathbf{N}^1\mathbf{N}^2},\vec{v}_{\mathbf{N}^1\mathbf{N}^2})=(\rho\tilde{\mathbf{v}}_{\mathbf{N}^1\mathbf{N}^2},\mathbf{v}_{\mathbf{N}^1\mathbf{N}^2})_{\mathbf{L}^2(\Omega)}$, $c_{\mathbf{N}^{1,2}}(\vec{\tilde{v}}_{\mathbf{N}^1\mathbf{N}^2},\vec{v}_{\mathbf{N}^1\mathbf{N}^2})=c(\tilde{\mathbf{v}}_{\mathbf{N}^1\mathbf{N}^2},\mathbf{v}_{\mathbf{N}^1\mathbf{N}^2})$, $\varepsilon_{N_\varphi^{1,2}\mathbf{N}^{1,2}}(\vec{\overline{\varphi}}_{N_\varphi^1N_\varphi^2},\vec{v}_{\mathbf{N}^1\mathbf{N}^2})=\varepsilon(\overline{\varphi}_{N_\varphi^1N_\varphi^2},\mathbf{v}_{\mathbf{N}^1\mathbf{N}^2})$, $b_{N_\psi^{1,2}\mathbf{N}^{1,2}}(\vec{\overline{\psi}}_{N_\psi^1N_\psi^2},\vec{v}_{\mathbf{N}^1\mathbf{N}^2})=b(\overline{\psi}_{N_\psi^1N_\psi^2},$ $\mathbf{v}_{\mathbf{N}^1\mathbf{N}^2})$, $\lambda_{N_\theta^{1,2}\mathbf{N}^{1,2}}(\vec{\overline{\theta}}_{N_\theta^1N_\theta^2},\vec{v}_{\mathbf{N}^1\mathbf{N}^2})=\lambda(\overline{\theta}_{N_\theta^1N_\theta^2},\mathbf{v}_{\mathbf{N}^1\mathbf{N}^2})$, $d_{N_\varphi^{1,2}}(\vec{\tilde{\varphi}}_{N_\varphi^1N_\varphi^2},\vec{\overline{\varphi}}_{N_\varphi^1N_\varphi^2})=d(\tilde{\varphi}_{N_\varphi^1N_\varphi^2},\overline{\varphi}_{N_\varphi^1N_\varphi^2})$, $a_{N_\varphi^{1,2}N_\psi^{1,2}}(\vec{\overline{\varphi}}_{N_\varphi^1N_\varphi^2},\vec{\overline{\psi}}_{N_\psi^1N_\psi^2})$ $=a(\overline{\varphi}_{N_\varphi^1N_\varphi^2},\overline{\psi}_{N_\psi^1N_\psi^2})$, $a_{N_\psi^{1,2}N_\varphi^{1,2}}(\vec{\overline{\psi}}_{N_\psi^1N_\psi^2},\vec{\overline{\varphi}}_{N_\varphi^1N_\varphi^2})=a(\overline{\psi}_{N_\psi^1N_\psi^2},\overline{\varphi}_{N_\varphi^1N_\varphi^2})$, $\zeta_{N_\psi^{1,2}}(\vec{\tilde{\psi}}_{N_\psi^1N_\psi^2},\vec{\overline{\psi}}_{N_\psi^1N_\psi^2})=\zeta(\tilde{\psi}_{N_\psi^1N_\psi^2},\overline{\psi}_{N_\psi^1N_\psi^2})$, $R^\theta_{N_\theta^{1,2}}(\vec{\tilde{\theta}}_{N_\theta^1N_\theta^2},\vec{\overline{\theta}}_{N_\theta^1N_\theta^2})=(\chi\tilde{\theta}_{N_\theta^1N_\theta^2},\overline{\theta}_{N_\theta^1N_\theta^2})_{L^2(\Omega)}$, $\eta_{N_\theta^{1,2}}(\vec{\tilde{\theta}}_{N_\theta^1N_\theta^2},\vec{\overline{\theta}}_{N_\theta^1N_\theta^2})=\eta(\tilde{\theta}_{N_\theta^1N_\theta^2},\overline{\theta}_{N_\theta^1N_\theta^2})$, for all vector-functions $\vec{\tilde{v}}_{\mathbf{N}^1\mathbf{N}^2}$, $\vec{v}_{\mathbf{N}^1\mathbf{N}^2}\in\vec{V}_{\mathbf{N}^1\mathbf{N}^2}(I)$, $\vec{\tilde{\varphi}}_{N_\varphi^1N_\varphi^2},\vec{\overline{\varphi}}_{N_\varphi^1N_\varphi^2}\in\vec{V}^\varphi_{N_\varphi^1N_\varphi^2}(I)$, $\vec{\tilde{\psi}}_{N_\psi^1N_\psi^2},\vec{\overline{\psi}}_{N_\psi^1N_\psi^2}\in\vec{V}^\psi_{N_\psi^1N_\psi^2}(I)$, $\vec{\tilde{\theta}}_{N_\theta^1N_\theta^2},\vec{\overline{\theta}}_{N_\theta^1N_\theta^2}\in\vec{V}^\theta_{N_\theta^1N_\theta^2}(I)$, corresponding to $\tilde{\mathbf{v}}_{\mathbf{N}^1\mathbf{N}^2},\mathbf{v}_{\mathbf{N}^1\mathbf{N}^2}\in\mathbf{V}_{\mathbf{N}^1\mathbf{N}^2}(\Omega)$, $\tilde{\varphi}_{N_\varphi^1N_\varphi^2},\overline{\varphi}_{N_\varphi^1N_\varphi^2}\in V^\varphi_{N_\varphi^1N_\varphi^2}(\Omega)$, $\tilde{\psi}_{N_\psi^1N_\psi^2},\overline{\psi}_{N_\psi^1N_\psi^2}\in V^\psi_{N_\psi^1N_\psi^2}(\Omega)$, $\tilde{\theta}_{N_\theta^1N_\theta^2},\overline{\theta}_{N_\theta^1N_\theta^2}\in$ $V^\theta_{N_\theta^1N_\theta^2}(\Omega)$, respectively. The linear forms $L^{\mathbf{u}}_{\mathbf{N}^{1,2}}$, $L^\varphi_{N_\varphi^{1,2}}$, $L^\psi_{N_\psi^{1,2}}$ and $L^\theta_{N_\theta^{1,2}}$ are given by the following expressions:

