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Physics

States of ρ° and ω Mesons and ρ°−ω Mixing in the Relativistic Quark Model

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ρ° and ω meson states and ρ−ω mixing in the relativistic quark model with SU(2) symmetry breaking are considered. Mesons, as the $q\bar{q}$ quark-antiquark bound states, were previously studied within the framework of relativistic quark model with the condition of symmetry SU(2) conservation, i.e. $m_u = m_d$. With this consideration, the masses of ρ° and ω mesons are equal. But the charge, or isotop symmetry SU(2), is broken because of the small mass difference between the up and down current quarks. As a consequence, the physical ρ° and ω mesons are not isospin eigenstates, but contain a small admixture of states with different isospins. This phenomenon is known as ρ−ω mixing. In addition, the masses of physical ρ° and ω vector mesons are different. Since the masses of the current u_0 and d_0 quarks are different, it can be assumed that the corresponding constituent quarks masses are also different $m_u \neq m_d$. The states vectors of physical mesons as a linear combinations of the states vectors of $u\bar{u}$ and $d\bar{d}$, bound systems with different masses, are represented, in which mixing is expressed by the presence of a small mixing parameter ε. Expressions are obtained for the mass mixing parameter ε_m, the real quantity, and for the meson states mixing parameter ε, complex quantity. Both parameters are expressed by the masses of mesons and bound systems $u\bar{u}$ and $d\bar{d}$. Expressions are obtained also for quark charges $\bar{e}_{\rho^0}$ and $\bar{e}_\omega$, which depend on the isotopic states of mesons. The meson decay constant is generalized to the case of breaking of isotopic symmetry. The calculations were made using the Salpeter equation with the oscillator potential for the confinement. Several sets of constituent quark masses and the potential parameters were found, which describe well the states of ρ° and ω mesons, ρ−ω mixing and give the leptonic decay widths of ρ° and ω vector mesons, which are in very good agreement with experimental data. The obtained results, sets of quark masses and potential parameters, can be used in considering the states of other mesons.

Bound systems, mesons, relativistic quark model, Salpeter equation, mixing parameters, lepton decay width

Mesons, as the quark-antiquark $q\bar{q}$ bound states, were previously studied within the framework of Relativistic quark model (RQM) based on Bete-Salpeter equation in its instantaneous approximation, i.e. Salpeter equation (SE) [1,2] and many their properties are successfully described with the condition of

SU(2) symmetry conservation, $m_u = m_d$ (see, e.g. [3-6]). With this consideration, the masses of ρ^o and ω mesons are equal. However, there is a small difference between the masses of u_o and d_o current quarks, SU(2) symmetry is broken. As a consequence, the masses of physical ρ^o and ω vector mesons are different. They are also not eigenstates of isospin with values I = 1 and I = 0 and contain a small admixture of states with I = 0 and I = 1, respectively. This is one of the reasons for the $\rho^o - \omega$ mixing. Another reason for mixing in the quark model can be interaction which mixes the states $u\bar{u}$ and $d\bar{d}$.

States of ρ° and ω Mesons and ρ−ω Mixing

The observed breaking of isotopic symmetry is small, so the mixing of states can be represented as a linear combinations of the mesons states with isospins I = 1 and I = 0 containing a small mixing parameter ε [7]:

$$\begin{cases} |\rho\rangle = |\rho_I\rangle - \varepsilon|\omega_I\rangle, \\ |\omega\rangle = |\omega_I\rangle + \varepsilon^*|\rho_I\rangle. \end{cases} \tag{1}$$

Isospin eigenstates with values I = 1 and I = 0 and with projections $I_z = 0$ in the relativistic quark model with conservation of SU (2) symmetry are known $(m_u = m_d)$:

$$\begin{cases} |\rho_I^o, 10\rangle = |\varphi\rangle \frac{1}{\sqrt{2}}\left[|u\bar{u}\rangle - |d\bar{d}\rangle\right], \\ |\omega_I, 00\rangle = |\varphi\rangle \frac{1}{\sqrt{2}}\left[|u\bar{u}\rangle + |d\bar{d}\rangle\right]. \end{cases} \tag{2a}$$

$|\varphi\rangle$ describes the spatial states of bound systems and $|q\bar{q}\rangle$ are vectors in isotop space. Since the masses of the current u_o and d_o quarks are different, it can be assumed that the masses of the corresponding constituent quarks in RQM are also different $m_u \neq m_d$. Hence the state vectors (2a) can be taken as linear combination of the states vectors of $u\bar{u}$ and $d\bar{d}$ bound systems:

$$\begin{cases} |\rho_I^o\rangle = \frac{1}{\sqrt{2}}\left[|\varphi_{u\bar{u}}\rangle - |\varphi_{d\bar{d}}\rangle\right], \\ |\omega_I\rangle = \frac{1}{\sqrt{2}}\left[|\varphi_{u\bar{u}}\rangle + |\varphi_{d\bar{d}}\rangle\right]. \end{cases} \tag{2b}$$

