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Mathematics

Finite Difference Scheme for One System of Nonlinear Partial Differential Equations

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(Presented by Academy Member Elizbar Nadaraya)

In this paper, a finite difference scheme for one system of nonlinear partial differential equations is constructed and investigated. Investigated model is based on the well-known system of Maxwell's equations and represents some of its generalizations. The one-dimensional case with three-component magnetic field is considered. The convergence of the scheme under consideration is studied and estimate of the error of the approximate solution is obtained. *© 2022 Bull. Georg. Natl. Acad. Sci.*

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A great variety of applied problems are simulated by systems of nonlinear partial differential equations (NPDE). Such systems, based on the Maxwell's system, arise, for instance, at mathematical simulation of the process of penetrating magnetic field in the substance [1]. Nonlinear evolution equations as mathematical models are widely used in almost all scientific disciplines. For most nonlinear equations, it is very difficult to find exact solutions and there is no general solution available in the close form. It is known that constructing the exact solution for NPDE is possible only in some particular cases. That is why we are looking for the approximate solution using the direct methods with the corresponding theoretical background. Due to that it is almost impossible to find the exact analytical solution of the system studied in this work, we propose difference schemes for numerical solution.

In the domain $Q=(0;1)\times(0;\infty)$, let us consider the system:

$$\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}\left(S^{\alpha}\frac{\partial U}{\partial x}\right),\qquad \frac{\partial V}{\partial t}=\frac{\partial}{\partial x}\left(S^{\alpha}\frac{\partial V}{\partial x}\right),\qquad \frac{\partial W}{\partial t}=\frac{\partial}{\partial x}\left(S^{\alpha}\frac{\partial W}{\partial x}\right), \tag{1}$$

$$\frac{\partial S}{\partial t}=-aS^{\beta}+bS^{\gamma}\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}\right)^2+\left(\frac{\partial W}{\partial x}\right)^2\right] \tag{2}$$

with the following boundary and initial conditions:

$$U(0,t)=V(0,t)=W(0,t)=0, \tag{3}$$

$$U(1,t)=\psi_1>0, \qquad V(1,t)=\psi_2>0, \qquad W(1,t)=\psi_3>0,$$

$$U(x,0)=U_0(x), \quad V(x,0)=V_0(x), \quad W(x,0)=W_0(x), \quad S(x,0)=S_0(x). \tag{4}$$

Here $(x,t)\in Q;\ \alpha,\beta,\gamma\in R;\ a,b,\psi_1,\psi_2,\psi_3$ are positive constants and $U_0(x),\ V_0(x),\ W_0(x),\ S_0(x)$ are given functions.

Systems of (1), (2) types arise in mathematical modeling of many practical processes and in theoretical fields as well [1-5]. For more detailed informations see [6,7] and references therein. Some qualitative and structural properties of solutions of (1), (2) type systems are established in many works. System (1), (2) may be considered as one-dimensional analogue of the model of process of penetration of electromagnetic field into the substance. The questions of unique solvability, asymptotic behavior of solutions and approximate solutions for some cases of (1) - (4) type problems are studied in [1-17] and in the number of other works as well.

It is easy to check that if $a=0,\ \ b=1,\ \ \gamma=\alpha,\ \ U_0(x)=\psi_1 x,\ \ V_0(x)=\psi_2 x,\ \ W_0(x)=\psi_3 x$ and $S_0(x)=S_0=const>0,$ then when $\alpha\neq 1$ the solution of the problem (1) - (4) is [17]:

$$U(x,t)=\psi_1 x, \qquad V(x,t)=\psi_2 x, \qquad W(x,t)=\psi_3 x,$$
$$S(x,t)=\left[S_0^{1-\alpha}+(1-\alpha)\left(\psi_1^2+\psi_2^2+\psi_3^2\right)t\right]^{\frac{1}{1-\alpha}}. \tag{5}$$

As it can be seen from (5), for a finite value of time, namely, when

$$t_0=\frac{S_0^{1-\alpha}}{(\alpha-1)\left(\psi_1^2+\psi_2^2+\psi_3^2\right)}$$

and $\alpha>1,$ then the function $S(x,t)$ is not bounded.

