საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, ტ. 15, №2, 2021
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 15, no. 2, 2021

Mathematics

The Consistent Estimators of Charlier Statistical Structures in Banach Space of Measures

Zurab Zerakidze[*] and Mzevinar Patsatsia[**]

[*] *Faculty of Mathematics, Gori State Teaching University, Gori, Georgia*
[**] *Department of Mathematics, Faculty of Mathematics and Computer Sciences, Sokhumi State University, Tbilisi, Georgia*

(Presented by Academy Member Elizbar Nadaraia)

The statistical structure of Charlier is determined, and necessary and sufficient conditions for existence of consistent estimators of parameters in a Banach space of measures are given. A Banach space is constructed whose elements are measures, but this space is not a Banach space of measures. Charles's countable, strongly separable statistical structure is constructed, for which there is a consistent estimation of the parameter. In addition, Charlier continuum, strongly separable statistical structure is constructed, for which there is no consistent estimation of the parameter. A new definition of strong separability has been introduced and for the Charlier continuum structure of this type it has been shown that there is a consistent estimation of the parameter. *© 2021 Bull. Georg. Natl. Acad. Sci.*

Charlier statistical structures, singular structure, weakly separable structure, strongly separable structures

1. Introduction

In the general theory of the existence of statistical structures, the problem of determining strongly separable statistical structures and the problem of transition from weakly separable statistical structures to the corresponding strongly separable statistical structure often arises. Z. Zerakidze (see [1-4]) in terms of Zermelo-Frankel set theory and Martin's axiom proved that the existence of a continuous Borel weakly statistical structure implies the existence of a strongly statistical structure.

Let (E,S) be a measurable space and there be given the family of probability measures $\{\mu_i, i \in I\}$ defined on S. We recall the following definitions from [1-13].

Definition 1.1. The set of objects $\{E,S,\mu_i, i \in I\}$ is called a statistical structure.

Definition 1.2. A statistical structure $\{E,S,\mu_i, i \in I\}$ is called orthogonal (singular) if μ_i and μ_j are orthogonal for each $i \neq j$, $i \in I$, $j \in I$.

Definition 1.3. A statistical structure $\{E, S, \mu_i, i \in I\}$ is called weakly separable if there exists a family of S-measurable sets $\{X_i, i \in I\}$ such that the relations are fulfilled:

$$\mu_i(X_j) = \begin{cases} 1, & if \quad i = j; \\ 0, & if \quad i \neq j. \end{cases}$$

Definition 1.4. A statistical structure $\{E, S, \mu_i, i \in I\}$ is called separable if there exists a family of S-measurable sets $\{X_i, i \in I\}$ such that the relations are fulfilled:

$$\mu_i(X_j) = \begin{cases} 1, & if \quad i = j; \\ 0, & if \quad i \neq j. \end{cases} \qquad 2)\ card(X_i \cap X_j) < c, \quad if \quad i \neq j,$$

where c denotes the continuum power.

Definition 1.5. A statistical structure $\{E, S, \mu_i, i \in I\}$ is called strongly separable if there exists a disjoint family of S-measurable sets $\{X_i, i \in I\}$ such that the relations are fulfilled: $\mu_i(X_i) = 1, \quad \forall i \in I.$

Remark 1.1. It should be noted that strong separability implies weak separability, and weak separability implies orthogonality, but not vice versa.

Example 1.1. Let $E = [0,1] \times [0,1]$ and S be the Borel σ-algebra of subsets of E. Consider the S-measurable sets $X_i = \begin{cases} 0 \le x \le 1,\ y \in i, & if\ i \in (0,1]; \\ E, & if\ i = 0 \end{cases}$ and assume that l_i is a linear Lebesgue measure on X_i, $i \in (0,1]$, and l_0 is a plane Lebesgue measure on $[0,1] \times [0,1]$. Then the statistical structure $\{E, S, \mu_i, i \in [0,1]\}$ is orthogonal statistical structure, but not weakly separable statistical structure.

