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Informatics

Markov Renewal Type Stochastic Model for Structural Control of Complex Systems

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In the paper mathematical model in the form of closed two-line queuing system for complex, multi-unit redundant system with renewable (repairable) units is constructed. In this model the duration of replacement of failed main unit by redundant one is taken into consideration. Final form of the model is non-classical boundary-value problem of mathematical physics. Boundary conditions in that problem are nonlocal and represent the system of recursive integral equations. This problem is at present under investigation.

Structural control, stochastic model, reliability, maintenance, queuing system

The object of our study is any large-scale territorially distributed redundant technical system and its structural control subsystem. The controlled system consists of unreliable, repairable units (computer and telecommunication networks, gas and oil pipelines, power, defence and transport systems, etc.)

At the same time, telecommunication systems, and, in particular, Mobile Communication Networks are the most relevant (the most suitable) field for applications of our research. The controlled system is subject to random structural disturbances (failures of system's units), causing changes in its structure. To compensate for the above disturbances structural control (and management) is accomplished in the system.

The main aim of the control is to maintain the system's (given, initial) structure or to restore (recover) it in the case of its spontaneous change. Also, the reconfiguration of the existing structure may be the aim of the structural control.

Among many types of structural control we consider the basic ones: 1) control of redundancy; 2) renewal (repair) of failed units (both main and redundant ones); 3) replacement of failed main units by redundant ones; 4) reconfiguration of the system [1-7].

Naturally high-quality structural control is one of the basic tools for the high performance of the controlled system.

That is why at all the stages of the life cycle of the control system (designing (planning), exploitation, modernization) applied investigations are carried out to make optimal (or at least rational) decisions for the needs of the structural control.

The above said implies the analysis, evaluation and optimization of the dependability (reliability, availability, maintainability, survivability, etc.) and the performance of the considered system.

These issues traditionally are mainly studied in Reliability Theory (RT), specifically in Mathematical Theory of Reliability (MTR).

Note: During 70 years of its existence (development), the definitions of the term "reliability" have been changing often (in the literature, in the standards, etc.). Therefore, for the clarity of our project (text), keeping up with the tradition, under "reliability" we mainly imply a comparatively new term "dependability" (a general term covering availability, maintainability, security, safety, survivability, efficiency, effectiveness, performability, etc.).

MTR while studying complex systems applies the methods and models of the Queuing Theory – QT (repairman problem, Feller,1957) [1-5].

But while considering the above (mentioned) large-scale complex systems classical (traditional) MTR (including renewal theory, redundancy theory, maintenance problems, i.e. all dependability problems), and classical (traditional) QT proved to be less effective and often completely useless.

The point is that this theory is mainly equipment (machine) oriented [1-14]. The main results in classical MTR were obtained in 1950-1980-ies and they had been mainly concerning to single unit systems. Such systems belong to the class of binary systems, for which exists the simple reliability criteria of the type “operative or inoperative”.

In the cases when in classical RT complex systems were considered, even they in most cases were reduced to binary reliability criteria, such as k-out-of-n.

As for territorially distributed networks their standard investigation was reduced to the investigation of two-pole networks and connectivity analysis.

There exist separate successful cases of attempts to study non-binary systems, but this does not change main tendencies significantly.

It is not accidental that in 2004 at the MMR-2004 (Mathematical Methods in Reliability) conference, one of the most prominent specialists in Reliability Theory and recognized authority, Professor of The George Washington University Nozer Singpurwalla raised the topic: “Is Reliability theory still alive?” This question led to burning discussions among Reliability specialists that are continued till now.

On the one hand, the question is half-rhetorical, but on the other hand, stating this question and following discussions confirm that classical Reliability Theory has been in some kind of crisis for last decades.

Another prominent expert Igor Ushakov [7] commented that the question itself can be formulated in different way: “Is Reliability Theory still developed?”.

