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Mathematics

On Derived Functors of Semimodule-Valued Functors II

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A concept of a proper projective semimodule is introduced and proper projective resolutions are used to construct derived functors of additive functors from the category of cancellative semimodules to the category of cancellative semimodules. We investigate exactness of the long sequence of derived functors associated to a proper short exact sequence of semimodules. The right derived functors of the functor Hom are described as proper extensions of semimodules. *© 2023 Bull. Georg. Natl. Acad. Sci.*

semimodule, chain complex, projective resolution, derived functors, extension of semimodules

In the paper, we develop a new approach to construction of derived functors for semimodule-valued functors, which possesses some advantages over the old one presented in [1] (see Remarks 7, 16 and 18).

In what follows, Λ is an additively cancellative semiring with $1 \neq 0$, and all (left) Λ- semimodules are cancellative (for the definitions of semiring and semimodule and for the basic facts about them we refer to [2]).

Let B be a Λ- subsemimodule of a Λ- semimodule A. The quotient Λ- semimodule A/B is defined as the quotient Λ- semimodule of A by the smallest Λ- congruence on A some class of which contains B. This means that $cl(a_1) = cl(a_2)$ if and only if $a_1 + b_1 = a_2 + b_2$ for some $b_1, b_2 \in B$.

We denote by $K(\Lambda)$ the ring completion of Λ and by $K(A)$ the $K(\Lambda)$-module completion of a Λ-semimodule A. For any cancellative Λ- semimodule A, one may assume that $A \subseteq K(A)$ and each element c of $K(A)$ is a diference of two elements from A, i.e., $c = a_1 - a_2$, where $a_1, a_2 \in A$.

A Λ- semimodule A is called a Λ- module if $(A, +, 0)$ is an abelian group. One can easily see that A is a Λ- module if and only if A is a $K(\Lambda)$-module $((\lambda_1 - \lambda_2)a = \lambda_1 a - \lambda_2 a)$. Hence, if A is a Λ-module, then $K(A) = A$. For a Λ- semimodule A, by $U(A)$ we denote the maximal Λ- submodule of A, i.e., $U(A) = \{a \in A \mid a + a' = 0 \text{ for some } a' \in A\}$.

A sequence of Λ- semimodules and Λ- homomorphisms

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$$\cdots \longrightarrow L \xrightarrow{\alpha} M \xrightarrow{\beta} N \longrightarrow \cdots$$

is said to be exact at M if $\alpha(L) = Ker(\beta)$ and is called exact if it is exact at all places.

Definition 1 (cf. [3, definition of partially-free monoid on p. 322]). Given a Λ- semimodule F and subsets X and Y of F such that $X \cap Y = \varnothing$ and $X \subseteq U(F)$, we say that F is a *partially-free* Λ- *semimodule of type* (X,Y) if for any Λ- semimodule A and any map $f : X \cup Y \to A$ with $f(X) \subseteq U(A)$ there exists a unique Λ- homomorphism $f' : F \to A$ such that $f'|x \cup y = f$.

A partially-free Λ- semimodule is in fact the direct sum of a free $K(\Lambda)$-module and a free Λ-semimodule. More precisely, let $F(X,Y)$ denote a partially-free Λ- semimodule of type (X,Y). Then $F(X,Y) = F(X) \oplus F(Y)$, where $F(X)$ is a free $K(\Lambda)$-module over X and $F(Y)$ is a free Λ-semimodule over Y. In particular, $F(X,\varnothing)$ is a free $K(\Lambda)$-module on X, and $F(\varnothing,Y)$ is a free Λ-semimodule on Y.

Definition 2 (cf. Definition 2.1 of [1]). We say that a surjective Λ- homomorphism $\tau : B \to C$ is *proper* if there is a surjective Λ- homomorphism $\tau' : B' \to K(C)$ such that $(\tau')^{-1}(C) = B$ and $\tau'(b) = \tau(b)$ for all $b \in B$, and $\tau\big|_{U(B)} : U(B) \to U(C)$ is a surjective homomorphism of $K(\Lambda)$-modules.