$$L^{\mathbf{u}}_{\mathbf{N}^{1,2}}(\vec{v}_{\mathbf{N}^1\mathbf{N}^2})=\sum_{i=1}^{3}\sum_{r_i^1=0}^{N_i^1}\sum_{r_i^2=0}^{N_i^2}\left(r_i^1+\frac{1}{2}\right)\left(r_i^2+\frac{1}{2}\right)\left[\int_{h_3^-}^{h_3^+}\frac{1}{h_1h_2}\overset{r_i^1r_i^2}{v_{\mathbf{N}^1\mathbf{N}^2 i}}\left(\overset{r_i^1r_i^2}{f_i}+\overset{r_i^2}{g_i}\Big|_{\Gamma^{1,+}}\gamma^{1,+}+\overset{r_i^1}{g_i}\Big|_{\Gamma^{2,+}}\gamma^{2,+}\right.\right.$$
$$\left.\left.+\overset{r_i^2}{g_i}\Big|_{\Gamma^{1,-}}\gamma^{1,-}(-1)^{r_i^1}+\overset{r_i^1}{g_i}\Big|_{\Gamma^{2,-}}\gamma^{2,-}(-1)^{r_i^2}\right)dx_3+\sum_{x_3=h_3^-,\,h_1h_2>0}\frac{1}{h_1h_2}\overset{r_i^1r_i^2}{g_i}\,tr_{h_3^+}(\overset{r_i^1r_i^2}{v_{\mathbf{N}^1\mathbf{N}^2 i}})\right],$$