Here $|\varphi_{q\bar{q}}\rangle$ the product of spatial state vector of the $q\bar{q}$ bound system with total momentum J = 1 and isospin space vector $|q\bar{q}\rangle$. States (2b) are no loger eigenstates of isospin. State vectors $|\varphi_{u\bar{u}}\rangle$ and $|\varphi_{d\bar{d}}\rangle$ are orthogonal because the vectors $|u\bar{u}\rangle$ and $|d\bar{d}\rangle$ are orthogonal. We will not discuss the color state, it is known and the corresponding contribution will be included in the task parameters.

Now consider $\rho-\omega$ mixing. Let us introduce a new basis of real ρ^o and ω mesons states. The new basis vectors $|\psi_V\rangle$ can be represented following the formula (1):

$$\begin{cases} |\psi_{\rho^o}\rangle = |\varphi_{\rho_I^o}\rangle - \varepsilon|\varphi_{\omega_I}\rangle = \frac{1}{\sqrt{2}}\left[(1-\varepsilon)|\varphi_{u\bar{u}}\rangle - (1+\varepsilon)|\varphi_{d\bar{d}}\rangle\right], \\ |\psi_\omega\rangle = |\varphi_{\omega_I}\rangle + \varepsilon^*|\varphi_{\rho_I^o}\rangle = \frac{1}{\sqrt{2}}\left[(1+\varepsilon^*)|\varphi_{u\bar{u}}\rangle + (1-\varepsilon^*)|\varphi_{d\bar{d}}\rangle\right]. \end{cases} \tag{3}$$

ε is a small mixing parameter, complex number with real ε_r and imaginary ε_i parts. In calculations we will take into account only members of first order by parameter ε. In this approximation, the states (3) are also orthogonal.

SE for Calculations

For calculations, we use the RQM based on SE. The properties of this equation are well known (see for egs. [2,8]). Equations for the wave function (WF) of the bound state of two fermions with mass M_B in the c.m. frame ($\vec{p}_1 = -\vec{p}_2 = \vec{p}$) can be written as:

$$\varphi_B(\vec{p}) = G_o\left(M_B;\vec{p}\right)\int \frac{d\vec{p}'}{(2\pi)^3} V(\vec{p},\vec{p}')\varphi_B(\vec{p}') \tag{4}$$

and

$$\begin{aligned} G_o\left(M_B;\vec{p}\right) &= \left\{ \frac{\Lambda_{12}^{(++)}(\vec{p}_1,\vec{p}_2)}{M_B - (\omega_1+\omega_2) + i\varepsilon} - \frac{\Lambda_{12}^{(--)}(\vec{p}_1,\vec{p}_2)}{M_B + (\omega_1+\omega_2)} \right\} \gamma_1^o \otimes \gamma_2^o = \\ &= \left[M_B - h_1\left(\vec{p}_1\right) - h_2\left(\vec{p}_2\right) \right]^{-1} \Pi(\vec{p}) \gamma_1^o \otimes \gamma_2^o. \end{aligned} \tag{5}$$

Here: $V(\vec{p},\vec{p}')$ an interaction potential with a definite spin structure, $\Lambda_{12}^{(\pm\pm)}(\vec{p}_1,\vec{p}_2) = \Lambda_1^{(\pm)}(\vec{p}_1)\Lambda_2^{(\pm)}(\vec{p}_2)$ $\Lambda_i^{(\pm)}(p_i) = \dfrac{\omega_i \pm h_i(\vec{p}_i)}{2\omega_i}$, $\omega_i = \sqrt{m_i^2 + \vec{p}_i^{\,2}}$, $h_i(\vec{p}) = \gamma^o(\vec{\gamma}\vec{p} + m_i)$ is standard Dirac Hamiltonian and projection operator $\Pi(\vec{p}) = \left[\Lambda_{12}^{(++)}\left(\vec{p}_1,\vec{p}_2\right) - \Lambda_{12}^{(--)}\left(\vec{p}_1,\vec{p}_2\right) \right]$.