Let I be a set of parameters and let $B(I)$ be a σ-algebra of subsets of I which contains all its finite subsets.

Definition 1.6. We will say that the statistical structure $\{E, S, \mu_i, i \in I\}$ admits a consistent estimator of parameters $i \in I$, if there exists at least one measurable mapping $\delta : (E, S) \to (I, B(I))$, such that $\mu_i(\{x : \delta(x) = i\}) = 1, \quad \forall i \in I$.

Remark 1.2. If a statistical structure $\{E, S, \mu_i, i \in I\}$ admits a consistent estimator of parameters $i \in I$ then this statistical structure is strongly separable but not vice versa (see [12]).

Let M^σ be a real linear space of all alternating finite measures on S.

Definition 1.7. A linear subset $M_B \subset M^\sigma$ is called a Banach space of measures if:

1) the norm on M_B can be defined so that M_B is a Banach space with respect to this norm, and the inequality $\| \mu + \lambda \nu \| \ge \| \mu \|$ holds for any orthogonal measures $\mu, \nu \in M_B$ and a real number $\lambda \neq 0$;

2) if $\mu \in M_B$ and $|f(x)| \le 1$, then $\nu_f(A) = \int_A f(x)\mu(dx) \in M_B$ and $\| \nu_f \| \le \| \mu \|$;

3) if $\nu_n \in M_B$, $\nu_n > 0$, $\nu_n(E) < \infty$, $n = 1, 2, \ldots$ and $\nu_n \downarrow 0$, then for any linear functional $l^* \in M_B^*$: $\lim\limits_{n \to \infty} l^*(\nu_n) = 0$, where M_B^* conjugate to linear space M_B.

The definition and construction of a Banach space of measures and a Hilbert space of measures were given by Z. Zerakidze (see [11]).

Definition 1.8. Let I be a set of indexes and M_{B_i} be a Banach space for all $i \in I$. The Banach space $M_B = \{\{X_i\}_{i\in I} : X_i \in M_{B_i}, \forall i \in I, \sum_{i\in I} \| X_i \|_{M_{B_i}} < \infty\}$ with the norm $\left\|\{X_i\}_{i\in I}\right\| = \sum_{i\in I} \|X_i\|_{M_{B_i}}$ is called the direct sum of Banach space M_{B_i} and is denoted by

$$M_B = \underset{i\in I}{\oplus} M_{B_i} .$$

Remark 1.3. Obviously, any Banach space of measures is a Banach space, the elements of which are alternating measures, but not vice versa.

Example 1.2. Let $E = [0,1]$ and S be the Borel σ-algebra of subsets of E. Let l be the Lebesgue measure on $[0,1]$. Consider a set of alternating measures with bounded densities. More precisely, we define

$$\mu_g(A) = \int_A g(x) l(dx) , \quad A \in S , \quad g \in L_\infty([0,1]) ;$$
$$M_B = \{\mu_g : g \in L_\infty([0,1])\} ;$$

$$\| \mu_g \| = \underset{x\in[0,1]}{ess\sup} | g(x) | = \underset{x\in[0,1]}{vrai\sup} | g(x) | = \inf_N \sup_{x\in[0,1]\setminus N} | g(x) | , \quad \forall N , \quad l(N) = 0 .$$

The space M_B with the norm $\| \cdot \|$ is a Banach space.

Let us define the continuous functions

$$g_n(x) = \max(1 - nx, \frac{x}{n}) = \begin{cases} 1 - nx, & if\ x \in [0, \frac{n}{n^2+1}]; \\ \frac{x}{n}, & if\ x \in [\frac{n}{n^2+1}, 1]. \end{cases}$$

and the measures ν_n assuming $\nu_n(A) = \int_A g_n(x) l(dx)$, $A \in S$.

