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Mathematics

On Investigation of Nonclassical Three-Dimensional Model for Thermoelastic Solids with Three Phase-Lags

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In the present paper, a nonclassical dynamic three-dimensional model for thermoelastic solids, which depend on three phase-lag parameters, is considered. The initial-boundary value problem with mixed boundary conditions corresponding to the nonclassical model is studied, where on certain parts of the boundary displacement and temperature vanish and on the corresponding remaining parts of the boundary the densities of surface force and heat flux along the outward normal vector are given. Variational formulation of the initial-boundary value problem is obtained and the existence and uniqueness result and the continuous dependence of the solution on the given data in suitable spaces of vector-valued distributions is proved. *© 2023 Bull. Georg. Natl. Acad. Sci.*

nonclassical thermoelasticity, initial-boundary value problem, existence and uniqueness of solution

Several studies indicate that various modern technological processes, such as heating and drilling with ultrafast laser pulses [1-3], cannot be successfully described by the classical theory of thermoelasticity, which is based on Fourier's law of heat conduction and predicts the infinite speed of propagation of thermal disturbance. It was also experimentally observed that at low temperatures heat propagates as a thermal wave [4-6]. In order to eliminate the unrealistic behaviour of the classical model for thermoelastic solids several generalized theories of thermoelasticity have been developed. One of the first nonclassical models for thermoelastic solids was proposed by Lord and Shulman [7], where instead of the classical Fourier law of heat conduction the Maxwell-Cattaneo-Vernotte law was used, which is a generalization of Fourier's law and depends on one relaxation time parameter. The second well-known nonclassical theory of thermoelasticity, which eliminates the infinite speed of propagation of heat waves, was developed using a qualitatively different approach by Green and Lindsay [8], where the constitutive relations for the stress tensor and the entropy are generalized by introducing two different relaxation times. Later on, by using the dual-phase-lag heat conduction model proposed by Tzou [9] instead of Fourier's law Chandrasekharaiah

[10] constructed a nonclassical model for thermoelastic solids, which is an extension of the Lord-Shulman nonclassical model for thermoelastic bodies. By applying the method of potential and theory of integral equations the problems of stable and pseudo oscillations for the Lord-Shulman and Green-Lindsay nonclassical models were studied in [11]. The domain of influence result was obtained for the Lord-Shulman model in [12] and for the Green-Lindsay model in [13] in classical spaces of twice continuously differentiable functions. The initial-boundary value problems with mixed boundary conditions corresponding to the Lord-Shulman, Green–Lindsay and Chandrasekharaiah-Tzou linear dynamic three-dimensional models for thermoelastic solids were investigated in Sobolev spaces in [14-16].

Further generalization of the dual-phase-lag heat conduction model by Tzou was proposed by Roy Choudhuri [17], who used the Green-Naghdy model III of heat flow [18], which includes not only temperature gradient but also the thermal displacement gradient, and introduced a nonclassical model for thermoelastic solids, where the Fourier law of heat conduction was replaced by an approximation to a modification of the Fourier law with three phase-lags, or delay times, for the heat flux vector, the temperature gradient and the thermal displacement gradient. For the Roy Choudhuri nonclassical model with three phase-lags problems of propagation of waves, thermodynamic compatibility, uniqueness and continuous dependence problems, methods of numerical solution of corresponding problems and related topics are considered by many researchers (see [19-22] and the references given therein).

The present paper is devoted to an investigation of the nonclassical model for thermoelastic solids with three phase-lags proposed by Roy Choudhuri [17] in Sobolev spaces by applying a variational approach. We consider the linear three-dimensional initial-boundary value problem corresponding to the Roy Choudhuri model with general mixed boundary conditions, where, on certain parts of the boundary, the densities of surface force and heat flux along the outward normal vector are given, and, on the corresponding remaining parts, the displacement and temperature vanish. We obtain integral relations, which are equivalent to the original differential equations in the spaces of smooth enough functions, and on the basis of the integral relations we consider the variational formulation in suitable spaces of vector-valued distributions with respect to the time variable with values in Sobolev spaces of the initial-boundary value problem corresponding to the nonclassical dynamic three-dimensional model of thermoelastic solid. We investigate the existence, uniqueness and continuous dependence of the solution on the given data in suitable function spaces.