Let us introduce the "frequency components" of the unknown WF $\varphi_B(\vec{p})$ by the definition

$$\varphi_B(\vec{p}) = \sum_{\alpha_1,\alpha_2} \varphi_B^{(\alpha_1\alpha_2)}(\vec{p}),\ \ \varphi_B^{(\alpha_1\alpha_2)}(\vec{p}) = \Lambda_{12}^{(\alpha_1\alpha_2)}(\vec{p}_1,\vec{p}_2)\varphi_B(\vec{p}),\ (\alpha_1,\alpha_2 = +,-).$$

In the case of SE, the components $\varphi_{q\bar{q}}^{(\pm\mp)}(\vec{p}) = 0$, i.e. $\alpha_1 = \alpha_2 = \alpha$. Therefore, the WF is the sum:

$$\varphi_B(\vec{p}) = \varphi_B^{(++)}(\vec{p}) + \varphi_B^{(--)}(\vec{p}). \tag{6}$$

The solutions of the Salpeter equation satisfy the normalization condition:

$$\left\langle \varphi_B \middle| \varphi_B \right\rangle = \int \frac{d\vec{p}}{(2\pi)^3} \left[\left| \varphi_B^{(++)}(\vec{p}) \right|^2 - \left| \varphi_B^{(--)} \right|^2 \right] = 2M_B \tag{7}$$

SE (4) can be written as an equation for the eigenvalues of the mass M_B of a bound system:

$$\begin{aligned} &H_B\varphi_B(\vec{p}) = M_B\varphi_B(\vec{p}), \\ &H_B\varphi_B(\vec{p}) = \left[h_1\left(\vec{p}_1\right) + h_2\left(\vec{p}_2\right) \right]\varphi_B(\vec{p}) + \Pi(\vec{p})\int \frac{d\vec{p}'}{(2\pi)^3} \gamma_1^o \otimes \gamma_2^o V(\vec{p},\vec{p}')\varphi_B(\vec{p}'). \end{aligned} \tag{8}$$

For calculations we write down WF (2) and operators in SE (4) in matrix form. Potential of quark interaction in matrix form:

$$V(\vec{p},\vec{p}') = \begin{bmatrix} V_{u\bar{u}-u\bar{u}}(\vec{p},\vec{p}') & V_{u\bar{u}-d\bar{d}}(\vec{p},\vec{p}') \\ V_{d\bar{d}-u\bar{u}}(\vec{p},\vec{p}') & V_{d\bar{d}-d\bar{d}}(\vec{p},\vec{p}') \end{bmatrix}$$

takes into account potentials of all possible interactions of two fermions. But it can be assumed that the interaction that would mix $u\bar{u}$ and $d\bar{d}$ bound systems in the case of ρ^o and ω mesons should be weak enough due to the slight violation of isotopic symmetry and small difference between the masses of these mesons. It is also known that the main contribution to the mass of bound quark systems is made by the confinement potential (see eg.[4,5]). Therefore, we will consider the problem only taking into account the strong interaction. Let us write the equation (8) for states (2) in matrix form:

$$H\varphi_{\begin{pmatrix}\rho_I^o\\ \omega_I\end{pmatrix}}(\vec{p}) = H\begin{pmatrix}\varphi_{u\bar{u}}(\vec{p})\\ \mp\varphi_{d\bar{d}}(\vec{p})\end{pmatrix} = \left(\begin{bmatrix}H_{o,u\bar{u}}(\vec{p})\varphi_{u\bar{u}}(\vec{p}) & 0\\ 0 & H_{o,d\bar{d}}(\vec{p})\left(\mp\varphi_{d\bar{d}}(\vec{p})\right)\end{bmatrix}+\right.$$
$$\left.+\begin{pmatrix}\Pi_{u\bar{u}}(\vec{p}) & 0\\ 0 & \Pi_{d\bar{d}}(\vec{p})\end{pmatrix}\int\frac{d\vec{p}'}{(2\pi)^3}\gamma_1^o\otimes\gamma_2^o\begin{bmatrix}V_{u\bar{u}-u\bar{u}}(\vec{p},\vec{p}')\varphi_{u\bar{u}}(\vec{p}') & 0\\ 0 & +V_{d\bar{d}-d\bar{d}}(\vec{p},\vec{p}')\left(\pm\varphi_{d\bar{d}}(\vec{p})\right)\end{bmatrix}\right). \quad (9)$$

We get two equations:

$$M_{u\bar{u}}\varphi_{u\bar{u}}(\vec{p}) = H_{o,u\bar{u}}(\vec{p})\varphi_{u\bar{u}}(\vec{p}) + \Pi_{u\bar{u}}(\vec{p})\int\frac{d\vec{p}'}{(2\pi)^3}\gamma_1^o\otimes\gamma_2^o V_{u\bar{u}-u\bar{u}}(\vec{p},\vec{p}')\varphi_{u\bar{u}}(\vec{p}'),$$
$$M_{d\bar{d}}\varphi_{d\bar{d}}(\vec{p}) = H_{o,d\bar{d}}(\vec{p})\varphi_{d\bar{d}}(\vec{p}) + \Pi_{d\bar{d}}(\vec{p})\int\frac{d\vec{p}'}{(2\pi)^3}\gamma_1^o\otimes\gamma_2^o V_{d\bar{d}-d\bar{d}}(\vec{p},\vec{p}')\varphi_{d\bar{d}}(\vec{p}'). \quad (10)$$

They differ by masses of two interacting fermions. Masses of real mesons M_{ρ^o} and M_ω will be expressed by masses $M_{u\bar{u}}$ and $M_{d\bar{d}}$.