It is not difficult to see that $\nu_1 \ge \nu_2 \ge \cdots \ge \nu_n \ge \cdots$; $\nu_n([0,1]) < \infty, \forall n \in N$; $\nu_n \downarrow 0$; $\nu_n \in M_B$ and $\| \nu_n \| = 1$, $\forall n \in N$.

Now we construct the linear functional $l^* \in M_B^*$. For a continuous function $f : [0,1] \to R$ we define $\Phi(f) = f(0)$. Φ is a linear functional on $C([0,1])$ and $\|\Phi\| = 1$. By the Hann-Banach theorem Φ can be extended to $L_\infty([0,1])$, that is, there exists a functional $\tilde{\Phi} \in (L_\infty([0,1]))^*$, such that $\| \tilde{\Phi} \| = 1$ and for all $f \in C([0,1])$: $\tilde{\Phi}(f) = f(0)$. Now let's define the functional l^* as follows: $l^*(\mu_g) = \tilde{\Phi}(g)$, $g \in L_\infty([0,1])$. Then we have $l^* \in M_B^*$, $\| l^* \| = 1$ and $\lim_{n\to\infty} l^*(\nu_n) = 1$.

It is evident that M_B is not Banach space of measures.

2. The Charlier Statistical Structures

The normal distribution is symmetrical, i. e. normal distribution density function

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left\{ -\frac{(x-m)^2}{2\sigma^2} \right\}$$

symmetrical about the line $x = m$. But in practice, asymmetric distributions are often encountered. In the case when the asymmetry in absolute value is not very large, the density can be expressed with the help of the so-called Charlier law.

The density of Charlier law is determined by equality (see [13])

$$f_{Sh}(x) = f(x) + \frac{1}{\sigma}\left[\frac{S_k(x)}{6}\cdot z_u\cdot(u^3 - 3u) + \frac{E_k(x)}{24}\cdot z_u\cdot(u^4 - 6u^2 + 3)\right],$$

where $f(x)$ is a density of normal distribution, $S_k(x) = \frac{\mu_3}{\sigma^3}$ is asymmetry, $E_k(x) = \frac{\mu_4}{\sigma^4} - 3$ is kurtosis, $u = \frac{x-m}{\sigma}$ and $z_u = \frac{1}{\sqrt{2\pi}}e^{-\frac{u^2}{2}}$.

Thus, the second term in the right-hand side of $f_{Sh}(x)$ is a correction to the normal distribution law. Obviously, if $S_k(x) = 0$ and $E_k(x) = 0$ then the Charlier distribution coincides with the normal distribution.

Let $\mu(A) = \int_A f_{Sh}(x)dx$, $A \in L(R)$, is the probability of Charlier given on $(R, L(R))$, where $f_{Sh}(x)$ is the spectral density of Charlier and $L(R)$ is a Lebesgue σ-algebra in R.

Let $\{\mu_i, i \in I\}$ be the corresponding probability measures of Charlier.

Definition 2.1. The statistical structure $\{E, S, \mu_i, i \in I\}$, in which $\{\mu_i, i \in I\}$ is the probability measures of Charlier, will be called the statistical structure of Charlier.

Example 2.1. Let $E = R \times R$ and $B(R \times R)$ be a Borel σ-algebra of subsets of $R \times R$. As a set of parameters consider the set $I = Q^+$, where Q^+ is the set of positive rational numbers. Let $X_i = \{-\infty < x < +\infty, y = i, i \in Q^+\}$ and let $\mu_i(A) = \int_A f^i_{Sh}(x)dx$ be the Charlier linear measures on X_i, $i \in Q^+$.

The Charlier statistical structure $\{R \times R, B(R \times R), \mu_i, i \in Q^+\}$ is countable, strongly separable statistical structure (see definition 1.5).

If we now define the mapping $\delta : (R \times R, B(R \times R)) \to (Q^+, B(Q^+))$ by the formula $\delta(X_i) = i$, $\forall i \in Q^+$, we obtain $\mu_i(\{x : \delta(x) = i\}) = 1$, $\forall i \in Q^+$.