$$L^{\varphi}_{N_\varphi^{1,2}}(\vec{\overline{\varphi}}_{N_\varphi^1N_\varphi^2})=\sum_{r_\varphi^1=0}^{N_\varphi^1}\sum_{r_\varphi^2=0}^{N_\varphi^2}\left(r_\varphi^1+\frac{1}{2}\right)\left(r_\varphi^2+\frac{1}{2}\right)\left[\int_{h_3^-}^{h_3^+}\frac{1}{h_1h_2}\overset{r_\varphi^1r_\varphi^2}{\overline{\varphi}_{N_\varphi^1N_\varphi^2}}\left(\overset{r_\varphi^1r_\varphi^2}{f^\varphi}+\overset{r_\varphi^2}{g^\varphi}\Big|_{\Gamma^{1,+}}\gamma^{1,+}+\overset{r_\varphi^1}{g^\varphi}\Big|_{\Gamma^{2,+}}\gamma^{2,+}\right.\right.$$
$$\left.\left.+\overset{r_\varphi^2}{g^\varphi}\Big|_{\Gamma^{1,-}}\gamma^{1,-}(-1)^{r_\varphi^1}+\overset{r_\varphi^1}{g^\varphi}\Big|_{\Gamma^{2,-}}\gamma^{2,-}(-1)^{r_\varphi^2}\right)dx_3+\sum_{x_3=h_3^-,\,h_1h_2>0}\frac{1}{h_1h_2}\overset{r_\varphi^1r_\varphi^2}{g^\varphi}\,tr_{h_3^+}(\overset{r_\varphi^1r_\varphi^2}{\overline{\varphi}_{N_\varphi^1N_\varphi^2}})\right],$$

$$L^{\psi}_{N_\psi^{1,2}}(\vec{\overline{\psi}}_{N_\psi^1N_\psi^2})=\sum_{r_\psi^1=0}^{N_\psi^1}\sum_{r_\psi^2=0}^{N_\psi^2}\left(r_\psi^1+\frac{1}{2}\right)\left(r_\psi^2+\frac{1}{2}\right)\left[\int_{h_3^-}^{h_3^+}\frac{1}{h_1h_2}\overset{r_\psi^1r_\psi^2}{\overline{\psi}_{N_\psi^1N_\psi^2}}\left(\overset{r_\psi^1r_\psi^2}{f^\psi}+\overset{r_\psi^2}{g^\psi}\Big|_{\Gamma^{1,+}}\gamma^{1,+}+\overset{r_\psi^1}{g^\psi}\Big|_{\Gamma^{2,+}}\gamma^{2,+}\right.\right.$$
$$\left.\left.+\overset{r_\psi^2}{g^\psi}\Big|_{\Gamma^{1,-}}\gamma^{1,-}(-1)^{r_\psi^1}+\overset{r_\psi^1}{g^\psi}\Big|_{\Gamma^{2,-}}\gamma^{2,-}(-1)^{r_\psi^2}\right)dx_3+\sum_{x_3=h_3^-,\,h_1h_2>0}\frac{1}{h_1h_2}\overset{r_\psi^1r_\psi^2}{g^\psi}\,tr_{h_3^+}(\overset{r_\psi^1r_\psi^2}{\overline{\psi}_{N_\psi^1N_\psi^2}})\right],$$

