საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, ტ. 14, №3, 2020
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 14, no. 3, 2020

Informatics

A New Hesitant Fuzzy TOPSIS Approach in Multi-Attribute Group Decision Making

Gia Sirbiladze*, Irina Khutsishvili*, Anna Sikharulidze*, Teimuraz Manjapharashvili*, Roberto Santana**

* *Faculty of Exact and Natural Sciences, Ivane Javakhishvili Tbilisi State University, Tbilisi, Georgia*
** *Department of Computer Sciences and Artificial Intelligence of the Basque Country University, Spain.*

(Presented by Academy Member Elizbar Nadaraya)

The proposed multi-attribute decision making methodology applies the TOPSIS (Technique for Order Performance by Similarity to Ideal Solution) approach under hesitant fuzzy environment. The case when the information on the attributes weights is completely unknown is considered. The identification of the weights of attributes which is based on De Luca-Termini information entropy is presented in the context of hesitant fuzzy sets. The ranking of alternatives is performed in accordance with the proximity of their distances to the both fuzzy positive ideal solution (FPIS) and fuzzy negative ideal solution (FNIS). *© 2020 Bull. Georg. Natl. Acad. Sci.*

Multiple attribute group decision making, hesitant fuzzy sets, TOPSIS approach

In the proposed methodology the values of the attributes initially are given by all decision makers in the form of lingual expressions. Then, these lingual expressions are converted into the trapezoidal fuzzy numbers [1]. Decisions are made using TOPSIS method [2] constructed for the hesitant trapezoidal fuzzy environment [3].

The case when the information on the attributes weights is completely unknown is considered. The attributes weights are obtained by applying De Luca-Termini non-probabilistic entropy concept [4], which is considered in the context of hesitant fuzzy sets.

A hesitant fuzzy TOPSIS method is employed for ranking the alternatives. In the TOPSIS method the alternative with the nearest distance from the so-called fuzzy positive ideal solution (FPIS) and the farthest distance from the fuzzy negative ideal solution (FNIS) [2] is identified as optimal with respect to all attributes. Following the TOPSIS method's algorithm, a relative closeness coefficient is defined to determine the ranking order of all alternatives by calculating the distances to the both FPIS and FNIS. The developed approach was applied to evaluation of possible alternatives with the aim of their ranking in different decision making problems.

Preliminary concepts

A trapezoidal fuzzy number [1] $\tilde{A}$ can be determined by a quadruple $\tilde{A}=(a,b,c,d)$. Its membership function is defined as

$$\mu_{\tilde{A}}(x)=\begin{cases} 0, & if\ x<a, \\ \dfrac{x-a}{b-a} & if\ a\le x\le b, \\ 1, & if\ b\le x\le c, \\ \dfrac{d-x}{d-c} & if\ c\le x\le d, \\ 0 & if\ x>d, \end{cases}$$

where $a\le b\le c\le d$.

Let's $\tilde{A}=(a,b,c,d)$ be the trapezoidal fuzzy number. Using Graded Mean Integration Representation Method we can get following representation of $\tilde{A}$ by formula

$$p(\tilde{A})=(a+2b+2c+d)/6. \tag{1}$$

In HFS the degree of membership of an element to a reference set is presented by several possible fuzzy values. This allows to describe such situations when decision makers (DMs) have hesitancy in providing their preferences over alternatives. The HFS is defined as follows:

Definition 1 [3]: Let $X=\{x_1,x_2,...,x_n\}$ be a reference set. A hesitant fuzzy set H on X is defined in terms of a function $h_H(x)$, which, when applied to X, returns a subset of [0,1]:

$$H=\{<x,h_H(x)>|\,x\in X\},$$

where $h_H(x)$ is a set of some different values in [0,1], representing the possible membership degrees of the element $x\in X$ to H; $h_H(x)$ is called a hesitant fuzzy element (HFE).

Definition 2 [5]: Let M and N be two HFSs on $X=\{x_1,x_2,...,x_n\}$, then the distance measure between M and N is defined as $d(M,N)$, which satisfies the following properties:

1) $0\le d(M,N)\le 1$; 2) $d(M,N)=0$ iff $M=N$; 3) $d(M,N)=d(N,M)$.

It is clear that the number of values (length) for different HFEs may be different. Let $l(h_H(x))$ be the length of $h_H(x)$. After arranging the elements of $h_H(x)$ in a decreasing order, let $h_H^{\sigma(j)}(x)$ be the *j*th largest value in $h_H(x)$. To calculate the distance between M and N when $l(h_M(x_i))\ne l(h_N(x_i))$, it is necessary to extend the shorter one by adding any value in it, until both have the same length. The choice of this value depends on the expert's risk preferences. Optimist experts may add the maximum value from HFE, while pessimists may add the minimal value.