Obvious examples of proper surjective Λ- homomorphisms are: 1) Λ- isomorphisms; 2) projections $\pi_C : A \oplus C \to C$; 3) surjective homomorphisms of Λ- modules; 4) the canonical surjective Λ-homomorphism $A \to A/H$, where H is a Λ- submodule of a Λ- semimodule A.

If $\tau : B \to C$ is a proper surjective Λ- homomorphism, then $\tau^{-1}(X) = (\tau')^{-1}(X)$ for any subset X of the Λ- semimodule C. In particular, $Ker(\tau) = Ker(\tau')$.

Definition 3. We say that a Λ- homomorphism $\alpha : B \to C$ is *proper* if and only if the surjection $\bar{\alpha} : B \to \alpha(B)$ $(\bar{\alpha}(a) = \alpha(a))$ is proper.

For example, any injective Λ- homomorphism is proper.

Lemma 4. Given a proper Λ- homomorphism $\alpha : B \to C$. If $\alpha(b_1) = \alpha(b_2)$, then $b_1 + k_1 = b_2 + k_2$, where $k_1, k_2 \in Ker(\alpha)$.

This lemma says that any proper Λ- homomorphism is *k*-regular in the sense of Takahashi [4]. Note that the converse is not true.

Definition 5. Given a sequence $E : A \overset{\kappa}{\rightarrowtail} B \overset{\sigma}{\twoheadrightarrow} C$ of Λ- semimodules and Λ- homomorphisms, we call E a *proper short exact sequence* if κ is an injection, σ is a proper surjection and $\kappa(A) = Ker(\sigma)$.

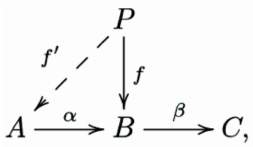
Definition 6. A Λ- semimodule P is *proper projective* if for every proper surjective Λ- homomorphism $\tau : B \to C$ and every Λ- homomorphism $\gamma : P \to C$, there is a Λ- homomorphism $\gamma' : P \to B$ such that $\tau\gamma' = \gamma$.

Remark 7. Every projective $K(\Lambda)$-module is a proper projective Λ- semimodule, but it is not proper projective in the sense of Definition 2.3 of [1].

Proposition 8 (cf. Proposition 2.4 of [1]). *A* Λ- semimodule P is a proper projective Λ- semimodule if and only if there exists an injective Λ- homomorphism $j: P \to F(X,Y)$, where $F(X,Y) = F(X) \oplus F(Y)$ is a partially-free Λ- semimodule of type (X,Y), and there is a $K(\Lambda)$-homomorphism $\eta: K(F(X,Y)) \to K(P)$ such that $\eta K(j) = 1$ and $\eta(F(X)) \subseteq U(P)$.

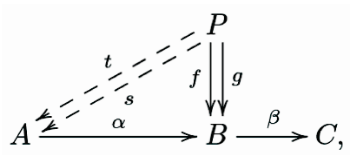
Proposition 9. For any Λ- semimodule C, there exists a proper surjective Λ- homomorphism $\tau: P \to C$ with P a proper projective Λ- semimodule.

Lemma 10. Given a diagram of Λ- semimodules and Λ- homomorphisms



where α is proper, $\alpha(A) = Ker(\beta)$ and P is a proper projective Λ- emimodule. If $\beta f = 0$, then there is a Λ- homomorphism $f': P \to A$ such that $f = \alpha f'$.