Thus, this Charlier strongly statistical structure admits a consistent estimator for parameters.

Let $\{E, S, \mu_i, i \in I\}$ be a Charlier orthogonal statistical structure and consider S-measurable functions $g_i(x)$, $\forall i \in I$, such that $\sum_{i \in I}\int_E |g_i(x)|\,\mu_i(dx) < +\infty$.

Let M_B be the set of measures defined by formula $\nu(B) = \sum_{i \in I_1}\int_B g_i(x)\mu_i(dx)$, where $I_1 \subset I$ is a countable subset in *I*, and define a norm on M_B by formula $\|\nu\| = \sum_{i \in I_1}\int_E |g_i(x)|\,\mu_i(dx)$, then M_B is a Banach space of measures and $M_B = \oplus_{i \in I} M_{B_i}$, where M_{B_i} is a Banach space of elements of the form $\nu(B) = \int_B g_i(x)\mu_i(dx),\ B \in S$, with the norm: $\|\nu\| = \int_B |g_i(x)|\,\mu_i(dx)$.

3. The Consistent Estimators of Charlier Statistical Structures in Banach Space of Measures

Let $\{\mu_i, i \in I\}$ be a Charlier probability measures defined on the measurable space (E,S) . For each $i \in I$ denote by $\overline{\mu}_i$ the completion of the measure μ_i, and denote by $dom(\overline{\mu}_i)$ the σ-algebra of all $\overline{\mu}_i$-measurable subsets of E . Let $S_1 = \bigcap_{i\in I} dom(\overline{\mu}_i)$.

Definition 3.1. The Charlier statistical structure $\{E, S_1, \overline{\mu}_i, i \in I\}$ is called strongly separable if there exists the family of S_1-measurable sets $\{Z_i, i \in I\}$ such that the relations are fulfilled:

1) $\overline{\mu}_i(Z_i) = 1$, $\forall i \in I$;
2) $Z_{i_1} \bigcap Z_{i_2} = \varnothing$, $\forall i_1 \neq i_2$, $i_1, i_2 \in I$;
3) $\bigcup_{i\in I} Z_i = E$.

Definition 3.2. We will say that the orthogonal charlier statistical structure $\{E, S_1, \overline{\mu}_i, i \in I\}$ admits a consistent estimator of parameters if there exists at least one measurable mapping $\delta : (E, S_1) \to (I, B(I))$, such that $\overline{\mu}_i(\{x : \delta(x) = i\}) = 1$, $\forall i \in I$.

Let $M_B = \bigoplus_{i\in I} M_{B_i}$ be a Banach space of measures, $cardI \le c$. Let E be a complete metric space, whose topological weights are not measurable in a wider sense, $S_1 = \bigcap_{i\in I} dom(\overline{\mu}_i)$ is a Borel σ-algebra on E .

Theorem 3.1. In order the Borel Charlier orthogonal statistical structure $\{E, S_1, \overline{\mu}_i, i \in I\}$ to be admitted as a consistent estimator of parameters in the theory of (ZFC)§(MA) it is necessary and sufficient that the correspondence $f \to l_f$ defined by the equality $\int_E f(x)\overline{\mu}_i(dx) = l_f(\overline{\mu}_i)$, $\overline{\mu}_i \in M_B$ was one-to-one (here l_f is a linear continuous functional on M_B, $f \in F(M_B)$, where we define $F = F(M_B)$ as a set of real functions f for which $\int_E f(x)\overline{\mu}_i(dx)$ is defined for all $\overline{\mu}_i \in M_B$).

We now recall the following lemmas.

Lemma 3.1 (see [10]). Let (E, ρ) be a complete separable metric space and let μ be a Borel probability measure defined on $(E, B(E))$. Let $\{X_i\}_{i\in I}$, $cardI \le c$, be a family of $B(E)$-measurable sets and $\mu(X_i) = 0$, $\forall i \in I$. Then $\mu^*(\bigcup_{i\in I} X_i) = 0$ in the theory (ZFC)§(MA).