In this work the hesitant weighted Hamming distance is used that is defined by following formula

$$d_{hwh}(M,N)=\sum_{i=1}^{n}w_i\left[\frac{1}{l_{x_i}}\sum_{j=1}^{l_{x_i}}\left|h_M^{\sigma(j)}(x_i)-h_N^{\sigma(j)}(x_i)\right|\right], \tag{2}$$

where $h_M^{\sigma(j)}(x_i)$ and $h_N^{\sigma(j)}(x_i)$ are the *j*th largest values in $h_M(x_i)$ and $h_N(x_i)$ respectively; $l_{x_i} = \max\{l(h_M(x_i)), l(h_N(x_i))\}$ for each $x_i \in X$; w_i $(i = 1,2,...,n)$ is the weight of the element $x_i \in X$ such that $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$.

Definition 3 [3]: For a HFE $h_H(x)$, the score function $s(h_H(x))$ is defined as follows:

$$s\left(h_H(x)\right) = \sum_{j=1}^{l(h_H(x))} h_H^{\sigma(j)}(x) \Big/ l(h_H(x)), \tag{3}$$

where $s\left(h_H(x)\right) \in [0,1]$.

Let h_1 and h_2 are two HFEs. Based on score function it is possible to make ranking of HFEs according to the following rules: $h_1 > h_2$, if $s\left(h_1\right) > s\left(h_2\right)$; $h_1 < h_2$, if $s\left(h_1\right) < s\left(h_2\right)$ and $h_1 = h_2$, if $s\left(h_1\right) = s\left(h_2\right)$.

Formulation of multi-attribute group decision making (MAGDM) problem in Hesitant Fuzzy Environment

Consider a general structure of MAGDM problem. Assume that there are m decision making alternatives $A = \{A_1, A_2, \dots, A_m\}$, and the group $E = \{e_1, e_2, \dots, e_k\}$ of k experts or decision makers (DM) evaluate them with respect to an n attributes $X = \{x_1, x_2, \dots, x_n\}$.

DMs provide evaluations over attributes in form of verbal assessments – linguistic terms. Then, these assessments are expressed in trapezoidal fuzzy numbers (TrFNs) using 5-point linguistic scale (Table 1):

Table 1. Linguistic scale for rating of alternatives

Linguistic term	Corresponding TrFNs
Very low (VL)	(0, 0.1, 0.2, 0.3)
Low (L)	(0.1, 0.2, 0.3, 0.4)
Medium (M)	(0.3, 0.4, 0.5, 0.6)
High (H)	(0.5, 0.6, 0.7, 0.8)
Very high (VH)	(0.7, 0.8, 0.9, 1.0)

After those transformations of lingual expressions, experts' joint assessments concerning each alternative represent HTrFS:

A HTrFS A_i of the *i*th alternative on X is given by

$$A_i = \left\{ < x_j, f_{A_i}(x_j) > | \, x_j \in X \right\},$$

where $f_{A_i}(x_j)$, $i = 1, 2, \dots, m;\ j = 1, 2, \dots, n$ indicates the possible membership degrees of the *i*th alternative A_i under the *j*th attribute x_j, and it can be expressed as a HTrFE $\tilde{t}_{ij}$. All HTrFEs create the aggregate fuzzy hesitant trapezoidal decision matrix $\tilde{T} = (\tilde{t}_{ij})_{m \times n}$.

Considering that the attributes have different importance degrees, we denote the vector of attributes weights by $w = \left(w_1, w_2, \dots, w_n\right)$, where $0 \le w_j \le 1$, $\sum_{j=1}^{n} w_j = 1$, and w_j is the importance degree of *j*th attribute.

Then a hesitant MAGDM problem can be expressed in matrix format as follows

$$\tilde{T} = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & \cdots & x_n \end{array} \\ \begin{bmatrix} \tilde{t}_{11} & \tilde{t}_{12} & \cdots & \tilde{t}_{1n} \\ \tilde{t}_{21} & \tilde{t}_{22} & \cdots & \tilde{t}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \tilde{t}_{m1} & \tilde{t}_{m2} & \cdots & \tilde{t}_{mn} \end{bmatrix} \end{array},$$

$$w = (w_1, w_2, \dots, w_n), \quad E = \{e_1, e_2, \dots, e_k\},$$

where $\tilde{T}$ is the hesitant trapezoidal fuzzy decision matrix, each element of which represents an HTrFE $\tilde{t}_{ij}$.