Lemma 11. Suppose we are given a diagram of Λ- semimodules and Λ- homomorphisms



where P is a proper projective Λ- semimodule and the following conditions hold:
(1) $\beta\alpha = 0$;
(2) if $\beta(b_1) = \beta(b_2)$, then there are $a_1, a_2 \in A$ such that $\alpha(a_1) + b_1 = \alpha(a_2) + b_2$;
(3) if $\beta(b) = 0$ and $b \in U(B)$, then there is $\alpha \in U(A)$ with $\alpha(a) = b$;
(4) $\beta f = \beta g$.
Then there exist Λ- homomorphisms $s, t: P \to A$ such that $\alpha s + f = \alpha t + g$.

A sequence of Λ- semimodules and Λ- homomorphisms

$$\boldsymbol{X}: \cdots \longrightarrow X_{n+1} \xrightarrow{\partial_{n+1}} X_n \xrightarrow{\partial_n} X_{n-1} \longrightarrow \cdots$$

is a chain complex if $\partial_n \partial_{n+1} = 0$ for each integer *n*. The *n-th homology* Λ- *semimodule* $H_n(\boldsymbol{X})$ of $\boldsymbol{X}$ is defined by $H_n(\boldsymbol{X}) = Ker(\partial_n) / \partial_{n+1}(X_{n+1})$. If $\boldsymbol{X} = \{X_n, \partial_n\}$ and $\boldsymbol{X}' = \{X'_n, \partial'_n\}$ are chain complexes, a *morphism* $f: \boldsymbol{X} \to \boldsymbol{X}'$ is a family $f = \{f_n\}$ of Λ- homomorphisms $f_n: X_n \to X'_n$ such that $f_{n-1}\partial_n = \partial'_n f_n$ for all *n*. The map $H_n(f): H_n(\boldsymbol{X}) \to H_n(\boldsymbol{X}')$ defined by $H_n(f)(cl(x)) = cl(f_n(x))$ is a Λ-homomorphism and thus each H_n is a covariant additive functor from the category of chain complexes and their morphisms to the category of Λ- semimodules.

Morphisms $f, g: \boldsymbol{X} \to \boldsymbol{X}'$ are *chain homotopic* if there exist Λ- homomorphisms $s_n, t_n: X_n \to X'_{n+1}$ such that

$$f_n + s_{n-1}\partial_n + \partial'_{n+1}s_n = g_n + t_{n-1}\partial_n + \partial'_{n+1}t_n$$

for all *n*. The family $\{s_n, t_n\}$ is called a *chain homotopy* from f to g and we write $(s,t): f \simeq g$.

Proposition 12 ([1]). If $(s,t): f \simeq g: \boldsymbol{X} \to \boldsymbol{X}'$, then $H_n(f) = H_n(g): H_n(\boldsymbol{X}) \to H_n(\boldsymbol{X}')$ for all n.

A morphism $f: \boldsymbol{X} \to \boldsymbol{X}'$ is *a chain homotopy equivalence* if there is a morphism $g: \boldsymbol{X}' \to \boldsymbol{X}$ and chain homotopies $(s,t): gf \simeq 1_{\boldsymbol{X}}$ and $(s',t'): fg \simeq 1_{\boldsymbol{X}'}$.

Corollary 13. If $f: \boldsymbol{X} \to \boldsymbol{X}'$ is a chain homotopy equivalence, then $H_n(f): H_n(\boldsymbol{X}) \to H_n(\boldsymbol{X}')$ is an isomorphism of Λ- semimodules for each n.

A *cochain complex* is a sequence of Λ- semimodules and Λ- homomorphisms

$$\boldsymbol{Y}: \cdots \longrightarrow Y^{n-1} \xrightarrow{\delta^{n-1}} Y^n \xrightarrow{\delta^n} Y^{n+1} \longrightarrow \cdots$$

with $\delta^n \delta^{n-1} = 0$ for all n. The *n-th cohomology* Λ-*semimodule* $H^n(\boldsymbol{Y})$ of $\boldsymbol{Y}$ is defined by $H^n(\boldsymbol{Y}) = Ker(\delta^n)/\delta^{n-1}(Y^{n-1})$. One obviously defines a *morphism* $g: \boldsymbol{Y} \to \boldsymbol{Y}'$ of cochain complexes, a Λ- homomorphism $H^n(g): H^n(\boldsymbol{Y}) \to H^n(\boldsymbol{Y}')$, a *cochain homotopy,* and a *cochain homotopy equivalence*. The validity of the statements dual to Proposition 12 and Corollary 13 is also obvious.