$$L^{\theta}_{N_\theta^{1,2}}(\vec{\overline{\theta}}_{N_\theta^1N_\theta^2})=\sum_{r_\theta^1=0}^{N_\theta^1}\sum_{r_\theta^2=0}^{N_\theta^2}\left(r_\theta^1+\frac{1}{2}\right)\left(r_\theta^2+\frac{1}{2}\right)\int_{h_3^-}^{h_3^+}\left[\frac{1}{h_1h_2}\overset{r_\theta^1r_\theta^2}{f^\theta}\,\overset{r_\theta^1r_\theta^2}{\overline{\theta}_{N_\theta^1N_\theta^2}}-\frac{1}{2h_1h_2}\sum_{\beta=1}^{2}\left((1+(-1)^{r_\theta^2+N_\theta^2+\beta})\overset{r_\theta^1,N_\theta^2+\beta}{f^\theta}\right.\right.$$

$$\left. +(1+(-1)^{r_\theta^1+N_\theta^1+\beta}) \overset{N_\theta^1+\beta,r_\theta^2}{f^\theta} \right) \overset{r_\theta^1 r_\theta^2}{\bar{\theta}}_{N_\theta^1 N_\theta^2} + \frac{1}{4h_1h_2}\sum_{\alpha,\beta=1}^{2}(1+(-1)^{r_\theta^1+N_\theta^1+\alpha})(1+(-1)^{r_\theta^2+N_\theta^2+\beta}) \overset{N_\theta^1+\alpha,N_\theta^2+\beta}{f^\theta} \left. \overset{r_\theta^1 r_\theta^2}{\bar{\theta}}_{N_\theta^1 N_\theta^2} \right] dx_3,$$

where $\gamma^{\alpha,\pm}=\sqrt{1+((h_\alpha^\pm)')^2}$, $\overset{r_1 r_2}{v}=\int_{h_1^-}^{h_1^+}\int_{h_2^-}^{h_2^+} vP_{r_1}(z_1)P_{r_2}(z_2)dx_1dx_2$, $\overset{r_\alpha}{v_\alpha}=\int_{h_\alpha^-}^{h_\alpha^+} v_\alpha P_{r_\alpha}(z_\alpha)dx_\alpha$, for all functions $v\in L^1(\Omega)$, $v_\alpha\in L^1(\Gamma^{3-\alpha,+})\cup L^1(\Gamma^{3-\alpha,-})$, $r_1,r_2\in\mathbf{N}\cup\{0\}$, $\alpha=1,2$.

For the constructed one-dimensional initial-boundary value problem (16)-(20) for thermo-electro-magneto-elastic bars, the following existence and uniqueness theorem is proved.

Theorem 1. Suppose that functions $h_1^\pm$ and $h_2^\pm$ are such that $\Omega\subset\mathbf{R}^3$ is a Lipschitz domain, $\rho,\chi\in L^\infty(\Omega)$, $\rho(x)>\alpha_\rho=const>0$, $\chi(x)>\alpha_\chi=const>0$, for almost all $x\in\Omega$, c_{ijpq}, ε_{pij}, b_{pij}, d_{ij}, ζ_{ij}, a_{ij}, η_{ij}, $\lambda_{ij}\in L^\infty(\Omega)$, $\partial\lambda_{ij}/\partial x_j\in L^3(\Omega)$, $i,j,p,q=1,2,3,$ and satisfy the symmetry conditions (10) and the following positive definiteness conditions

$$\sum_{i,j,p,q=1}^{3} c_{ijpq}\xi_{ij}\xi_{pq}\ge\alpha_c\sum_{i,j=1}^{3}(\xi_{ij})^2,\ \forall\xi_{ij}\in\mathbf{R},\xi_{ij}=\xi_{ji},\quad \sum_{i,j=1}^{3}\eta_{ij}\xi_j\xi_j\ge\alpha_\eta\sum_{i=1}^{3}(\xi_i)^2,\ \forall\xi_i\in\mathbf{R},$$
$$\sum_{i,j=1}^{3} d_{ij}\xi_j\xi_i+2\sum_{i,j=1}^{3}a_{ij}\bar{\xi}_j\xi_i+\sum_{i,j=1}^{3}\zeta_{ij}\bar{\xi}_j\bar{\xi}_i\ge\alpha_{da\zeta}\sum_{i=1}^{3}\left((\xi_i)^2+(\bar{\xi}_i)^2\right),\quad \forall\xi_i,\bar{\xi}_i\in\mathbf{R}, \tag{21}$$