Lemma 3.2 (see [4]). Let (E, ρ) be a complete metric space, whose topological weights are not measurable in a wider sense than in the theory (ZFC)§(MA) and let μ be an arbitrary Borel probability measure defined on $(E, B(E))$. Then there exists a closed separable subspace $E(\mu) \subseteq E$ such that $\mu(E(\mu)) = 1$ and $\mu(E \setminus E(\mu)) = 0$.

Lemma 3.3 (see [7]). Let (E, ρ) be a complete metric space, the topological weight of which is not measurable in the wider sense, then in the theory (ZFC)§(MA) satisfies the relation

$$(\forall I)(\forall \{X_i\}_{i\in I})((cardI \le c) \,\&\, \forall i(i \in I \Rightarrow \mu(X_i) = 0)) \Rightarrow \mu^*(\bigcup_{i\in I} X_i) = 0 .$$

Using Lemmas 3.1–3.3, we can prove Theorem 3.1 in the case of a complete separable metric space *E.*

Proof. Necessity. The existence of consistent estimators for parameters $\delta:(E,S_1)\to(I,B(I))$ implies that $\overline{\mu}_i(\{x:\delta(x)=i\})=1,\quad \forall i\in I$. Putting $X_i=\{x:\delta(x)=i\}$ for $i\in I$, we get: 1) $\overline{\mu}_i(X_i)=1,\quad \forall i\in I$; 2) $X_{i_1}\bigcap X_{i_2}=\varnothing$ for all different parameters i_1 and i_2 from I; 3) $\bigcup_{i\in I}X_i=E$.

Therefore, the Charlier statistical structure $\{E,S_1,\overline{\mu}_i,i\in I\}$ is strongly separable (see definition 3.1), hence, there exists S_1-measurable sets X_i, $\forall i\in I$, such that

$$\mu_i(X_{i'})=\begin{cases}1, & if \quad i=i';\\ 0, & if \quad i\neq i'.\end{cases}$$

Let l_{X_i} be a linear continuous functional that corresponds to the function $\overline{f}_1(x)=f_1(x)I_{X_i}(x)$. Then for any $\overline{\mu}_{i_1}\in M_B(\overline{\mu}_i)$ we have

$$\int_E\overline{f}_1(x)\overline{\mu}_{i_1}(dx)=\int_E f_1(x)f(x)I_{X_i}(x)\overline{\mu}_i(dx)=l_{\overline{f}_1}(\overline{\mu}_{i_1})=\|\overline{\mu}_{i_1}\|_{M_B(\overline{\mu}_i)}.$$

Let $\sum$ be the set of extensions of functionals satisfying the condition $l_f\le p(x)$ on the subspaces in which they are defined.

Let us introduce a partial ordering into $\sum$, assuming $l_{f_1}<l_{f_2}$ if l_{f_2} is defined on a largest than l_{f_1} and $l_{f_1}=l_{f_2}$, where both of them are defined. Let $\{l_{f_i}\}_{i\in I}$ be a linearity ordered subset in $\sum$, $M_B(\overline{\mu}_i)$ is the subspace, on which l_{f_i} is defined. We define $l_{f_i}\in\bigcup_{i\in I}M_B(\overline{\mu}_i)$ putting $l_f(\mu)=l_{f_i}(\mu)$ if $\mu\in M_B(\overline{\mu}_i)$. It is obvious that $l_{f_i}<l_{f'}$. Since any linearly ordered subset in $\sum$ has an upper bound, due to the Chorn lemma, $\sum$ contains the maximal element λ defined on some set X' satisfying the condition $\lambda\le p(x)$ for $x\in X'$. But X' must coincide with the entire space M_B because otherwise we could extend λ to a wider space by adding as above one more dimension. This contradicts the maximality of λ, and, hence, $X'=M_B$. Therefore, the extension of the functional is defined everywhere.