The above example shows that (1) - (4) does not have global solution at all. Thus, the solution of problem (1) - (4) with smooth initial and boundary conditions can be blown up at the finite time.

Note that if we add the following boundary conditions to (3):

$$\left.\frac{\partial S}{\partial x}\right|_{x=0}=\left.\frac{\partial S}{\partial x}\right|_{x=1}=0, \tag{6}$$

then $U,\ V,\ W$ and S defined by (5) are also solutions of the following system:

$$\frac{\partial U}{\partial t}=\frac{\partial}{\partial x}\left(S^\alpha\frac{\partial U}{\partial x}\right), \qquad \frac{\partial V}{\partial t}=\frac{\partial}{\partial x}\left(S^\alpha\frac{\partial V}{\partial x}\right), \qquad \frac{\partial W}{\partial t}=\frac{\partial}{\partial x}\left(S^\alpha\frac{\partial W}{\partial x}\right),$$
$$\frac{\partial S}{\partial t}=S^\gamma\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}\right)^2+\left(\frac{\partial W}{\partial x}\right)^2\right]+\frac{\partial^2 S}{\partial x^2} \tag{7}$$

with (3), (4), (6) boundary and initial conditions. We conclude that for $\alpha>1,$ the problem (3), (4), (6), (7) has no global solution neither.

In some cases, the linear stability of stationary solutions is studied for the above-mentioned problems. It is not difficult to show that if $\beta\neq\gamma$ the stationary solution (U_s,V_s,W_s,S_s) of problem (1) - (4) has the form:

$$U_s = \psi_1 x, \qquad V_s = \psi_2 x, \qquad W_s = \psi_3 x, \qquad S_s = \left[\frac{b}{a}\left(\psi_1^2+\psi_2^2+\psi_3^2\right)\right]^{\frac{1}{\beta-\gamma}}. \tag{8}$$

There appears the possibility of the Hopf bifurcation. Small perturbations may cause the transformation of solution (8) into periodic oscillations [18].

The study of similar problems in this area was first carried out in the article [8] for the two component (U,S) case. Many works [6, 7, 10, 12, 13, 15] are devoted to similar studies for two (U,S) and three components (U,V,S) cases of (1), (2) types systems. The (1), (2) system with (U,V,W,S) components are investigated in [17].

Let us study the problem of approximate solution of the problem (1) - (4).

If we introduce the following notation $E = S^{\frac{1}{2}}$ the problem will be as follows:

$$\frac{\partial U}{\partial t}-\frac{\partial}{\partial x}\left(E^{2\alpha}\frac{\partial U}{\partial x}\right)=0, \qquad \frac{\partial V}{\partial t}-\frac{\partial}{\partial x}\left(E^{2\alpha}\frac{\partial V}{\partial x}\right)=0, \qquad \frac{\partial W}{\partial t}-\frac{\partial}{\partial x}\left(E^{2\alpha}\frac{\partial W}{\partial x}\right)=0, \tag{9}$$

$$\frac{\partial E}{\partial t} = -\frac{a}{2}E^{2\beta-1}+\frac{b}{2}E^{2\gamma-1}\left[\left(\frac{\partial U}{\partial x}\right)^2+\left(\frac{\partial V}{\partial x}\right)^2+\left(\frac{\partial W}{\partial x}\right)^2\right], \tag{10}$$

$$\begin{gathered} U(0,t)=V(0,t)=W(0,t)=0, \\ U(1,t)=\psi_1, \qquad V(1,t)=\psi_2, \qquad W(1,t)=\psi_3, \end{gathered} \tag{11}$$

$$U(x,0)=U_0(x), \quad V(x,0)=V_0(x), \quad W(x,0)=W_0(x), \quad E(x,0)=\left[S_0(x)\right]^{\frac{1}{2}}. \tag{12}$$