for almost all $x\in\Omega$, where α_c, α_η, $\alpha_{da\zeta}$ are positive constants. If the given functions satisfy the following conditions

$$(h_1h_2)^{-1/2}\overset{r_i^1 r_i^2}{f_i}\in L^2(0,T;L^2(I)),\ (\gamma^{\alpha,+})^{3/4}h_{3-\alpha}^{-1/4}\frac{d^k}{dt^k}\overset{r_i^{3-\alpha}}{g_i}\Big|_{\Gamma^{\alpha,+}},(\gamma^{\alpha,-})^{3/4}h_{3-\alpha}^{-1/4}\frac{d^k}{dt^k}\overset{r_i^{3-\alpha}}{g_i}\Big|_{\Gamma^{\alpha,-}}\in L^2(0,T;L^{4/3}(I)),$$
$$(h_1h_2)^{-1/6}\frac{d^k}{dt^k}\overset{r_\varphi^1 r_\varphi^2}{f^\varphi}\in L^2(0,T;L^{6/5}(I)),\ (\gamma^{\alpha,+})^{3/4}h_{3-\alpha}^{-1/4}\frac{d^k}{dt^k}\overset{r_\varphi^{3-\alpha}}{g^\varphi}\Big|_{\Gamma^{\alpha,+}},(\gamma^{\alpha,-})^{3/4}h_{3-\alpha}^{-1/4}\frac{d^k}{dt^k}\overset{r_\varphi^{3-\alpha}}{g^\varphi}\Big|_{\Gamma^{\alpha,-}}\in L^2(0,T;L^{4/3}(I)),$$
$$(\gamma^{\alpha,+})^{3/4}h_{3-\alpha}^{-1/4}\frac{d^k}{dt^k}\overset{r_\psi^{3-\alpha}}{g^\psi}\Big|_{\Gamma^{\alpha,+}},(\gamma^{\alpha,-})^{3/4}h_{3-\alpha}^{-1/4}\frac{d^k}{dt^k}\overset{r_\psi^{3-\alpha}}{g^\psi}\Big|_{\Gamma^{\alpha,-}}\in L^2(0,T;L^{4/3}(I)),\quad k=0,1,$$
$$(h_1h_2)^{-1/6}\left(\overset{r_\theta^1 r_\theta^2}{f^\theta}-\frac{1}{2}\sum_{\beta=1}^{2}\left((1+(-1)^{r_\theta^2+N_\theta^2+\beta})\overset{r_\theta^1,N_\theta^2+\beta}{f^\theta}+(1+(-1)^{r_\theta^1+N_\theta^1+\beta})\overset{N_\theta^1+\beta,r_\theta^2}{f^\theta}\right)\right.$$
$$\left.+\frac{1}{4}\sum_{\alpha,\beta=1}^{2}(1+(-1)^{r_\theta^1+N_\theta^1+\alpha})(1+(-1)^{r_\theta^2+N_\theta^2+\beta})\overset{N_\theta^1+\alpha,N_\theta^2+\beta}{f^\theta}\right)\in L^2(0,T;L^{6/5}(I)),$$

where $0\le r_i^\alpha\le N_i^\alpha$, $0\le r_\varphi^\alpha\le N_\varphi^\alpha$, $0\le r_\psi^\alpha\le N_\psi^\alpha$, $0\le r_\theta^\alpha\le N_\theta^\alpha$, $\alpha=1,2$, $i=1,2,3$, and $\vec{u}_{\mathbf{N}^1\mathbf{N}^2 0}\in\vec{V}_{\mathbf{N}^1\mathbf{N}^2}(I)$, $\vec{u}_{\mathbf{N}^1\mathbf{N}^2 1}\in\vec{H}_{\mathbf{N}^1\mathbf{N}^2}(I)$, $\vec{\theta}_{N_\theta^1N_\theta^2 0}\in\vec{H}^\theta_{N_\theta^1N_\theta^2}(I)$, then the initial-boundary value problem (16)-(20) possesses a unique solution.