I.Ushakov continues his previous ideas and convincingly proves that one of the main reasons of the crisis in the field and urgency of the question Professor Singpurwalla put is that Reliability Theory is still mainly machine (component) oriented and it does not answer modern needs. Finally, he makes convincing conclusion that “it is clear that reliability problems moved to the system level rather than component level” [2-14]. This means that performance (effectiveness) analysis for large scale territorially distributed networks is a topic of the first rank importance.

Exactly in this direction we need to consider replacement of failed main unit by redundant one as separate maintenance operation; as a result **the bifurcation of arrivals arises**.

The fact is that, in traditional cases of redundancy, the main and redundant units, as a rule, are territorially concentrated at the same place and the replacement of the failed main unit with a redundant one means the latter’s switching over, which is often automatically performed and its duration is negligibly small. In other cases, for example, in two unit systems, main and redundant units are connected in parallel and both are operating, although only one is in actual service. In such cases, after failure of main unit, redundant one continues the service without any replacement.

In modern networks of the above type we have completely different situation. In particular, redundant units are not directly attached (linked) to main ones in above network. They are placed at specific storages and may be located at the distance of tens, hundreds and sometimes thousands of kilometers away from the active units. Therefore, the delivery time of the redundant unit to the place of the failed main one is quite essential.

In addition, the replacement operation, apart from the delivery of the redundant unit to the main unit's place, includes other sub-operations, which are necessary in order the redundant unit to continue the main unit's functions. Under such circumstances the mean replacement time is not insignificant, and it often reaches 10-40 % of the mean repair time. Moreover, the replacement operation, as a rule, is performed not by a repair facility, but by a special replacement channel. Therefore, the replacement of the failed main unit by the redundant one quite naturally becomes an independent maintenance operation, which is the subject of future investigation for large scale territorially distributed networks.

It is obvious from above description that the failure of main units generates two requests for service: 1) the replacement of failed unit by redundant one; 2) the repair of failed unit. That is called the **bifurcation of arrivals** (failures). Therewithal both of these maintenance operations should be performed in parallel mode [5-14].

The basic model in the form of a closed queuing system with two types of service operations is described in the following way.

A complex system with renewable (repairable) elements consists of main and standby identical elements. The numbers of main and standby elements are m and n and the failure intensities for those are α and β respectively.

When a main element fails, there arises the necessity for its replacement by the standby one.

If at that moment there is a serviceable standby in the system element and the replacement unit is free, the operation of replacement of failed main element begins; if not, the replacement begins after arising such possibility.

The failed main element, as well as a standby one, joins to queue for renewal (repair).

In case the service channels are busy, requests' queues for replacement or repair are formed. Thus, we have a queuing system with two types of services – replacement and repair. The request for replacement arises when the main unit fails. The same event generates request for repair. This means that at the time instance of main unit's failure, a bifurcation of failure stream occurs (input bifurcation). In other words, it means that the need of two parallel operations, replacement and repair arises at the moment of failure. This circumstance makes modelling of such systems significantly difficult.

The renewed (repaired) element becomes a standby one or joins to queue for replacement in the case of necessity.

There are two service (maintenance) units in the system - one for replacements, and another – for renewals.

The durations ξ and η for replacement and renewal are random variables with arbitrary distribution functions $F(x)=\int_0^x f(u)d(u)$ and $G(x)=\int_0^x g(u)d(u)$ respectively.

The intensities of replacement and renewal, $\lambda(x)$ and $\mu(x)$ are defined in the following way

$$\lambda(x)=\lim_{h\to 0}\left(\frac{1}{h}P\{\xi\in(x,x+h)\,|\,\xi>x\}\right).$$

$$\mu(x)=\lim_{h\to 0}\left(\frac{1}{h}|\,P\{\mu\in(x,x+h)\,|\,\mu>x\}\right).$$

It is known that, $\lambda(x)=f(x)/(1-F(x))$ and $\mu(x)=g(x)/(1-G(x))$.