Let us write the Hamiltonian matrix for the states of vector mesons ρ_I^o and ω_I:

$$H_{\alpha\beta} = \begin{bmatrix}H_{\rho\rho} & H_{\rho\omega}\\ H_{\omega\rho} & H_{\omega\omega}\end{bmatrix},\quad H_{\alpha\beta} = \langle\alpha_I|H|\beta_I\rangle. \quad (11)$$

Mixing Parameters of Mesons Masses and States

According to the WF normalization condition (7), in basis (2b) we obtain:

$$H_{\rho\rho} = \left\langle\varphi_{\rho_I^o}\middle|H\middle|\varphi_{\rho_I^o}\right\rangle = 2M^2_{\rho_I^o} = M^2_{u\bar{u}} + M^2_{d\bar{d}},$$
$$H_{\omega\omega} = \left\langle\varphi_{\omega_I}\middle|H\middle|\varphi_{\omega_I}\right\rangle = 2M^2_{\omega_I} = M^2_{u\bar{u}} + M^2_{d\bar{d}}. \quad (12)$$

The masses $M_{\rho_I^o}$ and M_{ω_I} are equal, as they should be. For masses of real mesons in basis (3) we obtain:

$$\left\langle\psi_{\rho^o}\middle|H\middle|\psi_{\rho^o}\right\rangle = 2M^2_{\rho^o} = \left(M^2_{u\bar{u}} + M^2_{d\bar{d}}\right) + 2\varepsilon_r\left(M^2_{d\bar{d}} - M^2_{u\bar{u}}\right),$$
$$\left\langle\psi_\omega\middle|H\middle|\psi_\omega\right\rangle = 2M^2_\omega = \left(M^2_{u\bar{u}} + M^2_{d\bar{d}}\right) - 2\varepsilon_r\left(M^2_{d\bar{d}} - M^2_{u\bar{u}}\right). \quad (13)$$

Masses M_{ρ^o} and M_ω contain only the real part of the mass mixing parameter. From the calculation results presented below, it will be seen that we can consider the mass mixing parameter itself as real [9]. We will denote it as $\varepsilon_m = \varepsilon_r$. From the sum and the difference of equalities (13) , we obtain:

$$M_\omega^2 + M_{\rho^o}^2 = \left(M_{u\bar{u}}^2 + M_{d\bar{d}}^2\right), \tag{14a}$$

$$M_\omega^2 - M_{\rho^o}^2 = -2\varepsilon_m \left(M_{d\bar{d}}^2 - M_{u\bar{u}}^2\right) \tag{14b}$$

and expression for the mass mixing parameter and the masses of mesons:

$$\varepsilon_m = -\frac{\left[M_\omega^2 - M_{\rho^o}^2\right]}{2\left[M_{d\bar{d}}^2 - M_{u\bar{u}}^2\right]}, \tag{15}$$

$$M_{\rho^o} = \sqrt{\left[\frac{1}{2}\left(M_{u\bar{u}}^2 + M_{d\bar{d}}^2\right) + \varepsilon_m \left(M_{d\bar{d}}^2 - M_{u\bar{u}}^2\right)\right]},$$

$$M_\omega = \sqrt{\left[\frac{1}{2}\left(M_{u\bar{u}}^2 + M_{d\bar{d}}^2\right) - \varepsilon_m \left(M_{d\bar{d}}^2 - M_{u\bar{u}}^2\right)\right]}.$$

The connections between the masses of mesons and the masses of $u\bar{u}$ and $d\bar{d}$ systems will allow to make a choice of masses of constitutuent quarks.

