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Geophysics

Scaling Features of Earthquakes Occurrences in the Equally Distributed Non-Overlapping Time Windows

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Present research is devoted to the analysis of scale-free features of distribution of the number of earthquakes occurred in the time windows of a given duration. We based our analysis on the well-known facts indicating the scale-free power law relationship in different characteristics of seismicity. From this point of view, it was interesting to know whether scale-free features would be retained, if we look at the distribution of earthquakes in a fixed size time windows, assuming that these windows are not empty and contain at least 30 events above selected magnitude thresholds. The obtained results show that, distribution of the number of earthquakes occurring in the fixed time windows is characterized by a scale-free features. The scale-free relation between the frequency of windows occurrence and the number of earthquakes occurring in time windows was observed for different magnitude thresholds. This is an interesting observation connecting two well-known scale-free features of seismic process, such as size-frequency distribution and temporal distribution of earthquakes. As it was mentioned above, we have found the scale-free features in these distributions in a mentioned sense and they are characteristic only for some ranges of number of earthquakes with certain magnitudes and for certain time windows. The result is clearly related to the original internal structure of seismic process and is absent when the original structure is intentionally destroyed.
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Seismology, earthquake catalogue, scaling, dynamics

Investigation of the well-known scaling relations characteristic of different aspects of earthquakes' distribution remain one of the important research tasks of contemporary earth sciences [1-6]. We can list plenty of researches devoted to the question of scaling features of seismic process in its spatial, temporal and energetic domains [5-10]. The importance of such analysis may be explained by the fact that assessment of scaling, or scale-free, characteristics of seismic data sets provides valuable information about the strength of correlations or about the extent of the regularity in the complicated process of earthquake generation. In most cases our knowledge about scaling features of seismic process is based on the analysis of waiting times, magnitudes or inter-earthquake distances sequences [see e.g. 5, 6, 8, 10, 11], the traditional objects of such analysis. At the same time, the logical possibility that other characteristics related with the process of earthquake generation may also pose scale-free properties (at least for certain ranges), still needs further proofing and documentation.

In the present research we aimed to look at the temporal features of seismic process from a new point of view and to analyze scaling features of data sets of number of earthquakes occurred in a consecutive non-overlapping time windows. In order to avoid statistical biases these windows were selected so that in each window occurred at least 30 earthquakes. We aimed to check whether a power-law relationship remains valid for earthquakes occurrence in a fixed size time windows at different magnitude thresholds.

Materials and Used Method of Analysis

In present research, we used southern California earthquake catalog freely available from http://www.isc.ac.uk/iscbulletin/search/catalogue. We selected 40 years time period from 1980 to 2020 (Fig. 1). The southern California (SC) earthquake catalog for the considered period is complete for events of magnitudes above M2.6, according to the Gutenberg–Richter relationship analysis [12].

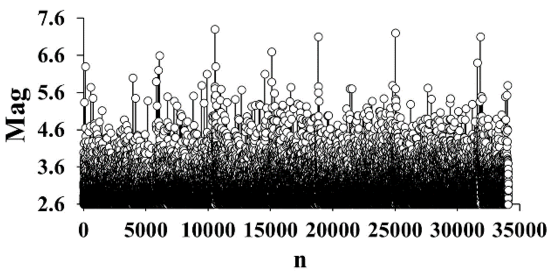

Fig. 1. The sequence of magnitudes in the South Californian earthquake catalogue, 1980-2020.

For different magnitude threshold values the numbers of earthquakes occurred in the consecutive non-overlapping time windows were calculated. As it was mentioned above, the time span of these windows were selected so that at least 30 earthquakes occurred in a time window. Thus, for the magnitude threshold M2.6 we have 220 windows of 96480-minute long (see Fig. 2). For M3.0 magnitude threshold, the number of 246240-minute long windows was 86. For M3.4 threshold, the number of 449280-minute long windows was 47. The data sets consisted of the sequence of number of earthquakes in these fixed time length windows were the subjects of our analysis. We named them the number of earthquakes (NEQ) data sets.

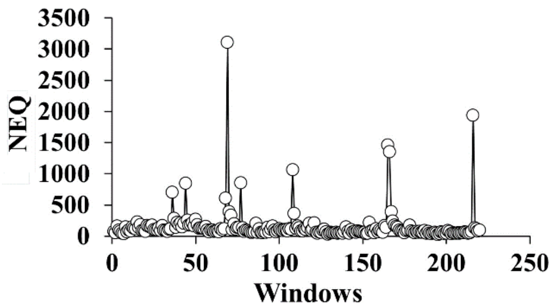

Fig. 2. Number of earthquakes (NEQ) occurred in non-overlapping 96480-minute long windows of SC earthquake catalogue above magnitude values M2.6.

