საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, ტ. 17, №1, 2023
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 17, no. 1, 2023

Physics

Gluon Propagator in the Infrared Region and the Problem of Potential between Quarks

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We present the results, derived in Tbilisi Group, performed during last decades on possibility of solving the Dyson-Schwinger equation (DSE) for the gluon propagator in the infrared region. The goal was attained only in the light-like gauge among all the classes of non-covariant axial gauges. This gauge has many advantages, mainly in independence of various structure functions from the gauge vector. This important property is a consequence of the used integration method, which guarantees homogeneity in the gauge vector at every stage of integration and is a peculiarity of the only light-like gauge $n^2 = 0$. Our investigations show that solution in this gauge exhibits the confining- like behavior for the gluon propagator in the truncated DSE. One gluon exchange effective potential between quarks constructed by this solution corresponds to the combination of the linearly rising Coulomb potentials welcomed by modern experimental data on hadron spectroscopy.

Dyson-Schwinger equation, gluon propagator, the infrared region

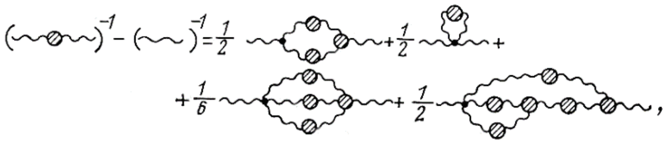
It is well-known that there are four interactions in Nature. Gravity and electromagnetism are of long-range character and are established experimentally (Newton, Coulomb), while a correct theoretical substantiation occurs after A.Einstein and J.Maxwell, correspondingly. The third type of interaction – weak is short range one provided by exchange of massive gauge-vector particles $W^{\pm}, Z$ (Schwinger, Weinberg, Salam, Glashow). As regards of the fourth interaction strong manifests itself in short distances (~10-13 cm) and is the interaction between the colored quarks by exchange of the colored gluons. Therefore, confinement phenomenon is proposed, because quarks and other colored objects are not encountered in free state in ordinary meaning. However, a confinement is not proved neither in a theory or by experiment.

The confinement hypothesis consists in the assumption that QCD interaction at large distances is arranged such that under any experimental condition one cannot observe free quarks, gluons and other open color states. In spite of strenuous efforts to prove the assertion in the framework of QCD nobody succeeded in this. The additional difficulty is the formulation of confinement itself in the language of quantum field theory. Some authors formulate conditions, e.g., Wilson's criterion or a criterion of color states propagators having no singularities. Generally speaking, we do not exclude a possibility of realizing other variants.

One of wide spreading method on this subject is searching of infrared (IR) singular q^{-4} behavior of the gluon propagator on the framework of Dyson-Schwinger equation (DSE) [1] invoking Slavnov-Taylor gauge identities [2-4]. The aim of this paper is to revise main results derived by Tbilisi group [5-13] in this direction during rather large period (1978-2010).

Our consideration consists in using the variety of axial gauges, $n\cdot A=0$ with $n^2=0$, which is named as light-like gauge. This gauge in problem, that is under the consideration, was used only by our group. This gauge has many advantages, among which we underline the independence of various invariant structure functions on the gauge vector. In general axial gauges with $n^2\neq 0$ this property is not realized and we have to make non-trivial assumptions. The Green's functions are gauge dependent quantities. Methods containing an explicit gauge dependence are often criticized for this. We emphasize that if one calculates correctly, the observable quantities, in any case, are gauge-independent.

DSE for the gluon propagator, which graphically looks like:



and may be written in the following form:

$$\Pi_{\mu\nu}^{ab}(p)-\Pi_{\mu\nu}^{(0)ab}(p)=\Pi_{\mu\nu}^{(1)ab}(p)+\Pi_{\mu\nu}^{(2)ab}(p). \tag{1}$$

Here $\Pi_{\mu\nu}^{ab}(p)$ is the polarization operator:

$$\Pi_{\mu\nu}^{ab}(p)=\delta^{ab}\Pi_{\mu\nu}(p)=i\delta^{ab}\left\{\pi_1(p^2)\left(g_{\mu\nu}-\frac{p_\mu p_\nu}{p^2}\right)+\pi_2(p^2)\left(g_{\mu\nu}-\frac{p_\mu n_\nu+p_\nu n_\mu}{(np)}+\frac{p^2 n_\mu n_\nu}{(np)^2}\right)\right\}. \tag{2}$$