We denote by $W^{r,2}(D) = H^r(D)$ and $H^{\hat{r}}(\hat{\Gamma})$, $r, \hat{r} \in \mathbf{R}$, $r \geq 0$, $0 \leq \hat{r} \leq 1$, the Sobolev spaces of orders $r, \hat{r}$ based on the spaces $H^0(D) = L^2(D)$ and $H^0(\hat{\Gamma}) = L^2(\hat{\Gamma})$ of square-integrable functions, respectively, where $D \subset \mathbf{R}^p$, $p \in \mathbf{N}$, is a bounded Lipschitz domain [23] and $\hat{\Gamma} \subset \partial D$ is a Lipschitz surface. We denote by $\mathbf{H}^r(D) = [H^r(D)]^3$, $\mathbf{H}^{\hat{r}}(\hat{\Gamma}) = [H^{\hat{r}}(\hat{\Gamma})]^3$, $\mathbf{L}^2(D) = [L^2(D)]^3$, $\mathbf{L}^s(\hat{\Gamma}) = [L^s(\hat{\Gamma})]^3$, $r \geq 0$, $0 \leq \hat{r} \leq 1$, $s \geq 1$, $r, \hat{r}, s \in \mathbf{R}$, the corresponding spaces of vector-valued functions. The trace operators are denoted by $tr_{\hat{\Gamma}} : H^1(D) \to H^{1/2}(\hat{\Gamma})$ and $\mathbf{tr}_{\hat{\Gamma}} : \mathbf{H}^1(D) \to \mathbf{H}^{1/2}(\hat{\Gamma})$. For a Banach space X, we denote by $C([0,T]; X)$ the space of continuous functions on $[0,T]$ with values in X. $L^q(0,T;X)$, $1 \leq q \leq \infty$, is the space of such measurable functions $g : (0,T) \to X$ that $\|g(t)\|_X \in L^q(0,T)$ and the generalized first, second, third and arbitrary $k \in \mathbf{N}$ -th order derivatives of g are denoted by $g' = dg/dt$, $g'' = d^2g/dt^2$, $g''' = d^3g/dt^3$ and $g^{(k)} = d^k g/dt^k$ [24].

Let us consider thermoelastic body with initial configuration $\overline{\Omega} \subset \mathbf{R}^3$, which consists of anisotropic homogeneous thermoelastic material. The body is clamped along a part Γ_0 of the Lipschitz boundary $\Gamma = \partial\Omega$ and, on the remaining part $\Gamma_1 = \Gamma \setminus \Gamma_0$, an applied surface force vector, with density $\mathbf{g} = (g_i): \Gamma_1 \times (0,T) \to \mathbf{R}^3$ is given, where $\Gamma = \Gamma_0 \cup \Gamma_1$ is a Lipschitz dissection [23] of Γ; the temperature θ vanishes along a part $\Gamma_0^\theta \subset \Gamma$ of the boundary and the heat flux along the outward normal vector of Γ, with density $\tilde{g}^\theta : \Gamma_1^\theta \times (0,T) \to \mathbf{R}$, is given on the remaining part $\Gamma_1^\theta = \Gamma \setminus \Gamma_0^\theta$ of the boundary, where $\Gamma = \Gamma_0^\theta \cup \Gamma_1^\theta$ is a Lipschitz dissection of Γ. The body is subjected to applied body force with density $\mathbf{f} = (f_i): \Omega \times (0,T) \to \mathbf{R}^3$ and heat source with density $\tilde{f}^\theta : \Omega \times (0,T) \to \mathbf{R}$. The nonclassical dynamic linear three-dimensional model of stress-strain state of the thermoelastic body $\overline{\Omega}$ obtained by Roy Choudhuri [17], which includes three different phase-lags, is given by the following initial-boundary value problem in differential form:

$$\rho \frac{\partial^2 u_i}{\partial t^2} - \sum_{j=1}^{3} \frac{\partial}{\partial x_j} \left(\sum_{p,q=1}^{3} \mu_{ijpq} e_{pq}(\mathbf{u}) + \eta_{ij}\theta \right) = f_i \qquad \text{in } \Omega \times (0,T), \quad i = 1,2,3, \tag{1}$$

$$\chi \left(\frac{\partial^2 \theta}{\partial t^2} + \tau_0 \frac{\partial^3 \theta}{\partial t^3} + \frac{\tau_0^2}{2} \frac{\partial^4 \theta}{\partial t^4} \right) - \sum_{p,q=1}^{3} \frac{\partial}{\partial x_p} \left(\lambda_{pq} \frac{\partial}{\partial x_q} \left(\frac{\partial \theta}{\partial t} + \tau_1 \frac{\partial^2 \theta}{\partial t^2} \right) \right) - \sum_{p,q=1}^{3} \frac{\partial}{\partial x_p} \left(\overline{\lambda}_{pq} \frac{\partial}{\partial x_q} \left(\theta + \tau_2 \frac{\partial \theta}{\partial t} \right) \right) -$$