Let C be a Λ- semimodule. A *chain complex over* C, or simply a *complex over* C, is a sequence of Λ- semimodules and Λ- homomorphisms

$$(\boldsymbol{X}, \varepsilon): \cdots \longrightarrow X_n \xrightarrow{\partial_n} X_{n-1} \longrightarrow \cdots \longrightarrow X_2 \xrightarrow{\partial_2} X_1 \xrightarrow{\partial_1} X_0 \xrightarrow{\varepsilon} C,$$

where

$$\boldsymbol{X}: \cdots \longrightarrow X_n \xrightarrow{\partial_n} X_{n-1} \longrightarrow \cdots \longrightarrow X_2 \xrightarrow{\partial_2} X_1 \xrightarrow{\partial_1} X_0 \longrightarrow 0$$

is a nonnegative chain complex and $\varepsilon \partial_1 = 0$. If each X_n is a proper projective Λ- semimodule, we say that $(\boldsymbol{X}, \varepsilon)$ is a *proper projective complex over* C. If $(\boldsymbol{X}, \varepsilon)$ is an exact sequence, ε is a proper surjective Λ- homomorphism and each ∂_n is a proper Λ- homomorphism, then we say that $(\boldsymbol{X}, \varepsilon)$ is a *proper resolution of* C. Finally, if $(\boldsymbol{X}, \varepsilon)$ is a proper resolution of C and each X_n is a proper projective Λ- semimodule, we say that $(\boldsymbol{X}, \varepsilon)$ is a *proper projective resolution of* C.

It follows from Proposition 9 that every Λ- semimodule has a proper projective resolution.

Given a Λ- homomorphism $\gamma: C \to C'$, a complex $(\boldsymbol{X}, \varepsilon)$ over C and a complex $(\boldsymbol{X}', \varepsilon')$ over C', we say that a morphism $f = \{f_n\}: \boldsymbol{X} \to \boldsymbol{X}'$ of chain complexes is a *lift of* $\gamma: C \to C'$, if $\gamma\varepsilon = \varepsilon' f_0$.

Using Lemmas 10 and 11, we have the following comparison theorem.

Theorem 14 (cf. Theorem 3.5 of [1]). If $\gamma: C \to C'$ is a Λ- homomorphism, $(\boldsymbol{P}, \varepsilon)$ is a proper projective complex over C and $(\boldsymbol{X}, \varepsilon')$ is a proper resolution of C', then there is a morphism of chain complexes $f = \{f_n\}: \boldsymbol{P} \to \boldsymbol{X}$ lifting $\gamma: C \to C'$. If $g = \{g_n\}: \boldsymbol{P} \to \boldsymbol{X}$ is another lift of γ, then f and g are chain homotopic.

As an immediate consequence of Theorem 14, we get:

Corollary 15. If $(\boldsymbol{P}, \varepsilon)$ and $(\boldsymbol{P}', \varepsilon')$ are proper projective resolutions of a Λ- semimodule C, then there are lifts $f = \{f_n\}: \boldsymbol{P} \to \boldsymbol{P}'$ and $g = \{g_n\}: \boldsymbol{P}' \to \boldsymbol{P}$ of $1_C: C \to C$ such that $gf \simeq 1_{\boldsymbol{P}}$ and $fg \simeq 1_{\boldsymbol{P}'}$.