Let l_f be a linear functional that corresponds to the function $f(x)=\sum_{i\in I}g_i(x)I_{X_i}(x)\in F(M_B)$. Then we have, $\int_E f(x)\mu(dx)=\|\mu\|=\sum_{i\in I}\|\overline{\mu}_i\|_{M_{B(\overline{\mu}_i)}}$, where $\mu(B)=\sum_{i\in I}\int_B g_i(x)\overline{\mu}_i(dx)$, $B\in S$.

Sufficiency. If for each $f\in F(M_B)$ the integral $\int_E f(x)\overline{\mu}_i(dx)$, $\forall\overline{\mu}_i\in M_B$, is defined, then there exists a countable subset I_f in I for which $\int_E f(x)\overline{\mu}_i(dx)=0$, if $i\notin I_f$; $\sum_{i\in I_f}\int_E|f(x)|\overline{\mu}_i(dx)<+\infty$ and for any countable subset $\tilde{I}\subset I$ and for the measure $\nu(C)=\sum_{i\in\tilde{I}}\int_C g_i(x)\overline{\mu}_i(dx)$ we have $\int_E f(x)\nu(dx)=\sum_{i\in I_f\cap\tilde{I}}\int_E f(x)g_i(x)\overline{\mu}_i(dx)$.

Let the correspondence $f\to l_f$ be defined by the equality $\int_E f(x)\overline{\mu}_i(dx)=l_f(\overline{\mu}_i)$, then for $\overline{\mu}_{i_1},\overline{\mu}_{i_2}\in M_B(\overline{\mu}_i)$ we have $\int_E f_{i_1}(x)\overline{\mu}_{i_2}(dx)=l_{f_{i_1}}(\overline{\mu}_{i_2})=\int_E f_1(x)f_2(x)\overline{\mu}_{i_1}(dx)=\int_E f_{i_1}(x)f_2(x)\overline{\mu}_{i_1}(dx)$.

Therefore, $f_{i_1}(x)=f_1(x)$ almost everywhere with respect to the measure $\overline{\mu}_{i1}$. Let $f_{\overline{\mu}_i}(x)>0$ almost everywhere with respect to $\overline{\mu}_i$ and $\int_E f_{\overline{\mu}_i}(x)\overline{\mu}_i(dx)<+\infty$. If we denote now $\overline{\mu}_i^*(C)=\int_C f_{\overline{\mu}_i}(x)\overline{\mu}_i(dx)$, then we obtain $\int_E f_{\overrightarrow{\mu}_i^*}(x)\overline{\mu}_{i'}(dx)=l_{f_{\overline{\mu}_i}}(\overline{\mu}_{i'})=0$, $\forall i'\neq i$, $\forall\overline{\mu}_{i'}\in M_B(\overline{\mu}_i)$.

Denote by $C_i = \left\{x : f_{\overline{\mu}_i^*}(x) > 0\right\}$. Then $\overline{\mu}_{i'}(C_i) = 0$, $\forall i' \neq i$. Therefore, there exist S_1-measurable sets X_i ($i \in I$) such that $\overline{\mu}_i(X_{i'}) = \begin{cases} 1, & if \quad i = i'; \\ 0, & if \quad i \neq i' \end{cases}$ and hence the Charlier statistical structure $\{E, S_1, \overline{\mu}_i, i \in I\}$ is weakly separable. We represent as an inductive sequence $\{\overline{\mu}_i < \omega_1\}$, where ω_1 denotes the first ordinal number of the power of the set I.

We define ω_1 sequence Z_i of parts of the E such that the following relations hold: 1) Z_i is a Borel subset of E, $\forall i < \omega_1$; 2) $Z_i \subset X_i$, $\forall i < \omega_1$; 3) $Z_i \cap Z_{i'} = \varnothing$ for all $i < \omega_1$, $i' < \omega_1$, $i \neq i'$; 4) $\overline{\mu}_i(Z_i) = 1$, $i < \omega_1$.