Determination of the attributes weights using De Luca-Termini entropy

The complexity and uncertainty of the investment decision problems leads to the information on attributes weights usually being incomplete or completely unknown. Here we consider a case when the attributes weights are unknown.

Suppose we have hesitant decision matrix $H = (h_{ij})_{m\times n}$, each element of which represents a HFE.

De Luca and Termini [4] defined a non-probabilistic entropy formula of a fuzzy set based on Shannon's function on a finite universal set X as:

$$E_{LT} = -k\sum_{i=1}^{n}\left[\mu_A(x_i)\ln\mu_A(x_i) + \left(1-\mu_A(x_i)\right)\ln\left(1-\mu_A(x_i)\right)\right], \quad k > 0,$$

where $\mu_A : X \to [0,1]$; k is a positive constant.

The attributes weights definition method based on the De Luca-Termini entropy can be described as follows:

Step1: Calculate the score matrix $S = \left(s_{ij}\right)_{m\times n}$ of hesitant decision matrix H, where $s_{ij} = s\left(h_{ij}\right)$ is the score value of h_{ij} (see formula (3)).

Step2: Calculate the normalized score matrix $S' = \left(s'_{ij}\right)_{m\times n}$, where

$$s'_{ij} = s_{ij} \Big/ \sum\nolimits_{i=1}^{m} s_{ij}\,. \tag{4}$$

Step3: Determine the attributes weights.

By using De Luca-Termini normalized entropy in context of hesitant fuzzy sets

$$E_j = -\frac{1}{m\ln 2}\sum_{i=1}^{m}\left(s'_{ij}\ln s'_{ij} + (1-s'_{ij})\ln(1-s'_{ij})\right), \; j = 1, 2, \dots, n, \tag{5}$$

the definition of the attributes weights is expressed by the formula

$$w_j = \frac{1-E_j}{\sum\nolimits_{j=1}^{n}(1-E_j)}, \; j = 1, 2, \dots, n, \tag{6}$$

where the value of w_j represents the relative intensity of x_j attribute importance.

Evaluation of the alternatives using TOPSIS approach

The idea of the TOPSIS method [2] as applied to the MAGDM problem consists in the choice of the best alternative in accordance with distances from both FPIS and FNIS, namely with the nearest distance from FPIS and the farthest from FNIS. Practical aspects of Fuzzy TOPSIS has been considered to decision making problems by authors of this paper in [6].

The algorithm of solving the constructed Fuzzy TOPSIS in MAGDM problem can be formulated as follows:

Step 1: Convert the experts verbal assessments into the assessments in a form of trapezoidal fuzzy numbers.

Step 2: Construct the aggregate hesitant trapezoidal decision matrix $\tilde{T}=(\tilde{t}_{ij})_{m\times n}$ based on the experts' hesitant trapezoidal evaluations.

Step 3: Transform aggregate hesitant trapezoidal decision matrix $\tilde{T}=(\tilde{t}_{ij})_{m\times n}$ into aggregate hesitant decision matrix $H=(h_{ij})_{m\times n}$ by using Graded Mean Integration Representation Method.

Step 4: Determine the criteria weights $w=(w_1,w_2,\dots,w_n)$ based on the method given in Section 2.

Step 5: Determine corresponding hesitant FPIS A^+ and hesitant FNIS A^-.

FPIS is composed of the best performance values for each attribute whereas FNIS consists of the worst performance values.

There are attributes of two types:

a) the benefit type attribute - this means that the bigger attribute's value the better;

b) the cost type attribute - that is, the smaller the attribute's value the better.

Calculate A^+ and A^- by formulas:

$$A^+=\left\{\max_i\left\langle h_{ij}^{\sigma(\lambda)}\right\rangle \mid j\in J';\ \min_i\left\langle h_{ij}^{\sigma(\lambda)}\right\rangle \mid j\in J''\right\}, \tag{7}$$

$$A^-=\left\{\min_i\left\langle h_{ij}^{\sigma(\lambda)}\right\rangle \mid j\in J';\ \max_i\left\langle h_{ij}^{\sigma(\lambda)}\right\rangle \mid j\in J''\right\},\ \lambda=1,2,\dots,n, \tag{8}$$

where J' is associated with a benefit attribute, and J'' - with a cost attribute.