A contravariant functor T from the category $CSMod_\Lambda$ of cancellative Λ- semimodules to the category $CSMod_{\Lambda'}$ of cancellative Λ'- semimodules is said to be additive if $T(\alpha + \beta) = T(\alpha) + T(\beta)$ for all Λ-

homomorphisms $\alpha, \beta : A \to B$ and all Λ- semimodules A and B. It follows from this definition that an additive contravariant functor $T : CSMod_{\Lambda} \to CSMod_{\Lambda'}$ sends trivial Λ- homomorphisms to trivial Λ'- homomorphisms and takes the trivial Λ- semimodule to the trivial Λ'- semimodule. Moreover, it sends Λ- modules to Λ'- modules.

We say that an additive contravariant functor $T : CSMod_{\Lambda} \to CSMod_{\Lambda'}$ is left exact with respect to proper short exact sequences if, for any proper short exact sequence $E : A \rightarrowtail^{\kappa} B \xrightarrow{\sigma} C,$ the sequence $T(E) : T(C) \xrightarrow{T(\sigma)} T(B) \xrightarrow{T(\kappa)} T(A)$ is exact at $T(B)$ and $T(\sigma)$ is an injective Λ'- homomorphism. If, in addition, $T(\kappa)$ is a surjective Λ'- homomorphism, then we say that T is exact w.r.t. proper short exact sequences.

For any additive contravariant functor T from the category $CSMod_{\Lambda}$ to the category $CSMod_{\Lambda'}$, we define the right derived functors $R^n T(n = 0,1,...)$ of T as follows. For a Λ- semimodule C, choose a proper projective resolution of C,

$$(\boldsymbol{P}, \varepsilon) : \cdots \longrightarrow P_n \xrightarrow{\partial_n} P_{n-1} \longrightarrow \cdots \longrightarrow P_2 \xrightarrow{\partial_2} P_1 \xrightarrow{\partial_1} P_0 \xrightarrow{\varepsilon} C.$$

Applying the functor T to $\boldsymbol{P}$ gives the cochain complex of Λ'- semimodules

$$T(\boldsymbol{P}) : 0 \longrightarrow T(P_0) \xrightarrow{T(\partial_1)} T(P_1) \xrightarrow{T(\partial_2)} T(P_2) \longrightarrow \cdots \longrightarrow T(P_{n-1}) \xrightarrow{T(\partial_n)} T(P_n) \longrightarrow \cdots,$$

and we define

$$(R^n T)(C) = H^n(T(\boldsymbol{P})), \quad n \geq 0.$$

Further, if $\gamma : C \to C'$ is a homomorphism of Λ- semimodules and $(\boldsymbol{P}', \varepsilon')$ is a proper projective resolution of C', then, by Theorem 14, there is a lift $f = \{f_n\} : \boldsymbol{P} \to \boldsymbol{P}'$ of γ. Applying the functor T to f gives the morphism of cochain complexes $T(f) = \{T(f_n)\} : T(\boldsymbol{P}') \to T(\boldsymbol{P})$, and we define

$$(R^n T)(\gamma) = H^n(T(f)) : (R^n T)(C') \to (R^n T)(C), \quad n \geq 0.$$

The additive contravariant functor T converts chain homotopies to cochain homotopies. Therefore, thanks to Theorem 14, Corollary 15 and the statements dual to Proposition 12 and Corollary 13, we can conclude that $(R^n T)(C)$ and $(R^n T)(\gamma)$ do not depend (up to an isomorphism) on the choice of proper projective resolutions of C and C' or on the choice of a lift of γ. Thus, for each $n \geq 0$, we have an additive contravariant functor $R^n T : CSMod_{\Lambda} \to CSMod_{\Lambda'}$ (well-defined up to a natural isomorphism), the *n-th right derived functor of* T.

Note that if F is an additive covariant functor from $CSMod_{\Lambda}$ to $CSMod_{\Lambda'}$, then one can define the left derived functors $L_n F(n = 0,1,...)$ of F by taking the homology of $F(\boldsymbol{P})$.