We introduce the random processes, which determine the states of considered system at the moment t.

i(t) – the number of elements missed in main group of elements;

j(t) – the number of nonserviceable (failed) elements in the system;

ξ(t) – the duration of time interval from starting of replacement to the moment t;

η(t) – the duration of time interval from starting of renewal to the moment t.

Denote,

$$p(t)=P\{i(t)=0;\, j(t)=0\}$$

$$q(i,t,x)=\lim_{h\to 0}\left(\frac{1}{h}P\{i(t)=i;\;\; j(t)=0;\;\; \xi(t)\in(x,x+h)\}\right);\qquad i=\overline{1,m};\,.$$

$$r(i,t,y)=\lim_{h\to 0}\left(\frac{1}{h}P\{i(t)=0;\;\; j(t)=j;\;\; \eta(t)\in(y,y+h)\}\right);\qquad j=\overline{1,n+j}\,;$$

$$z(i,j,t,x,y)=\lim_{h_1\to 0}\lim_{h_2\to 0}\left(\frac{1}{h_1}\frac{1}{h_2}P\{i(t)=i;\;\; j(t)=j;\;\; \xi(t)\in(x,x+h_1),\;\; \eta(t)\in)(y,y+h_2)\}\right);$$

$i=\overline{1,m}$; $j=\overline{1,n+j}$;

$$r(i,n+i,t,y)=\lim_{h_{01}\to 0}\left(\frac{1}{h}P\{i(t)=i;\;\; j(t)=n+i;\;\; (t),\;\; \eta(t)\in(y,y+h)\}\right)\;\; i=\overline{1,m}\,.$$

The last function reflects a situation, when there are missed elements in main group, but all of them, as well as all the standby ones are not serviceable (are failed) and therefore the replacement operation is not carried out.

Regarding the introduced functions we construct the system of integro-differential equations and partial differential equations.

They constitute four group of equations, describing the following states of the considered system: 1. Free state (there aren't requests in the system neither for replacement, nor for renewal); 2. Replacement (only replacement operation is carried out in the system); 3. Renewal (only renewal operation is carried out in the system); 4. Replacement and renewal (both replacement and renewal operations are carried out in the system).

Suppose that the function p(t) has derivative and the functions q(i,t,x), r(i,t,x), z(i,j,t,x,y) have continuous partial derivatives when t 0, x 0, y 0.

1. Free State

$$\frac{dP(t)}{dt}=-(m\alpha+n\beta)P(t)+\int_0^t q(1,t,x)\lambda(x)dx+\int_0^t r(1,t,x)\mu(y)dy\,. \tag{1.1}$$

2. Replacement

$$\frac{\partial q(i,t,x)}{\partial t}+\frac{\partial q(i,t,x)}{\partial x}=-\left[(m-i)\alpha+(n+i-1)\beta+\lambda(x)\right]q(i,t,x)+\int_0^t z(i,1,t,x,y)\mu(y)dy\,,$$

$$i=\overline{1,m}\,. \tag{2.1}$$

3. Renewal

$$\frac{\partial r(1,t,y)}{\partial t}+\frac{\partial r(1,t,y)}{\partial y}=-\left[m\alpha+(n-1)\beta+\mu(y)\right]r(1,t,y)+\int_0^t z(1,1,t,x,y)\lambda(x)dx\,, \tag{3.1}$$

$$\frac{\partial r(j,t,y)}{\partial t}+\frac{\partial r(j,t,y)}{\partial y}=-\left[m\alpha+(n-j)\beta+\mu(y)\right]r(j,t,y)+r(j-1,t,y)(n-j+1)+$$

$$+\int_0^t z(1,j,t,x,y)\lambda(x)dx,\ \ 2\le j\le n\ \ , \tag{3.2}$$

$$\frac{\partial r(i,n+i,t,y)}{\partial t}+\frac{\partial r(i,n+i,t,y)}{\partial y}=-\left[(m-i)\alpha+\mu(y)\right]r(i,n+i,t,y)+r(i-1,n+i-1,t,y)(m-i+1)+$$