In the following theorem we present the existence and uniqueness result for the three-dimensional initial-boundary problem and the result on the relationship between the obtained one-dimensional and the original three-dimensional problems, where we use the following anisotropic weighted Sobolev spaces

$$H^{s,s,2}_{h_{1,2}^\pm}(\Omega)=\{v\in H^1(\Omega);\partial_\alpha^{\tilde{k}}v\in H^2(\Omega),(h_\alpha^\pm)'\partial_1\partial_2\partial_\alpha^{\tilde{k}}v\in L^2(\Omega),0\le\tilde{k}\le s-2,$$
$$(h_\alpha^\pm)'\partial_\alpha^k v\in L^2(\Omega),1\le k\le s,\alpha=1,2\},\qquad s\in\mathbf{N},s\ge2,$$

which are Hilbert spaces equipped with the norms

$$\|v\|^2_{H^{s,s,2}_{h^\pm_{1,2}}(\Omega)}=\sum_{\alpha=1}^{2}\left(\sum_{\bar k=0}^{s-2}\left(\left\|\partial_\alpha^{\bar k}v\right\|^2_{H^2(\Omega)}+\left\|(h_\alpha^+)'\partial_1\partial_2\partial_\alpha^{\bar k}v\right\|^2_{L^2(\Omega)}+\left\|(h_\alpha^-)'\partial_1\partial_2\partial_\alpha^{\bar k}v\right\|^2_{L^2(\Omega)}\right)+\sum_{k=1}^{s}\left(\left\|(h_\alpha^+)'\partial_\alpha^k v\right\|^2_{L^2(\Omega)}+\left\|(h_\alpha^-)'\partial_\alpha^k v\right\|^2_{L^2(\Omega)}\right)\right).$$