Let us mention some properties of the right derived functors of an additive contravariant functor $T : CSMod_{\Lambda} \to CSMod_{\Lambda'}$:

(1) For a proper projective Λ- semimodule P, $(R^n T)(P) = 0$ for $n \geq 1$ and $(R^0 T)(P) = T(P)$.

(2) If T is left exact with respect to proper short exact sequences, then $R^0 T$ and T are isomorphic functors.

(3) If T is exact with respect to proper short exact sequences, then $(R^n T)(C) = 0$ for every Λ- semimodule C and all $n \geq 1$.

(4) If $T' : Mod_{\Lambda} \to Mod_{\Lambda'}$ is the restriction of T to the category of Λ- modules and $R^n T'$ are the usual derived functors of T', then $(R^n T)(C) = (R^n T')(C)$ for every Λ- module C and all $n \geq 1$.

Remark 16. Note that the last property, unlike Properties (1)-(3), does not hold for the derived functors introduced in [1]. More precisely, if $\mathfrak{R}^n T$ is the *n-th* right derived functor of T in the sense of [1] and C is a Λ- module, then $(\mathfrak{R}^n T)(C)$ need not coincide with $(R^n T')(C)$.

Given an additive contravariant functor $T : CSMod_{\Lambda} \to CSMod_{\Lambda'}$, let C be a Λ- semimodule and

$$\cdots \longrightarrow P_1 \xrightarrow{\partial_1} P_0 \xrightarrow{\varepsilon} C$$

be a proper projective resolution of C. By the universal property of kernels, we have a commutative diagram

$$\begin{array}{ccccc} & & T(C) & & \\ & \swarrow^{\rho(C)} & \downarrow{\scriptstyle T(\varepsilon)} & & \\ (R^0T)(C) & \xrightarrow{j} & T(P_0) & \xrightarrow{T(\partial_1)} & T(P_1), \end{array}$$

where $(R^0 T)(C) = Ker(T(\partial_1))$ and j is an inclusion. It is straightforward to see that $\rho : T \to R^0 T$ is a natural transformation. Then, following [5], we define the *proper projective stabilization* $\overline{T}$ of T, to be the kernel of ρ. We say that T is *proper projectively stable* if the inclusion $\overline{T} \to T$ is an isomorphism. It is easy to see that T is proper projectively stable if and only if $R^0 T = 0$.

A horseshoe lemma for proper projective resolutions and examining exactness of long sequences of cohomology semimodules associated to proper short exact sequences of cochain complexes lead to the following theorem.

Theorem 17. Suppose that T is an additive contravariant functor from the category $CSMod_{\Lambda}$ of cancellative Λ- semimodules to the category $CSMod_{\Lambda'}$ of cancellative Λ'- semimodules, and let $E : G \rightarrowtail^{\kappa} B \xrightarrow{\sigma} C$ be a proper short exact sequence of Λ- semimodules with G a Λ- module. Then there are connecting Λ'-homomorphisms $\beta^n(E) : R^n T(G) \to R^{n+1} T(C)$, natural with respect to E, giving rise to the sequence

$$0 \longrightarrow R^0T(C) \xrightarrow{R^0T(\sigma)} R^0T(B) \xrightarrow{R^0T(\kappa)} R^0T(G) \xrightarrow{\beta^0(E)} R^1T(C) \xrightarrow{R^1T(\sigma)} R^1T(B) \longrightarrow \cdots$$
$$\cdots \longrightarrow R^{n-1}T(G) \xrightarrow{\beta^{n-1}(E)} R^nT(C) \xrightarrow{R^nT(\sigma)} R^nT(B) \xrightarrow{R^nT(\kappa)} R^nT(G) \xrightarrow{\beta^n(E)} R^{n+1}T(C) \longrightarrow \cdots$$

with the following properties:

(1) It is exact at $R^n T(C)$ and $R^n T(\sigma)$ is *k*-regular.

(2) It is exact at $R^n T(B)$.