It is an inverse to full gluon propagator

$$\Delta_{\mu\nu}^{ab}(p)=\delta^{ab}\Delta_{\mu\nu}=-i\delta^{ab}\left\{\Delta_1(p^2)\left(g_{\mu\nu}-\frac{p_\mu n_\nu+p_\nu n_\mu}{(np)}\right)+\Delta_2(p^2)\frac{p^2 n_\mu n_\nu}{(np)^2}\right. \tag{3}$$

in the sense

$$\Delta_{\mu\lambda}^{ac}(p)\Pi_{\lambda\nu}^{cb}(p)=\delta^{ab}\left(g_{\mu\nu}-\frac{p_\mu n_\nu}{(np)}\right). \tag{4}$$

As a result, it follows relations between structure factors

$$\pi_1(p^2)=\frac{1}{\Delta_1(p^2)-\Delta_2(p^2)},\quad \pi_2(p^2)=-\frac{\Delta_2(p^2)}{\Delta_1(p^2)\left[\Delta_1(p^2)-\Delta_2(p^2)\right]}. \tag{5}$$

In DSE the free part $\Pi_{\mu\nu}^{(0)}(p)$ is:

$$\Pi_{\mu\nu}^{(0)ab}=i\delta^{ab}\left(p^2 g_{\mu\nu}-p_\mu p_\nu\right) \tag{6}$$

and $\Pi_{\mu\nu}^{(1),(2)}(p)$ are contributions from one- and two- loop diagrams, correspondingly. In loop integrations we use a dimensional regularization method in D $(D=2\omega=4+2\varepsilon,\varepsilon\to 0)$ – dimensional space, when the tadpole diagram contribution vanishes.

In the axial gauges $(nA)=0$ we must have $n_\mu\Delta_{\mu\nu}(p)=0$. It is seen from explicit form of two loop expression, that

$$n_\mu\Pi_{\mu\nu}^{(2)ab}(p)=0. \tag{7}$$

At the same time, using the Slavnov-Taylor gauge identities [1-4]

$$p_\lambda \Gamma_{\lambda\mu\nu}(p,q,r) = \Delta^{-1}_{\mu\nu}(q) - \Delta^{-1}_{\mu\nu}(r) \tag{8}$$

we see, that the polarization operator is orthogonal to momentum p_ν from the right-side

$$\Pi^{(1)ab}_{\mu\nu}(p)\, p_\nu = \Pi^{(2)ab}_{\mu\nu}(p)\, p_\nu = 0. \tag{9}$$

These properties give the following tensor content of one and two loop terms:

$$\Pi^{(1)}_{\mu\nu}(p) = \mathscr{P}^{(1)}_1(p)\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right) + \mathscr{P}^{(1)}_2(p)\left(g_{\mu\nu} - \frac{n_\mu p_\nu + p_\mu n_\nu}{(np)}\right) + \\ + \mathscr{P}^{(1)}_3(p)\left(\frac{p^2 n_\mu n_\nu}{(np)^2} - \frac{n_\mu p_\nu}{(np)}\right), \tag{10}$$

$$\Pi^{(2)}_{\mu\nu}(p) = \mathscr{P}^{(2)}_2(p)\left(g_{\mu\nu} - \frac{n_\mu p_\nu + p^2 n_\mu n_\nu}{(np)^2}\right) + \mathscr{P}^{(2)}_3(p)\left(\frac{p^2 n_\mu n_\nu}{(np)^2} - \frac{n_\mu p_\nu}{(np)}\right). \tag{11}$$

One can see from these expressions that in both terms of the polarization operator they appear to be non-symmetrical terms with respect to Lorentz indices.

From the definitions it is easy to show that the DSE are equivalent to the following scalar equations for the invariant structures:

$$\begin{aligned} &i\left[\pi_1(p^2) - p^2\right] = \mathscr{P}^{(1)}_1(p^2) \\ &i\pi_2(p^2) = \mathscr{P}^{(1)}_2(p^2) + \mathscr{P}^{(2)}_2(p^2) \\ &\mathscr{P}^{(1)}_3(p^2) + \mathscr{P}^{2}_3(p^{(2)}) = 0. \end{aligned} \tag{12}$$

The first of these equations coincides with $n_\mu n_\nu$ contracted DSE, which we studied in [8,9]. The last equation shows that the terms nonsymmetrical with Lorentz indices must be mutually cancelled.

Solution of Contracted Full DSE in the Light-Like Gauge

Investigation of the contracted DSE showed that the infrared asymptotics of the gluon propagator must be singular [8]. In doing so, the full propagator in the IR region was assumed to be proportional to the free one, $\Delta_{\mu\nu}(p) = Z(p^2)\Delta^{(0)}_{\mu\nu}(p)$, i.e.

$$\Delta_1(p^2) = \frac{Z(p^2)}{p^2}, \quad \Delta_2(p^2) = 0. \tag{13}$$

Consequently,

$$\pi_1(p^2) = p^2 Z^{-1}(p^2), \qquad \pi_2(p^2) = 0. \tag{14}$$

Since this restriction is not enough for providing $(p^2)^{-2}$ asymptotics in the full DSE [11], it seems natural to include other structures as well. In accordance with expectation, we take the following ansatz in (3)

$$\Delta_i(p^2) = \frac{1}{p^2}(a_i + b_i), \quad i = 1,2; \quad a_i, b_i = const. \tag{15}$$