From the condition of diagonality of matrix (11) in the basis of physical mesons (3), we obtain:

$$H_{\omega\rho} = \left\langle \varphi_{\omega_I} \middle| H \middle| \varphi_{\rho_I^o} \right\rangle \Rightarrow \left\langle \psi_\omega \middle| H \middle| \psi_{\rho^o} \right\rangle + \varepsilon\left[\left\langle \psi_{\rho^o} \middle| H \middle| \psi_{\rho^o} \right\rangle - \left\langle \psi_\omega \middle| H \middle| \psi_\omega \right\rangle\right] = 0$$

and define the states mixing parameter

$$\varepsilon = \frac{\left\langle \psi_\omega \middle| H \middle| \psi_\rho \right\rangle}{\left\langle \psi_\omega \middle| H \middle| \psi_\omega \right\rangle - \left\langle \psi_\rho \middle| H \middle| \psi_\rho \right\rangle}. \tag{16}$$

Taking into account the resonant nature of mesons, we make the replacement $M_V^2 \Rightarrow \left(M_V - \frac{i}{2}\Gamma_V\right)^2$.

Then the value of the state mixing parameter is defined as:

$$\varepsilon = \frac{\left\langle \psi_\omega \middle| H \middle| \psi_{\rho^o} \right\rangle}{2\left[\left(M_\omega^2 - iM_\omega \Gamma_\omega\right) - \left(M_{\rho^o}^2 - iM_{\rho^o}\Gamma_{\rho^o}\right)\right]} \tag{17}$$

and this is a complex quantity, in contrast to the mass mixing parameter ε_m .

Leptonic Decay of Vector Mesons

The solutions of equations (11), WFs $\varphi_{u\bar{u}}(\vec{p})$ and $\varphi_{d\bar{d}}(\vec{p})$, allow the calculation of the leptonic decay widths of ρ^o and ω mesons. Let us consider, the decays of mesons ρ_I^o and ω_I described by states (1). For the decay widths of a vector meson V in the lowest order of charge e , we have the expression (see for example [2,10,11]):

$$\Gamma\left(V \to e^+e^-\right) = \frac{4\pi\alpha^2}{3} M_V \left|\overline{f}_V\right|^2, \tag{18a}$$

where

$$\overline{f}_V = \frac{1}{M_V^{\,2}} \sqrt{3}\,\overline{e}_V f_V \tag{18b}$$

is decay constant of meson: $\overline{e}_V$ quark charge (in a proton charge unit) corresponding to the isotopic state of the meson and this: $\overline{e}_{\rho^o} = \frac{1}{\sqrt{2}}$, $\overline{e}_\omega = \frac{1}{3\sqrt{2}}$; $\frac{1}{M_V^{\,2}}$ contribution of photon. $f_V = f_{q\bar{q}}$ is the contribution of the meson, which is determined by the WF $\Psi_{V,q\bar{q}(LSJ\lambda)}(\vec{p})$, the solution of the SE written for the fermion and antifermion system [2,8].The connection between wave functions $\varphi_{q\bar{q}}(\vec{p})$ and $\Psi_{q\bar{q}}(\vec{p})$ is known (see e.g. work [2]). Three identical diagrams describe the meson decay process due to the three possible colors of the quarks, and for this reason in the decay constant appears $\sqrt{3}$ [12]. The WF of real mesons (3) are sums of WFs each term of which will contribute $f_{q\bar{q}}$ $\left(q\bar{q} = u\bar{u}, d\bar{d}\right)$ to the decay constant which can be written using the contribution $f_{d\bar{d}}$. In this case, the expressions for the isotopic structures of ρ^o and ω mesons that determine the charges of quarks are as follows: $\left(f_{u\bar{u}} / f_{d\bar{d}} \equiv t\right)$

$$\rho^o \Rightarrow \frac{1}{\sqrt{2}}\left[t(1-\varepsilon)\left|u\bar{u}\right\rangle - (1+\varepsilon)\left|d\bar{d}\right\rangle\right], \quad \omega \Rightarrow \frac{1}{\sqrt{2}}\left[t(1+\varepsilon)\left|u\bar{u}\right\rangle + (1-\varepsilon)\left|d\bar{d}\right\rangle\right].$$

For charges of quarks we get:

$$\overline{e}_{\rho^o} = \frac{1}{\sqrt{2}}\left[t(1-\varepsilon)\frac{2}{3} - (1+\varepsilon)\left(-\frac{1}{3}\right)\right], \quad \overline{e}_\omega = \frac{1}{\sqrt{2}}\left[t(1+\varepsilon)\frac{2}{3} + (1-\varepsilon)\left(-\frac{1}{3}\right)\right]. \tag{19}$$

If the isotopic symmetry is maintained $(\varepsilon = 0)$, we get a known result $\overline{e}_{\rho^o{}_I} = \frac{1}{\sqrt{2}}$ and $\overline{e}_{\omega_I} = \frac{1}{3\sqrt{2}}$.

By breaking the symmetry, the charges change and they already depend on the mixing parameter ε.