(3) $R^n T(\kappa)(R^n T(B)) \subseteq Ker(\beta^n(E))$.

Remark 18. Note that there is no analog of this theorem for the derived functors from [1].

Given an exact sequence

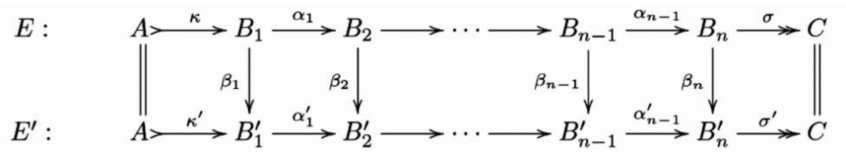
$$E: \quad A \overset{\kappa}{\rightarrowtail} B_1 \xrightarrow{\alpha_1} B_2 \longrightarrow \cdots \longrightarrow B_{n-1} \xrightarrow{\alpha_{n-1}} B_n \overset{\sigma}{\twoheadrightarrow} C$$

of Λ- semimodules and Λ- homomorphisms, we call E an *n-fold proper extension* of A by C if κ is an injection, σ is a proper surjection and $\alpha_1, ..., \alpha_{n-1}$ are proper homomorphisms.

We say that E is similar to an extension

$$E': \quad A \overset{\kappa'}{\rightarrowtail} B'_1 \xrightarrow{\alpha'_1} B'_2 \longrightarrow \cdots \longrightarrow B'_{n-1} \xrightarrow{\alpha'_{n-1}} B'_n \overset{\sigma'}{\twoheadrightarrow} C$$

if there exist Λ- homomorphisms $\beta_i : B_i \to B'_i \, (i = 1, ..., n)$ such that the diagram



commutes. Further we say that extensions E and E' are equivalent if there exist extensions $E_1, E_2, ...E_r$ such that $E_1 = E$, $E_r = E'$ and either E_i is similar to E_{i+1} or E_{i+1} is similar to E_i, $i = 1, 2, ..., r-1$.

Let A and C be Λ- semimodules and let $Ext_{\Lambda}^{n}(C, A)$ denote the set of all equivalence classes of *n*-fold proper extensions of A by C. As in [1], we can define the so-called *Baer sum* of extensions, which makes $Ext_{\Lambda}^{n}(C, A)$ an abelian monoid. Moreover, each $Ext_{\Lambda}^{n}(C, A)$ is a bifunctor from the category of cancellative Λ- semimodules to the category of cancellative abelian monoids, contravariant in C and covariant in A, and additive in both arguments. In addition, we let $Ext_{\Lambda}^{0}(C, A)$ denote $Hom_{\Lambda}(C, A)$.

Theorem 19 (cf. Theorem 4.14 of [1]). For every Λ- semimodule A, there exists an isomorphism of functors

$$R^n Hom_{\Lambda}(-, A) \cong Ext_{\Lambda}^{n}(-, A)$$

for each $n \geq 0$.

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მათემატიკა

წარმოებული ფუნქტორები ფუნქტორებისთვის მნიშვნელობებით ნახევრადმოდულებში II

ა. პაჭკორია

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შემოტანილია საკუთრივი პროექციული ნახევრადმოდულის ცნება და საკუთრივი პროექციული რეზოლვენტების გამოყენებით აგებულია წარმოებული ფუნქტორები ადიციური ფუნქტორებისთვის შეკვეცადი ნახევრადმოდულების კატეგორიიდან შეკვეცადი ნახევრადმოდულების კატეგორიაში. გამოკვლეულია ნახევრადმოდულების საკუთრივ მოკლე ზუსტ მიმდევრობასთან ასოცირებული წარმოებული ფუნქტორების გრძელი მიმდევრობის სიზუსტის საკითხი. ნახევრადმოდულების საკუთრივი გაფართოებების საშუალებით აღწერილია Hom ფუნქტორის მარჯვენა წარმოებული ფუნქტორები.

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