$$-\Theta_0 \sum_{p,q=1}^{3} \eta_{pq} e_{pq} \left(\frac{\partial^2 \mathbf{u}}{\partial t^2} + \tau_0 \frac{\partial^3 \mathbf{u}}{\partial t^3} + \frac{\tau_0^2}{2} \frac{\partial^4 \mathbf{u}}{\partial t^4} \right) = \frac{\partial \tilde{f}^\theta}{\partial t} + \tau_0 \frac{\partial^2 \tilde{f}^\theta}{\partial t^2} + \frac{\tau_0^2}{2} \frac{\partial^3 \tilde{f}^\theta}{\partial t^3} \qquad \text{in } \Omega \times (0,T), \tag{2}$$

$$\mathbf{u} = \mathbf{0} \quad \text{on } \Gamma_0 \times (0,T), \quad \sum_{j=1}^{3} \left(\sum_{p,q=1}^{3} \mu_{ijpq} e_{pq}(\mathbf{u}) + \eta_{ij}\theta \right) n_j = g_i \quad \text{on } \Gamma_1 \times (0,T),\ i = 1,2,3, \tag{3}$$

$$\theta = 0 \quad \text{on } \Gamma_0^\theta \times (0,T), \ -\sum_{p,q=1}^{3} \lambda_{pq} \frac{\partial}{\partial x_q} \left(\frac{\partial \theta}{\partial t} + \tau_1 \frac{\partial^2 \theta}{\partial t^2} \right) n_p - \sum_{p,q=1}^{3} \overline{\lambda}_{pq} \frac{\partial}{\partial x_q} \left(\theta + \tau_2 \frac{\partial \theta}{\partial t} \right) n_p = \frac{\partial \tilde{g}^\theta}{\partial t} \text{ on } \Gamma_1^\theta \times (0,T), \tag{4}$$

$$\left.\begin{array}{l} \mathbf{u}(x,0) = \mathbf{u}_0(x), \quad \dfrac{\partial \mathbf{u}}{\partial t}(x,0) = \mathbf{u}_1(x), \\ \theta(x,0) = \theta_0(x), \quad \dfrac{\partial \theta}{\partial t}(x,0) = \theta_1(x), \quad \dfrac{\partial^2 \theta}{\partial t^2}(x,0) = \theta_2(x), \quad \dfrac{\partial^3 \theta}{\partial t^3}(x,0) = \theta_3(x), \end{array}\right. \quad x \in \Omega, \tag{5}$$

where $e_{ij}(\mathbf{v}) = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right)$, $i,j = 1,2,3$, $\mathbf{v} = (v_i)_{i=1}^3 : \Omega \to \mathbf{R}^3$, is the linearized strain tensor, $\mathbf{n} = (n_i)_{i=1}^3$ is the unit outward normal to Γ, $\mathbf{u} = (u_i)_{i=1}^3 : \Omega \times (0,T) \to \mathbf{R}^3$ is the displacement vector-function of the thermoelastic body, $\theta : \Omega \times (0,T) \to \mathbf{R}$ is the temperature distribution, $\mathbf{u}_0 = (u_{0i})_{i=1}^3$ and $\mathbf{u}_1 = (u_{1i})_{i=1}^3$ are the initial displacement and velocity vector-functions, and θ_0 is the initial distribution of temperature, θ_1, θ_2 and θ_3 are the rate of change, the second and the third order derivatives of temperature with respect to the time variable at the initial moment of time, ρ is the mass density in the reference configuration, $(\mu_{ijpq})_{i,j,p,q=1}^3$ is the elasticity tensor, $(\eta_{pq})_{p,q=1}^3$ is the stress-temperature tensor, $\chi > 0$ is the volumetric heat capacity, $(\lambda_{pq})_{p,q=1}^3$ is the thermal conductivity tensor and $(\overline{\lambda}_{pq})_{p,q=1}^3$ is the Green-Naghdi thermal conductivity rate tensor included in the Green-Naghdi model III [18] to consider the dependence on thermal displacement gradient, $\Theta_0 > 0$ is the temperature of the thermoelastic body in natural state of no deformation, which is considered as a reference temperature, $\tau_0 \geq 0$, $\tau_1 \geq 0$ and $\tau_2 \geq 0$ stand for the heat flux, temperature gradient and thermal displacement gradient phase-lags, respectively. Note that in the case of $\tau_0 = \tau_1 = \tau_2 = 0$ the Roy Choudhuri model reduces to the Green-Naghdi model III; for $\overline{\lambda}_{pq} = 0$, $p,q = 1,2,3$, the considered nonclassical model with three phase-lags reduces to the Chandrasekharaiah-Tzou dual-phase-lag model [10, 16]; in the case of $\tau_1 = 0$, $\overline{\lambda}_{pq} = 0$, $p,q = 1,2,3$, if τ_0 is very small and we neglect the term with τ_0^2, then the Roy Choudhuri model reduces to the Lord-Shulman nonclassical model [7, 14]; in the case of $\tau_0 = \tau_1 = 0$ and $\overline{\lambda}_{pq} = 0$, $p,q = 1,2,3$, the nonclassical three-phase-lag model reduces to the classical linear three-dimensional model for thermoelastic solids.