Let us construct grids on $Q_T=[0,1]\times[0,T]$. Divide the intervals $[0,1]$ and $[0,T]$ into equal N and M parts respectively, and introduce the following notations [19]:

$$h=\frac{1}{N}, \quad \tau=\frac{T}{M}, \quad x_i = ih, \quad t_j = j\tau, \quad u_i^j = u\left(x_i,t_j\right),$$

$$\bar{\omega}_h = \left\{x_i = ih,\ i=0,1,...,N\right\}, \quad \omega_h^* = \left\{x_i = \left(i-\frac{1}{2}\right)h,\ i=0,1,...,N\right\},$$

$$\omega_\tau = \left\{\tau_j = j\tau,\ i=0,1,...,M\right\}, \quad \bar{\omega}_{h\tau} = \bar{\omega}_\tau \times \omega_\tau, \quad \omega_{h\tau}^* = \omega_h^* \times \omega_\tau,$$

$$u_x = \frac{u_{i+1}-u_i}{h}, \quad u_{\bar{x}} = \frac{u_i - u_{i-1}}{h}, \quad u_i = u_i^{j+1}, \quad u_\tau = \frac{u_i - u_i^j}{h}.$$

Let us also introduce standard scalar products and norms [19]:

$$(u,v) = \sum_{i=1}^{N-1} u_i v_i h, \qquad (u,v] = \sum_{i=1}^{N} u_i v_i h,$$

$$\|u\| = (u,u)^{\frac{1}{2}}, \quad \|u]| = (u,u]^{\frac{1}{2}}.$$

Let us construct an implicit finite difference scheme corresponding to the problem (9) - (12):

$$u_t^j = \left(e^{2\alpha}u_{\bar{x}}\right)_x, \qquad v_t^j = \left(e^{2\alpha}v_{\bar{x}}\right)_x, \qquad w_t^j = \left(e^{2\alpha}w_{\bar{x}}\right)_x, \tag{13}$$

$$e_t^j = -\frac{a}{2}e^{2\beta-1}+\frac{b}{2}e^{2\gamma-1}\left(u_{\bar{x}}^2+v_{\bar{x}}^2+w_{\bar{x}}^2\right), \tag{14}$$

$$u_0^j = v_0^j = w_0^j = 0, \quad u_N^j = \psi_1, \quad v_N^j = \psi_2, \quad w_N^j = \psi_3, \quad j = 0,1,...,M, \tag{15}$$

$$u_i^0 = U_0(x_i), \quad v_i^0 = V_0(x_i), \quad w_i^0 = W_0(x_i), \quad e_i^0 = \left[S_0\left(x_{i+\frac{1}{2}}\right)\right]^{\frac{1}{2}}, \quad i = 0,1,...,N-1, \tag{16}$$

where u, v and w functions are defined on the grid $\bar{\omega}_{h\tau}$ and grid function e is defined on the grid $\omega_{h\tau}^*$. Here and below the unindexed values mean that the grid functions are taken at the point (x_i, t_{j+1}) or $(x_{i-1/2}, t_{j+1})$.

It is not difficult to show that the approximations of the (13) - (16) scheme on the smooth solutions of the problem (9) - (12) are of the order $O(\tau + h^2)$.

For the errors $X = u - U$, $Y = v - V$, $Z = w - W$ and $R = e - E$ we have the following equations:

$$X_t^j = \left(e^{2\alpha} u_{\bar{x}} - E^{2\alpha} U_{\bar{x}}\right)_x + \phi_1, \tag{17}$$

$$Y_t^j = \left(e^{2\alpha} v_{\bar{x}} - E^{2\alpha} V_{\bar{x}}\right)_x + \phi_2, \tag{18}$$

$$Z_t^j = \left(e^{2\alpha} w_{\bar{x}} - E^{2\alpha} W_{\bar{x}}\right)_x + \phi_3, \tag{19}$$

$$R_t^j = -\frac{a}{2}\left(e^{2\beta-1} - E^{2\beta-1}\right) + \frac{b}{2}\left(e^{2\gamma-1} u_{\bar{x}}^2 - E^{2\gamma-1} U_{\bar{x}}^2 + e^{2\gamma-1} v_{\bar{x}}^2 - E^{2\gamma-1} V_{\bar{x}}^2 + e^{2\gamma-1} w_{\bar{x}}^2 - E^{2\gamma-1} W_{\bar{x}}^2\right) + \phi_4, \tag{20}$$

where $\phi_k = O(\tau + h^2)$, $k = 1,...,4.$

Assume that $\alpha = \gamma$ and $|\alpha| \le \frac{1}{2}$. Multiply the equations (17) - (20) scalarly on $2\tau X$, $2\tau Y$, $2\tau Z$ and $\frac{2}{b}\tau R$, respectively.