For the decay constants we obtain:

$$\overline{f}_{\rho^o} = \frac{1}{M_{\rho^o}^{\,2}} \overline{e}_{\rho^o} \sqrt{3} f_{d\bar{d}}, \quad \overline{f}_\omega = \frac{1}{M_\omega^{\,2}} \overline{e}_\omega \sqrt{3} f_{d\bar{d}}. \tag{20}$$

According to formula (18a), for the decay widths we obtain:

$$\Gamma\left(\rho^o \to e^+e^-\right) \equiv \Gamma_{\rho^o} = \frac{4\pi\alpha^2}{3} M_{\rho^o}\left|\overline{f}_{\rho^o}\right|^2 = \frac{4\pi\alpha^2}{3} M_{\rho^o}\left|\overline{e}_{\rho^o}\right|^2 3\left(\frac{f_{d\bar{d}}}{M_{\rho^o}^{\,2}}\right)^2, \tag{21a}$$

$$\Gamma\left(\omega \to e^+e^-\right) \equiv \Gamma_\omega = \frac{4\pi\alpha^2}{3} M_\omega\left|\overline{f}_\omega\right|^2 = \frac{4\pi\alpha^2}{3} M_\omega\left|\overline{e}_\omega\right|^2 3\left(\frac{f_{d\bar{d}}}{M_\omega^{\,2}}\right)^2. \tag{21b}$$

As in the case of the meson masses, only the real part of the mixing parameter ε appears in the expressions of the quark charges, and hence in the decay widths.

Calculations and Results

For potentials $V_{q\bar{q}-q\bar{q}}(\vec{p},\vec{p}')$ in SE we used the operator

$$V_c(\kappa;r) = \left(\kappa\gamma_1^o \otimes \gamma_2^o + (1-\kappa) I_1 \otimes I_2\right) V_c(r) \tag{22a}$$

with oscillatory confinement potential $V_c(r)$

$$V_c(r) = \left[V_o + \frac{\mu_{12}\omega_o^2}{2} r^2\right], \quad \mu_{12} = \frac{m_1 m_2}{m_1+m_2}, V_o<0 \tag{22b}$$

in mometum space. The operator $\left(\kappa\gamma_1^o \otimes \gamma_2^o + (1-\kappa) I_1 \otimes I_2\right)$ defines the spin structure of the potential in the Dirac space. The κ parameter determines the contribution of the fourth component of the vector and the $(1-\kappa)$ contribution of the scalar; V_0 and ω_0 denote parameters of the potential. In order to solve the system of equations (11) rewritten for the "frequency components" $\varphi_{q\bar{q}}^{(\pm\pm)}(\vec{p})$, we used the partial expansion of the equation and the expansion of the corresponding radial WFs $R_{q\bar{q},011}^{(\pm\pm)}(p)$ in a series in the basis of the oscillatory WFs [2,6,13].

For selection of potential parameters we use some consequences of early works, both ours and other authors. In particular, we use for the parameter κ value from an interval 0.3 - 0.6 [13]. The parameters of the potential V_0 and ω_o we determined from the condition (14a). In particular, we chose the values of the point "$r = z$", at which the potential is zero, from the interval $0.8\text{fm} \le z \le 1.0\text{fm}$ [3-5], and then determined $\omega_0 = \sqrt{-2V_0/\mu_{12}z^2}$. By solving equations (11), we obtained the masses $M_{u\bar{u}}$, $M_{d\bar{d}}$, which satisfy condition (14a), and radial WF $R_{u,011}^{(\pm\pm)}(p)$, $R_{d,011}^{(\pm\pm)}(p)$. The fulfillment of conditions (14) gives the values of the masses: $M_{\rho^o} = 775.26$ MeV and $M_\omega = 782.65$ MeV [PDG2020].

Calculations have shown that there are stable solutions of the SE for the certain quark masses with potential parameters $\kappa \ge 0.4$ and $0.9 \le z \le 1.0$ fm, as well as for $k = 0.3$ and z=1.0 fm. The small difference between masses of $K^\pm$ and $K^o\left(\overline{K}^o\right)$ mesons suggests a limited difference in the masses of constituent quarks $\Delta m = m_d - m_u$. We performed calculations in search of sets of quark masses and potential parameters that would give reasonable leptonic decay widths. The best calculation results are shown in the tables.