We assume that the elasticity tensor $(\mu_{ijpq})_{i,j,p,q=1}^3$, the stress-temperature tensor $(\eta_{pq})_{p,q=1}^3$, the thermal conductivity tensor $(\lambda_{pq})_{p,q=1}^3$ and the thermal conductivity rate tensor $(\overline{\lambda}_{pq})_{p,q=1}^3$ satisfy the following symmetry conditions:

$$\mu_{ijpq} = \mu_{ijqp} = \mu_{jipq}, \quad \eta_{ij} = \eta_{ji}, \quad \lambda_{pq} = \lambda_{qp}, \quad \overline{\lambda}_{pq} = \overline{\lambda}_{qp}, \qquad i,j,p,q = 1,2,3. \tag{6}$$

If $\mathbf{u} = (u_i)_{i=1}^3$ and θ are smooth enough, then by multiplying equations (1) by arbitrary continuously differentiable functions $v_i : \overline{\Omega} \to \mathbf{R}$ $(i = 1,2,3)$, which vanish on Γ_0, and equation (2) by a continuously differentiable function $\varphi : \overline{\Omega} \to \mathbf{R}$, such that $\varphi = 0$ on Γ_0^θ, by integrating on Ω, by using the Green identity [23], and by taking into account the symmetry properties (6) and the boundary conditions (3), (4), we obtain the following integral relations:

$$\sum_{i=1}^3 \int_\Omega \rho \frac{\partial^2 u_i}{\partial t^2} v_i dx + \sum_{i,j,p,q=1}^3 \int_\Omega \mu_{ijpq} e_{pq}(\mathbf{u}) e_{ij}(\mathbf{v}) dx + \sum_{i,j=1}^3 \int_\Omega \eta_{ij} \theta e_{ij}(\mathbf{v}) dx = \sum_{i=1}^3 \int_\Omega f_i v_i dx + \sum_{i=1}^3 \int_{\Gamma_1} g_i v_i d\Gamma, \tag{7}$$

$$\int_\Omega \chi \left(\frac{\partial^2 \theta}{\partial t^2} + \tau_0 \frac{\partial^3 \theta}{\partial t^3} + \frac{\tau_0^2}{2} \frac{\partial^4 \theta}{\partial t^4} \right) \varphi dx + \sum_{p,q=1}^3 \int_\Omega \lambda_{pq} \frac{\partial}{\partial x_q} \left(\frac{\partial \theta}{\partial t} + \tau_1 \frac{\partial^2 \theta}{\partial t^2} \right) \frac{\partial \varphi}{\partial x_p} dx + \sum_{p,q=1}^3 \int_\Omega \overline{\lambda}_{pq} \frac{\partial}{\partial x_q} \Big(\theta +$$

$$+ \tau_2 \frac{\partial \theta}{\partial t} \Big) \frac{\partial \varphi}{\partial x_p} dx - \Theta_0 \sum_{p,q=1}^3 \int_\Omega \eta_{pq} e_{pq} \left(\frac{\partial^2 \mathbf{u}}{\partial t^2} + \tau_0 \frac{\partial^3 \mathbf{u}}{\partial t^3} + \frac{\tau_0^2}{2} \frac{\partial^4 \mathbf{u}}{\partial t^4} \right) \varphi dx = \int_\Omega f^\theta \varphi dx - \int_{\Gamma_1^\theta} g^\theta \varphi d\Gamma, \tag{8}$$

where $f^\theta = \dfrac{\partial \tilde{f}^\theta}{\partial t} + \tau_0 \dfrac{\partial^2 \tilde{f}^\theta}{\partial t^2} + \dfrac{\tau_0^2}{2} \dfrac{\partial^3 \tilde{f}^\theta}{\partial t^3}$, $g^\theta = \dfrac{\partial \tilde{g}^\theta}{\partial t}$. Therefore, if $\mathbf{u} = (u_i)_{i=1}^3$ and θ are solutions of equations (1), (2) and satisfy boundary conditions (3), (4), then $\mathbf{u} = (u_i)_{i=1}^3$ and θ are solutions of (7), (8). Conversely, if $\mathbf{u} = (u_i)_{i=1}^3$ and θ are smooth enough solution of equations (7), (8), then by using the Green identity and density arguments we obtain that $\mathbf{u} = (u_i)_{i=1}^3$ and θ satisfy equations (1), (2) and boundary conditions (3), (4). So, the initial-boundary value problem (1)-(5) corresponding to the nonclassical dynamic three-dimensional model of thermoelastic solid with three phase-lags is equivalent to equations (7), (8), together with initial conditions (5) in the spaces of smooth enough functions.