Using the discrete analog of the formula of integration by part and the identities:

$$2\tau(g_t, g) = \|g\|^2 - \|g^j\|^2 + \tau^2\|g_t\|^2,$$

$$2\tau(G_t, G] = \|G]|^2 - \|G^j]|^2 + \tau^2\|G_t]|^2,$$

where g and G are any grid functions, we obtain (for details see [19]):

$$\|X\|^2 - \|X^j\|^2 + \tau^2\|X_t\|^2 = -2\tau\left[e^\delta, u_{\bar{x}}^2 - e^\delta + E^\delta, u_{\bar{x}} U_{\bar{x}} + E^\delta, U_{\bar{x}}^2 - (\phi_1, X)\right],$$

$$\|Y\|^2 - \|Y^j\|^2 + \tau^2\|Y_t\|^2 = -2\tau\left[e^\delta, v_{\bar{x}}^2 - e^\delta + E^\delta, v_{\bar{x}} V_{\bar{x}} + E^\delta, V_{\bar{x}}^2 - (\phi_2, Y)\right],$$

$$\|Z\|^2 - \|Z^j\|^2 + \tau^2\|Z_t\|^2 = -2\tau\left[e^\delta, w_{\bar{x}}^2 - e^\delta + E^\delta, w_{\bar{x}} W_{\bar{x}} + E^\delta, W_{\bar{x}}^2 - (\phi_3, Z)\right],$$

$$\frac{1}{b}\|R]|^2 - \|R^j]|^2 + \tau^2\|R_t]|^2 = -\frac{a}{b}\tau\left(e^{2\beta-1} - E^{2\beta-1}\right)(e - E) + \tau\Big(\left(e^\delta - e^{\delta-1}E, u_{\bar{x}}^2\right] - \left(E^{\delta-1}e - E^\delta, U_{\bar{x}}^2\right] +$$

$$+\left(e^\delta - e^{\delta-1}E, v_{\bar{x}}^2\right] - \left(E^{\delta-1}e - E^\delta, V_{\bar{x}}^2\right] + \left(e^\delta - e^{\delta-1}E, w_{\bar{x}}^2\right] - \left(E^{\delta-1}e - E^\delta, W_{\bar{x}}^2\right]\Big) + \frac{2\tau}{b}(\phi_4, R].$$

The following notation is introduced here $2\alpha = \delta$.

By adding these equations and after simple transformations, assuming that $\beta \geq \frac{1}{2}$, then we have

$$\|X\|^2-\left\|X^j\right\|^2+\|Y\|^2-\left\|Y^j\right\|^2+\|Z\|^2-\left\|Z^j\right\|^2+\frac{\tau}{b}\left(\|R]|^2-\frac{1}{b}\left\|R^j\right]|^2\right) \leq$$

$$\leq -2\tau\left[\left(\frac{e^\delta+e^{\delta-1}E}{2}u_{\bar{x}}^2+\frac{E^\delta+E^{\delta-1}e}{2}U_{\bar{x}}^2,1\right]-\left(e^\delta+E^\delta,u_{\bar{x}}^2U_{\bar{x}}^2\right]+\right.$$

$$+\left(\frac{e^\delta+e^{\delta-1}E}{2}v_{\bar{x}}^2+\frac{E^\delta+E^{\delta-1}e}{2}V_{\bar{x}}^2,1\right]-\left(e^\delta+E^\delta,v_{\bar{x}}^2V_{\bar{x}}^2\right]+$$

$$+\left(\frac{e^\delta+e^{\delta-1}E}{2}w_{\bar{x}}^2+\frac{E^\delta+E^{\delta-1}e}{2}W_{\bar{x}}^2,1\right]-\left(e^\delta+E^\delta,w_{\bar{x}}^2W_{\bar{x}}^2\right]-$$