Theorem 2. If $\Omega\subset\mathbf{R}^3$ is a bounded domain with Lipschitz boundary, $\mathbf{u}_0\in\mathbf{V}(\Omega)$, $\mathbf{u}_1\in\mathbf{L}^2(\Omega)$, $\theta_0\in L^2(\Omega)$, $\mathbf{f}\in L^2(0,T;\mathbf{L}^2(\Omega))$, $\mathbf{g},\mathbf{g}'\in L^2(0,T;\mathbf{L}^{4/3}(\Gamma_1))$, $f^\varphi,(f^\varphi)'\in L^2(0,T;L^{6/5}(\Omega))$, $g^\varphi,(g^\varphi)'\in L^2(0,T;L^{4/3}(\Gamma_1))$, $g^\psi,(g^\psi)'\in L^2(0,T;L^{4/3}(\Gamma_1))$, $f^\theta\in L^2(0,T;L^{6/5}(\Omega))$, $\rho,\chi\in L^\infty(\Omega)$, $\rho(x)>\alpha_\rho=const>0$, $\chi(x)>\alpha_\chi=const>0$, for almost all $x\in\Omega$, $c_{ijpq},\varepsilon_{pij},b_{pij},d_{ij},\zeta_{ij},a_{ij},\eta_{ij},\lambda_{ij}\in L^\infty(\Omega)$, $\partial\lambda_{ij}/\partial x_j\in L^3(\Omega)$, $i,j,p,q=1,2,3$, the symmetry and positive definiteness conditions (10), (21) are fulfilled, and the functions $\mathbf{u}_{\mathbf{N}^1\mathbf{N}^2 0}\in\mathbf{V}_{\mathbf{N}^1\mathbf{N}^2}(\Omega)$, $\mathbf{u}_{\mathbf{N}^1\mathbf{N}^2 1}\in\mathbf{H}_{\mathbf{N}^1\mathbf{N}^2}(\Omega)$, $\theta_{N_\theta^1N_\theta^2 0}\in H^\theta_{N_\theta^1N_\theta^2}(\Omega)$, corresponding to the initial conditions $\vec u_{\mathbf{N}^1\mathbf{N}^2 0}\in\vec V_{\mathbf{N}^1\mathbf{N}^2}(I)$, $\vec u_{\mathbf{N}^1\mathbf{N}^2 1}\in\vec H_{\mathbf{N}^1\mathbf{N}^2}(I)$, $\vec\theta_{N_\theta^1N_\theta^2 0}\in\vec H^\theta_{N_\theta^1N_\theta^2}(I)$ of the one-dimensional problems, tend to $\mathbf{u}_0$, $\mathbf{u}_1$ and θ_0 in the spaces $\mathbf{H}^1(\Omega)$, $\mathbf{L}^2(\Omega)$ and $L^2(\Omega)$, respectively, as $N_{\min}=\min\limits_{1\le i\le 3}\{N^{1,2}_{1,2,3},N^{1,2}_\varphi,N^{1,2}_\psi,N^{1,2}_\theta\}\to\infty$, then the three-dimensional problem (11)-(15) possesses a unique solution and the functions $\mathbf{u}_{\mathbf{N}^1\mathbf{N}^2}(t)$, $\varphi_{N_\varphi^1N_\varphi^2}(t)$, $\psi_{N_\psi^1N_\psi^2}(t)$, $\theta_{N_\theta^1N_\theta^2}(t)$, restored from the solutions $\vec u_{\mathbf{N}^1\mathbf{N}^2}$, $\vec\varphi_{N_\varphi^1N_\varphi^2}$, $\vec\psi_{N_\psi^1N_\psi^2}$, $\vec\theta_{N_\theta^1N_\theta^2}$ of the problem (16)-(20), possess the following properties

$$\begin{aligned}&\mathbf{u}_{\mathbf{N}^1\mathbf{N}^2}(t)\to\mathbf{u}(t)\ \text{ in }\mathbf{H}^1(\Omega), && \varphi_{N_\varphi^1N_\varphi^2}(t)\to\varphi(t)\ \text{ in }H^1(\Omega),\\ &\mathbf{u}'_{\mathbf{N}^1\mathbf{N}^2}(t)\to\mathbf{u}'(t)\ \text{ in }\mathbf{L}^2(\Omega), && \qquad\qquad\qquad\text{for all } t\in[0,T],\ \text{as } N_{\min}\to\infty.\\ &\theta_{N_\theta^1N_\theta^2}(t)\to\theta(t)\ \text{ in }L^2(\Omega), && \psi_{N_\psi^1N_\psi^2}(t)\to\psi(t)\ \text{ in }H^1(\Omega),\end{aligned}$$

In addition, if
$d^r\mathbf{u}/dt^r\in L^2(0,T;\ (H^{s_r,s_r,2}_{h^\pm_{1,2}}(\Omega))^3)$, $s_r\in\mathbf{N}$, $r=0,1,2$, $s_0\ge s_1\ge s_2\ge 2$, $s_1\ge 3$, $\varphi,\varphi'\in L^2(0,T;H^{s_1^\varphi,s_1^\varphi,2}_{h^\pm_{1,2}}(\Omega))$, $\psi,\psi'\in L^2(0,T;H^{s_1^\psi,s_1^\psi,2}_{h^\pm_{1,2}}(\Omega))$, $s_1^\varphi,s_1^\psi\in\mathbf{N}$, $s_1^\varphi\ge 3$, $s_1^\psi\ge 3$, $d^{\bar r}\theta/dt^{\bar r}\in L^2(0,T;H^{s^\theta_{\bar r},s^\theta_{\bar r},2}_{h^\pm_{1,2}}(\Omega))$, $s^\theta_{\bar r}\in\mathbf{N}$, $\bar r=0,1$, $s_0^\theta\ge s_1^\theta\ge 2$, $s_0^\theta\ge 3$, then, for suitable initial data $\vec u_{\mathbf{N}^1\mathbf{N}^2 0}$, $\vec u_{\mathbf{N}^1\mathbf{N}^2 1}$ and $\vec\theta_{N_\theta^1N_\theta^2 0}$, the following estimate is valid