For all the calculations carried out, the ratio of contributions f_{uu}/f_{dd} is close to unity. The widths of lepton decays of ρ^o and ω mesons [PDG2020] are equal to $\Gamma_{\rho^o} = 7.04 \pm 0.06$ KeV, $\Gamma_\omega = 0.6 \pm 0.02$ KeV and their ratio is determined in the interval

$$11.26 \le \Gamma_\rho / \Gamma_\omega \le 12.24. \tag{23}$$

For lepton decays $\rho^o{}_I$ and ω_I mesons (ε =0, $M_{\omega_I} = M_{\rho^o{}_I}$), the mass of which are equal, this ratio is equal to 9, as well as the ratio of squares of these quark charges $\bar{e}_{\rho^o{}_I}^2$ and $\bar{e}_{\omega_I}^2$. For real ρ^o and ω mesons, according to formulas (23), this ratio depends on the masses of mesons, which are no longer equal, and the quantities $\left|\bar{e}_{\rho^o}\right|^2$ and $\left|\bar{e}_\omega\right|^2$. As a result, from the tables we see that the ratio of decay widths depends on the value Δm and varies from about 10 at $\Delta m=5$ MeV to about 12 at $\Delta m=11$ MeV. In the case of $z=0.9$ fm, this ratio reaches 12 already under $\Delta m=10$ MeV.

Table 1. Mixing Parameters of Masses and States

m_d	m_u	Δm	$\lvert V_o \rvert$	ω_o	$-\varepsilon_m$	$-\varepsilon_r$	ε_i	$\kappa; z$
297	287	10	575.95	555.19	0.1651	0.0167	0.1582	0.3;1
297	286	11	572.69	554.10	0.1500	0.0184	0.1741	0.3;1
271	261	10	539.70	563.10	0.1678	0.0165	0.1557	0.4;1
272	261	11	543.76	564.70	0.1526	0.0181	0.1712	0.4;1
243	233	10	512.84	580.32	0.1685	0.0164	0.1550	0.5;1
244	233	11	517.94	582.62	0.1533	0.0180	0.1705	0.5;1
222	213	9	565.20	708.10	0.1655	0.0167	0.1579	0.4;0.9
222	212	10	557.96	704.40	0.1489	0.0185	0.1755	0.4;0.9

Table 2. Meson Leptonic Decay Widths

m_d	m_u	Δm	$\lvert \bar{e}_{\rho^o} \rvert^2$	$\lvert \bar{e}_{\omega} \rvert^2$	Γ_ρ	Γ_ω	$\Gamma_\rho / \Gamma_\omega$	$\kappa; z$
297	287	10	0.4795	0.0423	7.019	0.60	11.67	0.3;1
297	286	11	0.4774	0.0410	6.976	0.58	11.97	0.3;1
271	261	10	0.4793	0.0423	6.983	0.60	11.65	0.4;1
272	261	11	0.4774	0.0411	7.035	0.59	11.94	0.4;1
243	233	10	0.4789	0.0422	7.017	0.60	11.67	0.5;1
244	233	11	0.4768	0.0410	7.085	0.59	11.96	0.5;1
222	213	9	0.4772	0.0416	7.107	0.60	11.80	0.4;0.9
222	212	10	0.4746	0.0402	7.016	0.58	12.153	0.4;0.9
EXPERIMENT					7.04	0.6		

Conclusions

The description of ρ^o and ω mesons states and $\rho-\omega$ mixing in the relativistic quark model is considered taking into account the violation of SU (2) symmetry.

Expressions have been defined and calculated: a) parameters of masses mixing ε_m and states mixing ε of mesons; b) charge values $\bar{e}_{\rho^o}$ and $\bar{e}_{\omega}$ corresponding to the changed isotopic states of mesons; c) lepton decay widths of vector mesons in the case of SU (2) symmetry breaking.

Several sets of values of constituent quark masses and the corresponding potential parameters are determined, which describe well the states of ρ^o and ω mesons, $\rho-\omega$ mixing and give the values of the leptonic decay widths of mesons which are in good agreement with experiment.

The obtained sets of quark masses and potential parameters can be used in considering the states of other mesons.