$$\left.-\left(\phi_1,X\right)-\left(\phi_2,Y\right)-\left(\phi_3,Z\right)-\frac{1}{b}\left(\phi_4,R\right]\right] \leq$$

$$\leq -2\tau\left\{\left(\left[\left(e^\delta+e^{\delta-1}E\right)\left(E^\delta+E^{\delta-1}e\right)\right]^{\frac{1}{2}}-e^\delta-E^\delta,\left|u_{\bar{x}}\right|\left|U_{\bar{x}}\right|\right]+\right.$$

$$+\left(\left[\left(e^\delta+e^{\delta-1}E\right)\left(E^\delta+E^{\delta-1}e\right)\right]^{\frac{1}{2}}-e^\delta-E^\delta,\left|v_{\bar{x}}\right|\left|V_{\bar{x}}\right|\right]-$$

$$+\left(\left[\left(e^\delta+e^{\delta-1}E\right)\left(E^\delta+E^{\delta-1}e\right)\right]^{\frac{1}{2}}-e^\delta-E^\delta,\left|w_{\bar{x}}\right|\left|W_{\bar{x}}\right|\right]-$$

$$\left.-\left(\phi_1,X\right)-\left(\phi_2,Y\right)-\left(\phi_3,Z\right)-\frac{1}{b}\left(\phi_4,R\right]\right\}. \tag{21}$$

Note that,

$$\left(e^\delta-e^{\delta-1}E\right)\left(E^\delta-E^{\delta-1}e\right)-\left(e^\delta+E^\delta\right)^2=$$

$$=2e^\delta E^\delta+e^{\delta+1}E^{\delta-1}+e^{\delta-1}E^{\delta+1}+e^{2\delta}-2e^\delta E^\delta-E^{2\delta}=\left(e^{\delta+1}-E^{\delta+1}\right)\left(E^{\delta-1}-e^{\delta-1}\right). \tag{22}$$

Since $|\delta| \leq 1,$ then:

$$\left(e^{\delta+1}-E^{\delta+1}\right)\left(E^{\delta-1}-e^{\delta-1}\right) \geq 0.$$

Using the last inequality and (21), (22) relations we finally get

$$\|X\|^2+\|Y\|^2+\|Z\|^2+\frac{1}{b}\|R]|^2 \leq$$

$$\leq\left\|X^j\right\|^2+\left\|Y^j\right\|^2+\left\|Z^j\right\|^2+\frac{1}{b}\left\|R^j\right]|^2+2\tau\left(\left(\phi_1,X\right)+\left(\phi_2,Y\right)+\left(\phi_3,Z\right)+\frac{1}{b}\left(\phi_4,R\right]\right),$$

from which we have the following identity

$$\|X\|+\|Y\|+\|Z\|+\|R]|=O\left(\tau+h^2\right).$$

The following statement is therefore fair.

Theorem. If $\beta \geq \frac{1}{2}$, $\alpha=\gamma$, $|\alpha| \leq \frac{1}{2}$, then the finite difference scheme (13) - (16) converges to smooth solution of the problem (9) - (12) in the grid functions space $L_2\left(\omega_h\right)$ and the order of convergence is $O\left(\tau+h^2\right)$.

მათემატიკა

სასრულ-სხვაობიანი სქემა არაწრფივ კერძოწარმოებულებიან დიფერენციალურ განტოლებათა ერთი სისტემისთვის

თ. ჯანგველაძე

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(წარმოდგენილია აკადემიის წევრის ე. ნადარაიას მიერ)

ნაშრომში აგებული და გამოკვლეულია არაწრფივ კერძოწარმოებულებიან დიფერენციალურ განტოლებათა ერთი სისტემისთვის სასრულ-სხვაობიანი სქემა. მოდელი დაფუძნებულია მაქსველის ცნობილ განტოლებათა სისტემაზე და წარმოადგენს მის გარკვეულ განზოგადებას. განიხილება ერთგანზომილებიანი შემთხვევა სამკომპონენტიანი მაგნიტური ველით. შესწავლილია განხილული სქემის კრებადობა და მიღებულია მიახლოებითი ამონახსნის ცდომილების შეფასება.

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