$$\begin{aligned}&\left\|\mathbf{u}-\mathbf{u}_{\mathbf{N}^1\mathbf{N}^2}\right\|_{C([0,T];\mathbf{H}^1(\Omega))}+\left\|\mathbf{u}'-\mathbf{u}'_{\mathbf{N}^1\mathbf{N}^2}\right\|_{C([0,T];\mathbf{L}^2(\Omega))}+\left\|\varphi-\varphi_{N_\varphi^1N_\varphi^2}\right\|_{C([0,T];H^1(\Omega))}+\left\|\psi-\psi_{N_\psi^1N_\psi^2}\right\|_{C([0,T];H^1(\Omega))}\\ &+\left\|\theta-\theta_{N_\theta^1N_\theta^2}\right\|_{C([0,T];L^2(\Omega))}+\left\|\theta-\theta_{N_\theta^1N_\theta^2}\right\|_{L^2(0,T;H^1(\Omega))}\le\frac{1}{(N_{\min})^s}\,o(T,h_1^\pm,h_2^\pm,\mathbf{N}^1,\mathbf{N}^2,N_\varphi^1,N_\varphi^2,N_\psi^1,N_\psi^2,N_\theta^1,N_\theta^2),\end{aligned}$$

where $s=\min\{s_2,s_1-1,s_1^\varphi-1,s_1^\psi-1,s_1^\theta,s_0^\theta-1\}$, $o(T,h_1^\pm,h_2^\pm,\mathbf{N}^1,\mathbf{N}^2,N_\varphi^1,N_\varphi^2,N_\psi^1,N_\psi^2,N_\theta^1,N_\theta^2)\to 0$, as $N_{\min}\to\infty$.

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მათემატიკა

თერმო-ელექტრო-მაგნიტო-დრეკადი ძელების ერთგანზომილებიანი მოდელების გამოკვლევის შესახებ

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(წარმოდგენილია აკადემიის წევრის ე. ნადარაიას მიერ)

წარმოდგენილ ნაშრომში განხილულია ანიზოტროპული არაერთგვაროვანი მასალისაგან შემდგარი თერმო-ელექტრო-მაგნიტო-დრეკადი ძელების წრფივი დინამიკური მოდელი. აგებულია შესაბამისი სამგანზომილებიანი საწყის-სასაზღვრო ამოცანის ერთგანზომილებიანი ამოცანებით აპროქსიმაციის ალგორითმი, როცა ძელის გვერდით პირით ზედაპირებზე მოცემულია ზედაპირული ძალის სიმკვრივე და ელექტრული გადაადგილების და მაგნიტური ველის ინდუქციის მდგენელები ზედაპირის გარე ნორმალის გასწვრივ. მიღებული ერთგანზომილებიანი საწყის-სასაზღვრო ამოცანები გამოკვლეულია სათანადო ფუნქციონალურ სივრცეებში. ამავე დროს, დამტკიცებულია აგებული ერთგანზომილებიანი ამოცანების ამონახსნებიდან აღდგენილი სამი სივრცითი ცვლადის ვექტორ-ფუნქციების მიმდევრობის შესაბამის ფუნქციონალურ სივრცეში კრებადობა საწყისი სამგანზომილებიანი საწყის-სასაზღვრო ამოცანის ამონახსნისაკენ და დამატებით პირობებში შეფასებულია კრებადობის რიგი.

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