Suppose that $Z_{i_0} = X_{i_0}$. Suppose that the partial sequence $\{Z_{i'}\}_{i' < i}$ is already defined for $i < \omega_1$. It is clear that $\mu^*(\bigcup_{i' < i} Z_{i'}) = 0$ (see [12]). Thus, there exists a Borel subset Y_i of the space E such that the following relations are valid: $\bigcup_{i' < i} Z_{i'} \subset Y_i$ and $\mu^*(Y_i) = 0$.

Assuming that $Z_i = X_i \setminus Y_i$, we construct the ω_1 sequence $\{Z_i\}_{i < \omega_1}$ of disjunctive measurable subsets of the space E. Therefore, $\mu_i(Z_i) = 1$, $\forall i < \omega_1$ and the Charlier statistical structure $\{E, S_1, \overline{\mu}_i, i \in I\}$, $card I = c$, is strongly separable because there exists a family of elements of the σ-algebra $S_1 = \bigcap_{i \in I} dom(\overline{\mu}_i)$ such that: 1) $\overline{\mu}_i(Z_i) = 1$, $\forall i \in I$; 2) $Z_i \cap Z_{i'} = \varnothing$, $\forall i \neq i'$, $i, i' \in I$; 3) $\bigcup_{i \in I} Z_i = E$.

For $x \in E$, we put $\delta(x) = i$, where i is the unique parameter from the set I for which $x \in Z_i$. The existence of such a unique parameter from I can be proved using conditions 2), 3).

Let now $Y \in B(I)$. Then $\{x : \delta(x) \in Y\} = \bigcup_{i \in Y} Z_i$. We must show that for all $i \in I$: $\{x : \delta(x) \in Y\} \in dom(\overline{\mu}_i)$. If $i_0 \in I$, then $\{x : \delta(x) \in Y\} = \bigcup_{i \in Y} Z_i = Z_{i_0} \cup (\bigcup_{i \in Y \setminus \{i_0\}} Z_i)$. On the other hand, from the validity of the condition 1)-3) it follows that $Z_{i_0} \in S_1 = \bigcap_{i \in I} dom(\overline{\mu}_i) \subseteq dom(\overline{\mu}_{i_0})$. On the other hand the validity of the condition $\bigcup_{i \in Y \setminus \{i_o\}} Z_i \subseteq (E \setminus Z_{i_o})$ implies that $\overline{\mu}_{i_0}\left(\bigcup_{i \in Y \setminus \{i_0\}} Z_i\right) = 0$. The last equality yields that $\bigcup_{i \in Y \setminus \{i_0\}} Z_i \in dom(\overline{\mu}_{i_0})$. Since $dom(\overline{\mu}_{i_0})$ is a σ-algebra, we deduce that $\{x : \delta(x) \in Y\} \in dom(\overline{\mu}_{i_0})$.

If $i_0 \notin Y$, then $\{x : \delta(x) \in Y\} = \bigcup_{i \in Y} Z_i \subseteq (E \setminus Z_{i_0})$ and we conclude that $\overline{\mu}_{i_0}(\{x : \delta(x) \in Y\}) = 0$. The last relation implies that $\{x : \delta(x) \in Y\} \in dom(\overline{\mu}_{i_0})$.

We have shown that the map $\delta : (E, S_1) \to (I, B(I))$ is a measurable map. Since $B(I)$ contains all singletons of I we ascertain that $\overline{\mu}_i(\{x : \delta(x) = i\}) = \overline{\mu}_i(Z_i) = 1, \quad \forall i \in I$.

Remark 3.1. It follows from Theorem 3.1 that, in accordance with the above correspondence, a certain function $f \in F(M_B)$ corresponds to each linear continuous functional l_f. If in $F(M_B)$ we identify functions that coincide in measures $\{\overline{\mu}_i, i \in I\}$, the correspondence will be bijective.