Introducing these alternations into the corresponding equations, such as the Slavnov-Taylor identity (8) and the Kim-Baker expressions for three-gluon vertices [14], after tedious but straightforward calculations one can prove that it is possible to obey DSE in the infrared region by the following form of the gluon propagator

$$\Delta_{\mu\nu}^{ab}(p) = -i\frac{\delta^{ab}}{p^2}\left\{\left(\frac{a_1}{p^2}+b_1\right)\left(g_{\mu\nu}-\frac{p_\mu n_\nu + p_\nu n_\mu}{(np)}\right)+\left(\frac{a_2}{p^2}+b_2\right)\frac{p^2 n_\mu n_\nu}{(np)^2}\right\}. \qquad (16)$$

In this manner we conclude that in the choosed gauge $n^2 = 0$ the most general behavior of the gluon propagator is given by the Eq. (16). In the nonrelativistic limit, constructed one gluon exchange potential is the linear combination of Coulomb and linearly rising potentials [10]. As compared to [8], now the coefficients a_i, b_i are not fixed.

Important information about properties of interaction between quarks for comparatively large spread of distances is provided by the study of states of heavy quarks: $J/\psi, \Upsilon$ and their excitations. Here due to large quark masses the potential description comes out to be applicable with sufficient accuracy. It comes out at least at the interval (10-14 cm to 10-13 cm) the potential is universal, i.e. does not depend on the quark flavor and its form may be described by the expression:

$$V(r) = -\frac{4}{3}\frac{\alpha_S(r)}{r} + a^2 r, \qquad (17)$$

where $a = 0.42\,GeV$ and $\alpha_S(r)$ is defined from the well-known expression of asymptotic freedom

$$\alpha_S(Q^2) = \frac{4\pi}{\left(11-\frac{2n_f}{3}\right)\log\left(-\frac{Q^2}{\Lambda^2}\right)}; \qquad |Q^2| \gg \Lambda^2 \qquad (18)$$

with the aid of Fourier transformation. One sees that for small r the potential is dominated by the Coulomb term which corresponds to the validity of asymptotic freedom. Beyond $r = 3\cdot 10^{-14}\,\text{cm}$ the main contribution is made by the increasing term, which is by no means, connected with perturbation theory. For the system in which the potential description is possible the confinement condition consists in an unlimited increase of the potential with the distance increasing:

$$V(r) \to \infty, \quad r \to \infty. \qquad (19)$$

In particular, the potential, following from the above found solution obviously satisfies the condition provided. It is applicable for very large distances.

ფიზიკა

გლუონის პროპაგატორი ინფრაწითელ არეში და კვარკებს შორის პოტენციალის პრობლემა

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ნაშრომში წარმოდგენილია თბილისის ჯგუფის მიერ უკანასკნელ დეკადებში მიღებული შედეგები, რომლებიც ეხება შვინგერ-დაისონის განტოლების ამოხსნის შესაძლებლობას ინფრაწითელ არეში. მიზანი მიიღწევა მხოლოდ სინათლისმაგვარ კალიბრებში აქსიალური კლასების ყველა არაკოვარიანტულ კალიბრებს შორის. ამ კალიბრებს აქვს ბევრი უპირატესობა, მათ შორის, ძირითადია სხვადასხვა სტრუქტურული ფუნქციის დამოუკიდებლობა კალიბრების ვექტორისგან. ეს მნიშვნელოვანი თვისება გამომდინარეობს ინტეგრაციის მეთოდიდან, რომელიც უზრუნველყოფს ერთგვაროვნებას კალიბრული ვექტორის მიმართ ინტეგრაციის ყველა ეტაპზე და წარმოადგენს მხოლოდ სინათლისმაგვარი $n^2 = 0$ კალიბრების თავისებურებას. ჩვენი კვლევები აჩვენებს, რომ ამოხსნა ამ კალიბრებში ამჟღავნებს კონფაინმენტის ტიპის ყოფაქცევას გლუონის პროპაგატორისთვის ინფრაწითელ არეში, რომელიც მიღებულია შემოკლებული დაისონ-შვინგერის განტოლებიდან. აღნიშნული ამონახსნით აგებული ერთგლუონიანი გაცვლის ეფექტური პოტენციალი შეესაბამება წრფივად ზრდადი და კულონური პოტენციალების კომბინაციას, რაც ეთანხმება თანამედროვე ექსპერიმენტების მონაცემებს ჰადრონულ სპექტროსკოპიაში.

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Received November, 2022