$$+\int_0^t z(i+1,n+i,t,x,y)\lambda(x)dx,\ \ i=1,m-1\,; \tag{3.3}$$

$$\frac{\partial r(m,m+n,t,y)}{\partial t}+\frac{\partial r(m,m+n,t,y)}{\partial y}=--\mu(y)r(m,m+n,t,y)+r(m-1,m+n-1,t,y)\alpha\,. \tag{3.4}$$

4. Replacement and Renewal

$$\frac{\partial z(i,1,1,t,x,y)}{\partial t}+\frac{\partial z(i,1,1,t,x,y)}{\partial x}+\frac{\partial z(i,1,1,t,x,y)}{\partial y}=$$

$$-\left[(m-i)\alpha+(m+i-2)\beta+\lambda(x)+\mu(y)\right]z(i,1,t,x,y),\ \ i=\overline{1,m}\,; \tag{4.1}$$

$$\frac{\partial z(1,j,t,x,y)}{\partial t}+\frac{\partial z(1,j,t,x,y)}{\partial y}+\frac{\partial z(1,j,t,x,y)}{\partial y}=-\left[(m-1)\alpha+(n-j)\beta+\lambda(x)+\mu(y)\right]z(1,j,t,x,y)+$$

$$+z(1,j-1,t,x,y)(n-j+1)\beta,\ \ j=\overline{2,n}\,; \tag{4.2}$$

$$\frac{\partial z(i,j,t,x,y)}{\partial t}+\frac{\partial z(i,j,t,x,y)}{\partial x}+\frac{\partial z(i,j,t,x,y)}{\partial y}=-\left[(m-i)\alpha+(n+i-j-1)\beta+\lambda(x)+\mu(y)\right]z(i,j,t,x,y)+$$

$$+z(i-1,j-1,t,x,y)(m-i+1)\alpha+z(i,j-1,t,x,y)(n+i-j+1)\beta,\ \ i=\overline{2,m};\ \ j=\overline{2,n+i-1}\,; \tag{4.3}$$

Boundary Conditions

2b. Replacement

$$q(i,t,0)=\int_0^t q(i+1,t,x)\lambda(x)dx,\ \ i=\overline{1,m-1}\,; \tag{2b.1}$$

3b. Renewal

$$r(1,t,0)=P(t)n\beta+\int_0^t r(2,t,y)\mu(y)dy\,; \tag{3b.1}$$

$$q(j,t,0)=\int_0^t r(j+1,t,y)\mu(y)dy,\ \ j=\overline{2,n-1};\tag{3b.2}$$

$$r(i,n+i,t,0)=0,\ \ i=\overline{0,m};\tag{3b.3}$$

4b. Replacement and Renewal

$$z(i,1,t,0,y)=\int_0^t z(i+1,1,t,x,y)\lambda(x)dx,\ \ i=\overline{1,m-1};\tag{4b.1}$$

$$z(m,1,t,0,y)=0;\tag{4b.2}$$

$$z(1,1,t,x,0)=\int_0^t z(1,2,t,x,y)\mu(y)dy+q(1,t,x)n\beta;\tag{4b.3}$$

$$z(i,1,t,x,0)=\int_0^t z(i,2,t,x,y)\mu(y)dy+q(i-1,t,x)(m-i+1)\alpha+q(i,t,x)(n+i-1)\beta,\ \ i=\overline{2,m};\tag{4b.4}$$

$$z(1,j,t,0,y)=\int_0^t z(i,j,t,x,y)\lambda(x)dx+r(j-1,t,y)m\alpha,\ \ j=\overline{1,n};\tag{4b.5}$$