As far as we aimed to asses the scaling features of NEQ data sets, it seemed preferable to use standard methods like detrended fluctuation analysis (DFA), which is often used for similar research purposes. Unfortunately, for our data sets this was impossible because, as we reported earlier, DFA can be used correctly when there are at least 500 data in the window [6]. Thus, we decided to apply a power law features testing of targeted NEQ data sets.

For the last several decades testing of power law scaling relationship has been used to represent and model the relationship between different dependent (*Y*) and independent (*X*) variables: $Y \sim X^b$. This relationship usually is rewritten using a logarithmic transformation. Graphically, this is equivalent to plotting Y and X in a *log–log* space, where parameter b (the slope) can be identified either graphically or numerically, under the assumptions of linear regression. In practice, to identify power-law behavior in either natural or man-made systems the simplest way is to use a histogram method for certain data sets, and see if the relationship in log-log scale looks straight [13]. Often recommended procedure here is to vary the width of the bins, after normalization of sample counts by the width of the bins and to calculate a cumulative probability or frequency of distribution. Thus, instead of plotting a simple histogram of the data, one makes a plot of the probability, or frequency, that X has a value greater or equal to the given threshold value. Logarithm of the relation between the amount of windows and the threshold values of the number of earthquakes in these windows are given in the plots below.

Results and Discussions

We started from the analysis of NEQ data sets from the original SC seismic catalogue. As mentioned in the previous section NEQ data sets have been constructed so that they contained earthquakes above the following magnitude values M2.6, M3.0 and M3.4. It needs to be underlined that, on the log-log plot of the relation between frequency of windows (which we name as N) and threshold values, n (the number of earthquakes in windows), close to the linear relationship is revealed just for the certain range of n values. Here we focus only on these linear parts presented in Fig. 3, where relation between frequency of earthquakes occurrences and threshold values (of earthquakes occurred in time windows) are given, from the left to right (curves in Fig. 3 are shifted to the right side for better visibility). First, we observed that in the southern California for a considered period the minimal time span, when at least 30 earthquakes above magnitude threshold M2.6 occur, is 67 days. Linear part of *LogN-Logn* relationship, meanwhile, is observed for the cases when the number of occurred earthquakes was in the range from 85 to 189 in each window (we point here again that this fact can not be seen in Fig. 3, because curves are shifted for better visibility). Portion of the number of such windows which formed a linear part of the *LogN-Logn* relationship varied from 12% to 54% of all windows (220 in case of M2.6). In the case of M3.0 magnitude threshold a minimal time span, when at least 30 earthquakes occur in SC catalogue, is 171 days. Linear part of *LogN-Logn* relationship is observed when the number of occurred earthquakes was in the range of 74 to 166 in each window. Portion of the number of windows in a linear range varied from 15% to 65% of all windows. The last magnitude threshold value we analyzed is M3.4, for which the minimal time span, when at least 30 earthquakes occur, is 312 days. The linear part of *LogN-Logn* relationship is observed when the number of occurred earthquakes was in the range from 38 to 94 in each window. Portion of the number of windows forming mostly linear part of *LogN-Logn* relationship varied from 19% to 91% of all windows.

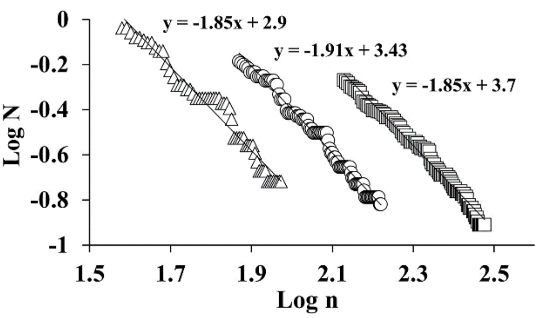

Fig. 3. Relation between the frequency of occurrences of fixed size time windows and threshold values of the number of earthquakes occurred in windows in the original SC catalogue.

Relationship between the number of windows and number of occurred earthquakes corresponding mostly to linear part of curves in Fig. 3, are shown in Fig. 4. Here we see that as the earthquake magnitude threshold increases, the portion of windows revealing scale-free features of relationship with the number of occurred earthquakes also increases. It is interesting that at the same time the minimal number of such windows almost do not change.