მათემატიკა

შარლეს სტატისტიკური სტრუქტურების ძალდებული შეფასებები ბანახის ზომათა სივრცეში

ზ. ზერაკიძე* და მ. ფაცაცია**

**გორის სახელმწიფო სასწავლო უნივერსიტეტი, მათემატიკის ფაკულტეტი, გორი, საქართველო*
***სოხუმის სახელმწიფო უნივერსიტეტი, მათემატიკის დეპარტამენტი, მათემატიკისა და კომპიუტერული მეცნიერებების ფაკულტეტი, თბილისი, საქართველო*

(წარმოდგენილია აკადემიის წევრის ე. ნადარაიას მიერ)

განსაზღვრულია შარლეს სტატისტიკური სტრუქტურა და დადგენილია პარამეტრების ძალდებული შეფასებების არსებობის აუცილებელი და საკმარისი პირობები ბანახის ზომათა სივრცეში. აგებულია ბანახის სივრცე, რომლის ელემენტებია ზომები, მაგრამ ეს სივრცე არ არის ბანახის ზომათა სივრცე. აგებულია შარლეს თვლადი, ძლიერად განცალებადი სტატისტიკური სტრუქტურა, რომლისთვისაც არსებობს პარამეტრის ძალდებული შეფასება. გარდა ამისა, აგებულია შარლეს კონტინუალური, ძლიერად განცალებადი სტატისტიკური სტრუქტურა, რომლისთვისაც არ არსებობს პარამეტრის ძალდებული შეფასება. შემოღებულია ძლიერად განცალებადობის ახალი განმარტება და ასეთი ტიპის შარლეს კონტინუალური სტრუქტურისთვის ნაჩვენებია, რომ არსებობს პარამეტრის ძალდებული შეფასება.

REFERENCES

1. Borovkov A. A. (1984) Mathematical statistics. Estimation of parameters. Testing of hypotheses. Moscow; English transl. (1998) Mathematical statistics. Gordon &Breach, Amsterdam.
2. Ibramkhalilov I. and Skorokhod A. (1980) Sostoiatel'nye otsenki parametrov sluchainykh protsessov. Naukova Dumka, Kiev (in Russian).
3. Jech T. (2003) Set Theory. Springer-Verlag, Berlin.
4. Kharazishvili A. (1984) Topologicheskie aspekty teorii mery. Naukova Dumka, Kiev.
5. Kharazishvili A. (2004) Nonmeasurable sets and functions. North-Holland Mathematics Studies, Elsevier, Amsterdam.
6. Shiryaev A. (1980) Veroiatnost'. M. (in Russian).
7. Pantsulaia G. (1989) Ob ortogonal'nykh semeistvakh veroyatnostnykh mer. Trudy gruzinskogo politekhnicheskogo instituta 3501: 106-112 (in Russian).
8. Pantsulaia G. (2003) On separation properties for families of probability measures, *Georgian Mathematical Journal*, **10(2)**: 335-342.
9. Ibragimov I. and Rozanov Yu. (1970) Gaussian Random processes, M. (in Russian).
10. Zerakidze Z. (1984) On weakly divisible and divisible families of probability measures (in Russian), *Bull. Acad. Sci. Georg. SSR*, **113 (2)**: 273-275 (in Russian).

11. Zerakidze Z. (1991) Banach space of measures. In: Grigolionies B. et al (eds.) Probability theory and mathematical statistics, *Proceedings of the Fifth Vilnius Conference, VSP/Mokslas*, **2**: 609-616.
12. Zerakidze Z. and Patsatsia M. (2018) Extreme points and consistent criteria for hypothesis testing in a Banach space, *Bull. Georg. Natl. Acad. Sci.*, **12**(4): 33-39.
13. Danko P. E., Popov A. G., Kozhevnikova T. I., Danko S. P. (2009) Vysshaia matematika v uprazhneniiakh i zadachakh, II. M. (in Russian).

Received April, 2021