We identify the unknown vector-function $\mathbf{u}$ and the function θ with vector-functions defined on $[0,T]$ with values in suitable spaces of functions defined on Ω, and by applying the integral relations (7), (8) we obtain the following variational formulation of the initial-boundary value problem (1)-(5) in the spaces of vector-valued distributions: Find the unknown vector-function $\mathbf{u},\mathbf{u}',\mathbf{u}'',\mathbf{u}'''\in C([0,T];\mathbf{V}(\Omega))$, $\mathbf{u}^{(4)}\in L^2(0,T;\mathbf{V}(\Omega))$, $\theta,\theta',\theta''\in C([0,T];V^\theta(\Omega))$, $\theta'''\in L^2(0,T;V^\theta(\Omega))$, $\theta^{(4)}\in L^2(0,T;L^2(\Omega))$, which satisfy the following equations in the sense of distributions on $(0,T)$,

$$\rho(\mathbf{u}'',\mathbf{v})_{\mathbf{L}^2(\Omega)}+a(\mathbf{u},\mathbf{v})+b(\theta,\mathbf{v})=(\mathbf{f},\mathbf{v})_{\mathbf{L}^2(\Omega)}+(\mathbf{g},\mathbf{tr}_{\Gamma_1}(\mathbf{v}))_{\mathbf{L}^2(\Gamma_1)},\qquad \forall\mathbf{v}\in\mathbf{V}(\Omega), \tag{9}$$

$$\chi\left(\theta''+\tau_0\theta'''+\frac{\tau_0^2}{2}\theta^{(4)},\varphi\right)_{L^2(\Omega)}+a_1^\theta(\theta'+\tau_1\theta'',\varphi)+a_2^\theta(\theta+\tau_2\theta',\varphi)-$$

$$-\Theta_0 b^\theta\left(\mathbf{u}''+\tau_0\mathbf{u}'''+\frac{\tau_0^2}{2}\mathbf{u}^{(4)},\varphi\right)=(f^\theta,\varphi)_{L^2(\Omega)}-(g^\theta,tr_{\Gamma_1^\theta}(\varphi))_{L^2(\Gamma_1^\theta)},\qquad \forall\varphi\in V^\theta(\Omega), \tag{10}$$

together with the initial conditions

$$\mathbf{u}(0)=\mathbf{u}_0,\ \ \mathbf{u}'(0)=\mathbf{u}_1,\ \ \theta(0)=\theta_0,\ \ \theta'(0)=\theta_1,\ \ \theta''(0)=\theta_2,\ \ \theta'''(0)=\theta_3, \tag{11}$$

where $\mathbf{V}(\Omega)=\{\mathbf{v}\in\mathbf{H}^1(\Omega);\mathbf{tr}_\Gamma(\mathbf{v})=\mathbf{0}\ \text{ on }\Gamma_0\}$, $V^\theta(\Omega)=\{\varphi\in H^1(\Omega);tr_\Gamma(\varphi)=0\ \text{ on }\Gamma_0^\theta\}$,

$$a(\tilde{\mathbf{v}},\mathbf{v})=\int_\Omega\sum_{i,j,p,q=1}^3\mu_{ijpq}e_{pq}(\tilde{\mathbf{v}})e_{ij}(\mathbf{v})dx,\qquad \forall\mathbf{v},\tilde{\mathbf{v}}\in\mathbf{H}^1(\Omega),$$

$$a_1^\theta(\tilde\varphi,\varphi)=\int_\Omega\sum_{p,q=1}^3\lambda_{pq}\frac{\partial\tilde\varphi}{\partial x_q}\frac{\partial\varphi}{\partial x_p}dx,\quad a_2^\theta(\tilde\varphi,\varphi)=\int_\Omega\sum_{p,q=1}^3\overline{\lambda}_{pq}\frac{\partial\tilde\varphi}{\partial x_q}\frac{\partial\varphi}{\partial x_p}dx,\qquad \forall\varphi,\tilde\varphi\in H^1(\Omega),$$

$$b(\varphi,\mathbf{v})=b^\theta(\mathbf{v},\varphi)=\int_\Omega\varphi\sum_{p,q=1}^3\eta_{pq}e_{pq}(\mathbf{v})dx,\qquad \forall\varphi\in L^2(\Omega),\mathbf{v}\in\mathbf{H}^1(\Omega),$$

$(.,.)_{\mathbf{L}^2(\Omega)}$, $(.,.)_{L^2(\Omega)}$, $(.,.)_{\mathbf{L}^2(\Gamma_1)}$ and $(.,.)_{L^2(\Gamma_1^\theta)}$ are scalar products in the spaces $\mathbf{L}^2(\Omega)$, $L^2(\Omega)$, $\mathbf{L}^2(\Gamma_1)$ and $L^2(\Gamma_1^\theta)$, respectively.