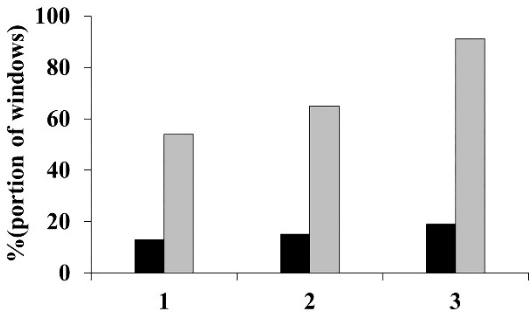

Fig. 4. Portion of windows which form the linear part of Log*N*-Log*n* relation at magnitude thresholds: 1) M2.6, 2) M3.0 and M3.4 (minimal number of windows – black, maximal number of windows-grey).

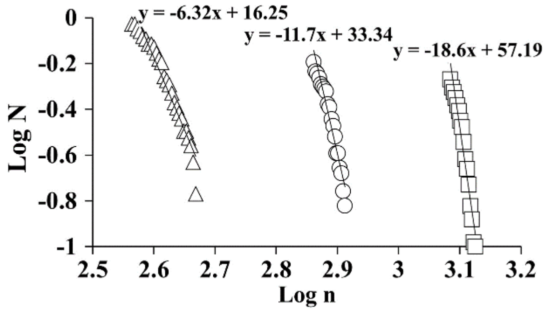

Fig. 5. Relation between the frequency of occurrences of fixed size time windows and threshold values of the number of earthquakes occurred in windows in the randomized SC catalogue.

In order to be convinced that the observed relationship is an internal property of earthquake time distribution we carried out the same analysis for earthquake catalogue, where the original time structure was intentionally destroyed by the procedure of shuffling. As Fig. 5 shows, *LogN* vs *Logn* relationship for the time structure destroyed southern Californian seismic catalogue, it is hard to select parts of these curves, which can be regarded as acceptably linear. In any case the slope of these curves is clearly larger compared to the original case, that points to a practical absence of the scale-free features for more or less accountable range of number of earthquakes in the analyzed fixed size windows.

Conclusion

According to the results of our analysis the scale-free features are characteristic for the sequences of the number of earthquakes occurred in the fixed time windows in SC catalogue. This is interesting observation connecting two well-known scale-free features of the seismic process such as the size-frequency distribution and time distribution of earthquakes. We found that the scale-free features in the mentioned sense are characteristic only for some ranges of the number of earthquakes with certain magnitudes and for certain time windows. This result is clearly related to the original internal structure of seismic process and is absent in the case, when the original structure is intentionally destroyed.

გეოფიზიკა

მიწისძვრების მოხდენის სკეილინგური თავისებურებები თანაბრად განაწილებულ არაგადამფარავ დროით ფანჯრებში

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აღნიშნულ კვლევაში განხილულია მოცემული სიგრძის დროის ინტერვალების ფანჯრებში მიწისძვრების რაოდენობის სკეილინგური მახასიათებლების განაწილება. ჩვენი ანალიზი ეფუძნება ცნობილ ფაქტებს, რომლებიც გვიჩვენებენ სკეილინგური ხარისხოვანი კანონის კავშირს სეისმურობის სხვადასხვა მახასიათებლებში. ასეთი მიდგომით საინტერესო იყო გაგვერკვია შენარჩუნდება თუ არა სკეილინგური მახასიათებლები ფიქსირებული ზომის დროით ფანჯრებში, თუ ვივარაუდებთ, რომ ეს ფანჯრები არ არის ცარიელი და შეიცავს გარკვეული მაგნიტუდის მნიშვნელობის 30 ან მეტ მოვლენას. მიღებული შედეგებიდან ჩანს, რომ ფიქსირებული დროით ფანჯრებში მიწისძვრების რაოდენობის განაწილება ხასიათდება სკეილინგური თავისებურებებით. განხილულ იქნა სხვადასხვა მაგნიტუდების დროს სკეილინგური კავშირი ფანჯრების წარმოქმნის სიხშირის და ფანჯრებში მომხდარი მიწისძვრების რაოდენობას შორის. კვლევა აერთიანებს კარგად ცნობილ სეისმური პროცესის ისეთ სკეილინგურ მახასიათებლებს, როგორიც არის მაგნიტუდების განაწილების სიხშირე და მიწისძვრების დროში განაწილება. როგორც აღინიშნა, გამოვლინდა, რომ სკეილინგური თავისებურებები ხასიათდება მხოლოდ მიწისძვრების რაოდენობის რამდენიმე დიაპაზონისთვის გარკვეული მაგნიტუდისა და დროის ინტერვალების ფანჯრებისათვის. მიღებული შედეგი ნათლად არის დაკავშირებული სეისმური პროცესის თავდაპირველ შიდა სტრუქტურასთან და არ ვლინდება ორიგინალური სტრუქტურის განზრახ გაიშვიათების შემდეგ.

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