$$z(1,i,t,x,0)=\int_0^t z(1,j+1,x,y)\mu(y)dy,\ \ j=\overline{1,n-1};\tag{4b.6}$$

$$z(1,n,t,x,0)=0;\tag{4b.7}$$

$$z(1,n+1,t,x,0)=0;\tag{4b.8}$$

$$z(i,j,t,0,y)=\int_0^t z(i+1,j,t,x,y)\lambda(x)dx,\ \ i=\overline{2,m-1};\ j=\overline{2,n+i-1};\tag{4b.9}$$

$$z(m,j,t,0,y)=0,\ \ j=\overline{2,m+n-1};\tag{4b.10}$$

$$z(i,j,t,x,0)=\int_0^t z(i,j+1,t,x,y)\mu(y)dy,\ \ i=\overline{2,m};\ \ j=\overline{2,n+i-1};\tag{4b.11}$$

$$z(i,n+i-1,t,x,0)=0;\tag{4b.12}$$

$$z(1,1,t,0,0)=P(t)m\alpha;\tag{4b.13}$$

$$z(i,j,t,0,0)=0,\ \ i+j>2;\tag{4b.14}$$

Hence, we have obtained the mathematical model in the form of non-classical boundary-value problem of mathematical physics. Boundary conditions in this problem are nonlocal and represent the system of recursive integral equations. The correct statement of this problem and its further investigation with analytical and numerical methods is a complicated and interesting problem from the point of view of the applied mathematics, as well as from the points of view of the reliability theory and the queuing theory.

Conclusion

Both in the classical reliability theory and classical queuing theory the afore mentioned problem was not investigated in general. Namely, only very particular cases were examined: 1) $m = 1$, $n = 1$; 2) $m = 1$, $n = 2$; 3) M/M/N i.e. the distribution function of repair time is exponential and replacement time is equal to zero (instant replacement); 4) several similar statements were investigated.

Only in publications of experts of the Georgian Technical University, for the last 17-18 years many Markov and semi-Markov models have been examined for arbitrary m and arbitrary n cases. It means that possibilities of Markov and semi-Markov models are largely utilized.

In this paper, for the first time in scientific literature, we have constructed Markov renewal type stochastic model, which is further generalization of the above Markov and semi-Markov ones. The investigation of the model is, obviously, very difficult problem. We hope, the probabilistic method, offered in [19, 20] will be greatly useful in the matter. At the same time, if one of the functions F and G is exponential, two mutually dual semi-Markov models are obtained as the special cases of our model. For them we surely say, that these semi-Markov models can be perfectly investigated by the method in [19, 20].

The author of the paper continues further research of the problem and calls upon the interested colleagues to join this work.

ინფორმატიკა

განახლების ტიპის მარკოვული სტოქასტური მოდელი რთული სისტემების სტრუქტურული მართვისათვის

რ. ხუროძე

აკადემიის წევრი, საქართველოს მეცნიერებათა ეროვნული აკადემია; საქართველოს ტექნიკური უნივერსიტეტი, თბილისი, საქართველო

წარმოდგენილ ნაშრომში აგებულია სტოქასტური მოდელი, ჩაკეტილი ორარხიანი მომსახურების სისტემის სახით, რთული მრავალელემენტიანი დარეზერვებული სისტემისათვის. ეს სისტემა შედგება მტყუნებადი აღდგენადი ელემენტებისგან. ამ მოდელში მტყუნებადი ძირითადი ელემენტის ჩანაცვლება სარეზერვო ელემენტით განხილულია როგორც მომსახურების დამოუკიდებელი ოპერაცია. საბოლოო ფორმით მოდელი წარმოადგენს მათემატიკური ფიზიკის არაკლასიკურ სასაზღვრო ამოცანას. სასაზღვრო პირობები აღნიშნულ ამოცანაში არალოკალურია და წარმოდგენილია რეკურსიული ინტეგრალური განტოლებების სისტემის სახით. ამჟამად, მიმდინარეობს ამ ამოცანის გამოკვლევა.

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