For problem (9)-(11), which is equivalent to the initial-boundary value problem (1)-(5) in the spaces of smooth enough functions, the following theorem is valid.

Theorem. Suppose that Ω is a bounded Lipschitz domain and the parameters characterizing elastic and thermal properties of the thermoelastic body satisfy the symmetry conditions (6), the positive definiteness conditions

$$\sum_{i,j,p,q=1}^3\mu_{ijpq}\varepsilon_{pq}\varepsilon_{ij}\ge c_\mu\sum_{i,j=1}^3(\varepsilon_{ij})^2,\ \ \forall\varepsilon_{ij}\in\mathbf{R},\varepsilon_{ij}=\varepsilon_{ji},\ \ i,j=1,2,3,\ \ c_\mu>0,$$

$$\sum_{p,q=1}^3\lambda_{pq}\varepsilon_p\varepsilon_q\ge c_\lambda\sum_{p=1}^3(\varepsilon_p)^2,\ \ \sum_{p,q=1}^3\overline{\lambda}_{pq}\varepsilon_p\varepsilon_q\ge c_{\overline{\lambda}}\sum_{p=1}^3(\varepsilon_p)^2,\ \forall\varepsilon_p\in\mathbf{R},\ p=1,2,3,\ \ c_\lambda>0,c_{\overline{\lambda}}>0$$

and $\tau_0>0$, $\tau_1>0$, $\tau_2>0$. If $\mathbf{f}\in C([0,T];\mathbf{H}^3(\Omega))$, $\mathbf{f}'\in C([0,T];\mathbf{H}^2(\Omega))$, $\mathbf{f}''\in C([0,T];\mathbf{H}^1(\Omega))$, $\mathbf{f}''',\mathbf{f}^{(4)}\in L^2(0,T;\mathbf{L}^2(\Omega))$, $\mathbf{g},\mathbf{g}',\mathbf{g}'',\mathbf{g}''',\mathbf{g}^{(4)},\mathbf{g}^{(5)}\in L^2(0,T;\mathbf{L}^{4/3}(\Gamma_1))$, $f^\theta,f^{\theta'}\in L^2(0,T;L^2(\Omega))$, $g^\theta,g^{\theta'}$,

$g^{\theta''} \in L^2(0,T;L^{4/3}(\Gamma_1^\theta))$, and the initial conditions $\mathbf{u}_0 \in \mathbf{H}^5(\Omega) \cap \mathbf{V}(\Omega)$, $\mathbf{u}_1 \in \mathbf{H}^4(\Omega) \cap \mathbf{V}(\Omega)$, $\theta_0 \in H^4(\Omega) \cap V^\theta(\Omega)$, $\theta_1 \in H^3(\Omega) \cap V^\theta(\Omega)$, $\theta_2 \in H^2(\Omega) \cap V^\theta(\Omega)$, $\theta_3 \in V^\theta(\Omega)$, satisfy the following compatibility conditions

$$\left.\begin{array}{l} g_i(0) = tr_{\Gamma_1}\left(\sum\limits_{j=1}^{3}\left(\sum\limits_{p,q=1}^{3} \mu_{ijpq} e_{pq}(\mathbf{u}_0) + \eta_{ij}\theta_0\right) n_j\right), \; g_i'(0) = tr_{\Gamma_1}\left(\sum\limits_{j=1}^{3}\left(\sum\limits_{p,q=1}^{3} \mu_{ijpq} e_{pq}(\mathbf{u}_1) + \eta_{ij}\theta_1\right) n_j\right), \\ g_i''(0) = tr_{\Gamma_1}\left(\sum\limits_{j=1}^{3}\left(\sum\limits_{p,q=1}^{3} \mu_{ijpq} e_{pq}(\mathbf{u}_2) + \eta_{ij}\theta_2\right) n_j\right), \; g_i'''(0) = tr_{\Gamma_1}\left(\sum\limits_{j=1}^{3}\left(\sum\limits_{p,q=1}^{3} \mu_{ijpq} e_{pq}(\mathbf{u}_3) + \eta_{ij}\theta_3\right) n_j\right), \end{array}\right. \quad i = 1,2,3,$$

$$g^\theta(0) = -tr_{\Gamma_1^\theta}\left(\sum_{p,q=1}^{3} \lambda_{pq} \frac{\partial}{\partial x_q}\left(\theta_1 + \tau_1\theta_2\right) n_p + \sum_{p,q=1}^{3} \bar{\lambda}_{pq} \frac{\partial}{\partial x_q}\left(\theta_0 + \tau_2\theta_1\right) n_p\right),$$