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ფიზიკა

ρ° და ω მეზონების მდგომარეობები და ρ°–ω შერევა რელატივისტურ კვარკულ მოდელში

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განხილულია ρ° და ω მეზონების მდგომარეობები და ρ–ω შერევა რელატივისტურ კვარკულ მოდელში SU(2) სიმეტრიის დარღვევით. მეზონები, როგორც კვარკისა და ანტიკვარკის ბმული მდგომარეობები, ადრე იყო შესწავლილი რელატივისტური კვარკული მოდელის ფარგლებში SU(2) სიმეტრიის შენახვით, ანუ, $m_u = m_d$. ასეთ განხილვაში ρ° და ω მეზონების მასები ტოლია. მაგრამ მუხტური, SU(2) სიმეტრია u_0 და d_0 დენის კვარკების მასების მცირე განსხვავების გამო დარღვეულია. შედეგად, ρ° და ω მეზონები არ წარმოადგენენ იზოტოპური სპინის საკუთარ მდგომარეობებს და შეიცავენ განსხვავებული იზოტოპური სპინის მქონე მდგომარეობების მცირე მინარევებს. ეს ფენომენი ცნობილია როგორც ρ–ω შერევა. ასევე, ρ° და ω მეზონების მასები განსხვავებულია. რადგან დენის კვარკების მასები განსხვავებულია, შეიძლება ვიგულისხმოთ, რომ განსხვავებულია ასევე შესაბამისი კონსტიტუენტური კვარკების მასებიც $m_u \neq m_d$. ფიზიკური მეზონების ბმული მდგომარეობები წარმოდენილია როგორც განსხვავებული მასების $u\bar{u}$ და $d\bar{d}$ ბმული მდგომარეობების წრფივი კომბინაცია, რომელშიც შერევა მოცემულია შერევის მცირე ε პარამეტრით. მიღებულია მასების შერევის ε_m, რეალური სიდიდე და მდგომარეობების შერევის ε, კომპლექსური სიდიდე. ორივე პარამეტრი გამოსახულია მეზონების და $u\bar{u}$ და $d\bar{d}$ ბმული მდგომარეობების მასებით. მიღებულია ასევე მეზონების იზოტოპურ მდგომარეობებზე დამოკიდებული კვარკების მუხტების $\bar{e}_{\rho^0}$ და $\bar{e}_{\omega}$ გამოსახულებები. მეზონის დაშლის კონსტანტა განზოგადებულია იზოტოპური სიმეტრიის დარღვევის შემთხვევისათვის. დათვლები ჩატარებულია სოლპიტერის განტოლების გამოყენებით კონფაინმენტის ოსცილატორული პოტენციალით. განსაზღვრულია კონსტიტუენტური კვარკების მასების და პოტენციალის შესაბამისი პარამეტრების ნაკრებები, რომლებიც კარგად აღწერენ ρ° და ω მეზონების მდგომარეობებს, ρ–ω შერევას და იძლევიან მეზონების ლეპტონური დაშლებისათვის შედეგებს, რომლებიც კარგ თანხმობაშია ექსპერიმენტულ მონაცემებთან. მიღებული შედეგები, კვარკების მასების და პოტენციალის შესაბამისი პარამეტრების ნაკრებები, შეიძლება გამოყენებულ იქნეს სხვა მეზონების მდგომარეობების განხილვისას.

REFERENCES

1. Itzykson C. and Zuber C. (1980) Quantum Field theory. New York.
2. Kopaleishvili T. (2001) Bound $q\bar{q}$ systems in the framework of different versions of 3D reductions of the Bethe-Salpeter Equation. *Phys. Part. Nucl*., **32:** 560.
3. Chachkhunashvili M.Sh. and Kopaleishvili T.I. (1989) Bound q\bar q and qqq systems with light quarks within the framework of Salpeter equations: quark-antiquark systems. *Few-Body Systems*, **6**:1.
4. Münz C.R. et al. (1995) A Bethe-Salpeter model for light mesons: spectra and decays. *Phys. Rev*., **C52**, 4: 1211.
5. Ricken R. et al. (2000) The meson spectrum in a Covariant Quark Model. *Eur. Phys. J*., **A9**: 221.
6. Archvadze A. et al. (1995) On the mass spectrum of bound systems in the framework of the Salpeter Equation. *Nucl. Phys*. **A 581**: 460.
7. O'Connel H.B. et al. (1997) Rho-omega mixing, vector meson dominance and the pion form-factor. *Prog. Part. Nucl. Phys*. **39**:201.
8. Resag J. et al. (1994) Analysis of the instantaneous Bethe-Salpeter equation for $q\bar{q}$ bound-states. *Nucl. Phys*., **A578**: 397.
9. Sachs Robert G. and Willemsen Jorge F. (1970) Two-Pion Decay Mode of the ω and ρ-ω mixing. *Phys. Rev*., **D2**, 1: 133.
10. Bhatnagar Shashank (2005) Leptonic decays of vector mesons. *Int. J. of Modern Physics.* **E. 14**: 909.
11. Jena S.N. et al. (2015) Leptonic decay width and decay constants of vector mesons in a relativistic potential model. *Chinese Journal of Physics*., **53**: 1.
12. Halsen Frencis, Martin Alan D. (1987) Kvarki i leptony. Vvedenie v phiziku chastits. M. (in Russian).
13. Babutsidze T., Kopaleishvili T. and Ruseteky A. (1999) Bound q\bar q systems in the framework of the different versions of the 3-dimensional reductions of the Bethe-Salpeter equation. *Phys. Rev*., **C59**, 2: 976.

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