Step 6: Using (2) calculate the separation measures d_i^+ and d_i^- of each alternative A_i from the hesitant FPIS A^+ and the hesitant FNIS A^-, respectively:

$$d_i^+=\sum\nolimits_{j=1}^{n} d(h_{ij},h_j^+)\,w_j\ =\sum_{j=1}^{n} w_j\left[\frac{1}{l}\sum_{j=1}^{l}\left|h_{ij}^{\sigma(j)}-(h_j^{\sigma(j)})^+\right|\right], \tag{9}$$

$$d_i^-=\sum\nolimits_{j=1}^{n} d(h_{ij},h_j^-)\,w_j\ =\sum_{j=1}^{n} w_j\left[\frac{1}{l}\sum_{j=1}^{l}\left|h_{ij}^{\sigma(j)}-(h_j^{\sigma(j)})^-\right|\right],\ i=1,2,\dots,m. \tag{10}$$

Step 7: Calculate the relative closeness coefficient RC_i of each alternative A_i to the hesitant FPIS A^+:

$$RC_i=d_i^-\big/(d_i^++d_i^-),\ \ i=1,2,..,m. \tag{11}$$

Step 8: Perform the ranking of the alternatives A_i, $i=1,2,\dots,m$ according to the relative closeness coefficients RC_i, $i=1,2,\dots,m$ by the rule: for two alternatives A_α and A_β we say that A_α is more preferred than A_β, *i.e.* $A_\alpha \succcurlyeq A_\beta$, *if* $RC_\alpha \ge RC_\beta$, where $\succcurlyeq$ is a preference relation on A.

Conclusions

The new aspects in the TOPSIS approach have been used. We proposed a new attributes weighting method based on De Luca-Termini information entropy to express the relative intensities of attribute importance and determine the attributes weights.

Based on proposed methodology we have developed software package and used it to rank decision alternatives in the real decision making problems.

The work was supported by Shota Rustaveli National Science Foundation of Georgia (SRNSFG) [FR-18-466].

ინფორმატიკა

ახალი ჰესიტანტური ფაზი-TOPSIS მიდგომა მრავალატრიბუტული ჯგუფური გადაწყვეტილების მიღებაში

გ. სირბილაძე*, ი.ხუციშვილი*, ა. სიხარულიძე*, თ. მანჯაფარაშვილი*, რ. სანტანა**

** ივანე ჯავახიშვილის სახ. თბილისის სახელმწიფო უნივერსიტეტი, კომპიუტერულ მეცნიერებათა ფაკულტეტი, თბილისი, საქართველო*
*** კომპიუტერული მეცნიერების დეპარტამენტი და ბასკეთის ქვეყნის უნივერსიტეტის ხელოვნური ინტელექტი, ესპანეთი*

(წარმოდგენილია აკადემიის წევრის ე. ნადარაიას მიერ)

წარმოდგენილი მრავალ-ატრიბუტული გადაწყვეტილების მიღების მეთოდოლოგია იყენებს TOPSIS მიდგომას ჰესიტანტურ ფაზი-გარემოში. განიხილება შემთხვევა, როდესაც ინფორმაცია ატრიბუტების წონებზე სრულიად არ არსებობს. ატრიბუტების წონების იდენტიფიკაცია, დაფუძნებული დე ლუკა-ტერმინის ინფორმაციულ ენტროპიაზე, წარმოდგენილია ჰესიტანტური ფაზი-სიმრავლეების კონტექსტში. ალტერნატივების რანჟირება სრულდება ორივე – ფაზი დადებით იდეალურ გადაწყვეტილებასა (FPIS) და ფაზი უარყოფით იდეალურ გადაწყვეტილებასთან (FNIS) სიახლოვის შესაბამისად.

REFERENCES

1. Dubois D., Prade H. (1988) Possibility theory: an approach to computerized processing of uncertainty. Plenum Press, New-York.
2. Hwang C. L., Yoon K. (1981) Multiple attribute decision making: methods and applications. Verlag, Berlin-Heidelberg.
3. Torra V., Narukawa Y. (2009) On hesitant fuzzy sets and decision. In: The 18th IEEE International Conference on Fuzzy Systems, 1378-1382. Jeju Island, Korea.
4. De Luca A., Termini S. (1972) A definition of a nonprobabilistic entropy in the setting of fuzzy sets theory. *Information and Control*, **20** : 301-312.
5. Xu Z., Xia M. (2011) Distance and similarity measures for hesitant fuzzy sets. *Information Sciences*, **181** : 2128-2138.
6. Sirbiladze G., Khutsishvili I., Sikharulidze A., Ghvaberidze B. (2019) Hesitant fuzzy TOPSIS based facility location selection problem. *Bulletin of TICMI*, **23** (2) : 131-141.

Received May, 2020