where $\mathbf{n} = (n_i)_{i=1}^3$ is the unit outward normal vector to Γ,

$$\left.\begin{array}{l} u_{2i} = \dfrac{1}{\rho}\left(\sum\limits_{j=1}^{3} \dfrac{\partial}{\partial x_j}\left(\sum\limits_{p,q=1}^{3} \mu_{ijpq} e_{pq}(\mathbf{u}_0) + \eta_{ij}\theta_0\right) + f_i(0)\right), \\ u_{3i} = \dfrac{1}{\rho}\left(\sum\limits_{j=1}^{3} \dfrac{\partial}{\partial x_j}\left(\sum\limits_{p,q=1}^{3} \mu_{ijpq} e_{pq}(\mathbf{u}_1) + \eta_{ij}\theta_1\right) + f_i'(0)\right), \end{array}\right. \quad i = 1,2,3,$$

then the initial-boundary value problem (9)-(11) possesses a unique solution, which continuously depends on the given data, i.e. the mapping

$$(\mathbf{u}_0, \mathbf{u}_1, \theta_0, \theta_1, \theta_2, \theta_3, \mathbf{f}, \mathbf{f}', \mathbf{f}'', \mathbf{f}''', \mathbf{g}', \mathbf{g}'', \mathbf{g}''', \mathbf{g}^{(4)}, f^\theta, g^\theta, g^{\theta'}) \to (\mathbf{u}, \mathbf{u}', \mathbf{u}'', \mathbf{u}''', \mathbf{u}^{(4)}, \theta, \theta', \theta'', \theta''')$$

is linear and continuous from space

$$\begin{array}{l} \mathbf{H}^4(\Omega) \times \mathbf{H}^3(\Omega) \times H^3(\Omega) \times H^2(\Omega) \times V^\theta(\Omega) \times L^2(\Omega) \times C([0,T];\mathbf{H}^2(\Omega)) \times C([0,T];\mathbf{H}^1(\Omega)) \times \\ \times L^2(0,T;\mathbf{L}^2(\Omega)) \times L^2(0,T;\mathbf{L}^2(\Omega)) \times L^2(0,T;\mathbf{L}^{4/3}(\Gamma_1)) \times L^2(0,T;\mathbf{L}^{4/3}(\Gamma_1)) \times L^2(0,T;\mathbf{L}^{4/3}(\Gamma_1)) \times \\ \times L^2(0,T;\mathbf{L}^{4/3}(\Gamma_1)) \times L^2(0,T;L^2(\Omega)) \times L^2(0,T;L^{4/3}(\Gamma_1^\theta)) \times L^2(0,T;L^{4/3}(\Gamma_1^\theta)) \end{array}$$

to space

$$\begin{array}{l} C([0,T];\mathbf{V}(\Omega)) \times C([0,T];\mathbf{V}(\Omega)) \times C([0,T];\mathbf{V}(\Omega)) \times \left(C([0,T];\mathbf{L}^2(\Omega)) \cap L^2(0,T;\mathbf{V}(\Omega))\right) \times L^2(0,T;\mathbf{L}^2(\Omega)) \times \\ \times C([0,T];V^\theta(\Omega)) \times C([0,T];V^\theta(\Omega)) \times \left(C([0,T];L^2(\Omega)) \cap L^2([0,T];V^\theta(\Omega))\right) \times L^2(0,T;L^2(\Omega)). \end{array}$$

მათემატიკა

თერმოდრეკადი სხეულების სამფაზიანი დაგვიანებით არაკლასიკური სამგანზომილებიანი მოდელის გამოკვლევის შესახებ

გ. ავალიშვილი* და მ. ავალიშვილი**

** ივანე ჯავახიშვილის სახ. თბილისის სახელმწიფო უნივერსიტეტი, ზუსტ და საბუნებისმეტყველო მეცნიერებათა ფაკულტეტი, თბილისი, საქართველო*
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ნაშრომში განხილულია თერმოდრეკადი სხეულების არაკლასიკური დინამიკური სამგანზომილებიანი მოდელი, რომელიც დამოკიდებულია სამფაზიანი დაგვიანების პარამეტრზე. გამოკვლეულია არაკლასიკური მოდელის შესაბამისი საწყის-სასაზღვრო ამოცანა შერეული სასაზღვრო პირობებით, სადაც საზღვრის გარკვეულ ნაწილებზე გადაადგილება და ტემპერატურა ნულის ტოლია, ხოლო საზღვრის შესაბამის დარჩენილ ნაწილებზე მოცემულია ზედაპირული ძალის და გარე ნორმალის გასწვრივ სითბოს ნაკადის სიმკვრივეები. მიღებულია საწყის-სასაზღვრო ამოცანის ვარიაციული ფორმულირება და სათანადო ვექტორული მნიშვნელობების მქონე განაწილებების სივრცეებში დამტკიცებულია არსებობის და ერთადერთობის შედეგი და ამონახსნის უწყვეტად დამოკიდებულება მონაცემებზე.

REFERENCES

1. Zhu X., Villeneuve D.M., Naumov A.Yu., Nikumb S., Corkum P.B. (1999) Experimental study of drilling sub-10 μm holes in thin metal foils with femtosecond laser pulses. *Appl. Surf. Sci.*, **152**, 3-4: 138–148.
2. Wang X., Xu X. (2002) Thermoelastic wave in metal induced by ultrafast laser pulses. *J. Thermal Stresses*, **25**(5): 457–473.
3. Abdallah I.A. (2009) Dual phase lag heat conduction and thermoelastic properties of a semi-infinite medium induced by ultrashort pulsed laser. *Progress in Physics*, **5**(3): 60–63.
4. Ackerman C.C., Overton Jr. W. C. (1969) Second sound in solid helium-3. *Phys. Rev. Lett.*, **22**(15): 764–766.
5. Caviglia G., Morro A., Straughan B. (1992) Thermoelasticity at cryogenic temperatures. *Internat. J. Non-Linear Mech.*, **27**(2): 251–263.
6. Dreyer W., Struchtrup, H. (1993) Heat pulse experiments revisited. *Continuum Mech. Thermodyn.*, **5**: 3-50.
7. Lord H.W., Shulman Y. (1967) A generalized dynamical theory of thermoelasticity. *J. Mech. Phys. Solids*, **15**: 229-309.
8. Green A.E., Lindsay K.A. (1972) Thermoelasticity. *J. Elasticity*, **2**: 1–7.
9. Tzou D.Y. (1996) Macro to micro-scale heat transfer: The lagging behavior. Taylor & Francis, Washington.
10. Chandrasekharaiah D.S. (1998) Hyperbolic thermoelasticity: a review of recent literature. *Appl. Mech. Rev.* **51**(12): 705-729.
11. Burchuladze T.V., Gegelia T.G. (1985) Razvitie metoda potentsiala v teorii uprugosti. Metsniereba, Tbilisi (in Russian).
12. Ignaczak J., Carbonaro B., Russo R. (1986) Domain of influence theorem in thermoelasticity with one relaxation time. *J. Thermal Stresses*, **9**: 79-91.
13. Carbonaro B., Ignaczak J. (1987) Some theorems in temperature-rate dependent thermoelasticity for unbounded domains. *J. Thermal Stresses*, **10**: 193–220.
14. Avalishvili G., Avalishvili M. (2014) On approximation of Lord-Shulman model for thermoelastic plates with variable thickness by two-dimensional problems. *Bull. Georg. Natl. Acad. Sci.*, **8**(2): 4-14.
15. Avalishvili G., Avalishvili M., Müller W.H. (2018) An investigation of the Green-Lindsay three-dimensional model. *Math. Mech. Solids*, **23**(7): 987-1003.
16. Avalishvili G., Avalishvili M. (2013) On hierarchical models of prismatic shells within the framework of the Chandrasekharaiah-Tzou nonclassical theory of thermoelasticity. *Bull. Georg. Natl. Acad. Sci.*, **7**(2): 33-44.
17. Roy Choudhuri S.K. (2007) On a thermoelastic three-phase-lag model. *J. Thermal Stresses* **30**: 231-238.
18. Green A.E., Naghdi P.M. (1991) A re-examination of the basic postulates of thermomechanics. *Proc. R. Soc. London A*, **432**: 171-194.
19. Kumar R., Mukhopadhyay S. (2010) Analysis of the effects of phase-lags on propagation of harmonic plane waves in thermoelastic media. *Comp. Meth. Sci. Techn.*, **16**(1): 19-28.
20. Borgmeyer K., Quintanilla R., Racke R. (2014) Phase-lag heat conduction: decay rates for limit problems and well-posedness, *J. Evol. Equ.*, **14**: 863–884.
21. Zampoli V., Landi A. (2017) A domain of influence result about the time differential three-phase-lag thermoelastic model. *J. Thermal Stresses* **40**(1): 108-120.
22. Shakeriaski F., Ghodrat M., Escobedo-Diaz J., Behnia M. (2021) Recent advances in generalized thermoelasticity theory and the modified models: a review. *J. Comput. Des. Eng.*, **8**(1): 15-35.
23. McLean W. (2000) Strongly elliptic systems and boundary integral equations. Cambridge University Press, Cambridge.
24. Dautray R., Lions J.-L. (2000) Mathematical analysis and numerical methods for science and technology, vol. 5: Evolution problems I. Springer-Verlag, Berlin-Heidelberg.

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