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*Hydrology*

# Modeling of River Bed Evolution in Watercourses

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**The interaction of solid river sediments, their separate particles and generally, sediment and river bed are characterized by a great variety and difficulties. Despite this fact, the prolonged laboratory and field observations have resulted in development of a general model, which is based on ratio between average and non-scouring velocities of water flow. This model makes it possible to select the material linear scale according to its thickness and density, and under particular assumptions the thin material of the model may be replaced with a sand of higher diameter. In this way, there are obtained coefficients and quality indicators, which are different for different diameter of particles.**


river bed, sediment, non-scouring velocity, modeling

River bed evolution process belongs to the category of self-adjusting events. Water flow and river bed manage each other and via this interrelation they naturally establish the most likely dependence between geometric entities. In order to completely describe the interrelation between flow and river bed in rivers and large channels it is important to model the river bed evolution, and special studies, including laboratory trials are conducted for this purpose. In case of such studies, optimal selection of material and sediment composition for the model is of great importance.

It is known that misshapen river bed flows are completely similar in case of observance of geometric similarity of hydrodynamic, flow-restricting surface coarseness and river bed. In fact, such similarity is unattainable, so they draw attention to the observance of kinematic similarity. In such case, by selection of the appropriate sediment for the model, they carry out shift close to natural, which is made with observance of the following condition [1]:

$$\frac{V}{V_0^*} = idem, \tag{1}$$

where $V$ is an average velocity of flow passing, while $V_0^*$ is a non-scouring velocity.

Formula (1) is one of the main critical conditions of river bed evolution process, which makes it possible to select ground composition and scale for the model through entry of known expression of $V_0^*$. In particular, the following dependence is used for establishment of model scale:

$$\sigma_d = \frac{\rho_n^* - \rho}{\rho_m - \rho}\sigma, \tag{2}$$

where $\rho^*$ is sediment density; $\rho$ is water density.

As far as the current formulas for non-scouring velocity $V_0^*$ are empiric and they are mainly derived for wide river beds, critical conditions of modeling obtained by their means also have to be reliable for flows passing namely through such river beds. From this viewpoint, such expressions should be preferred, which take into account a broad variability of hydraulic and geometric characteristics of the flow and river bed. Theoretical values of non-scouring velocities in shorthand form are as follows [2]:

$$V_0^* = \left[\frac{\sigma C_s^2}{2g}\left(w_0^2 + \frac{T}{\rho}\right)\right]^{0.5}. \tag{3}$$

Plugging expression (3) in formula (1), we will get:

$$\frac{V_n^2}{V_m^2} = \frac{\sigma_n C_{s,n}^2 \left(w_0^2 + T\rho^{-1}\right)_n \cdot g_n}{\sigma_m C_{s,m} \left(w_0^2 + T\rho^{-1}\right)_m \cdot g_m} \tag{4}$$

Here, index „n" denotes actual (natural) value of characteristic, while index „m" – a model value. Let's entry the known scale multipliers: $\frac{V_n}{V_m} = \delta_v$; $\frac{C_{s,n}}{C_{s,m}} = \delta_c$; $\frac{\left(w_0^2 + T\rho^{-1}\right)_n}{\left(w_0^2 + T\rho^{-1}\right)_m} = \delta_{w_0T}$; $\frac{\sigma_n}{\sigma_m} = \delta_\sigma$; $\frac{g_n}{g_m} = \delta_g$. We get:

$$\sigma_v^2 = \delta_\sigma \cdot \delta_c^2 \cdot \delta_{w_0T}^2 \cdot \delta_g. \tag{5}$$

As far as $\sigma_g$ =1 and $\delta_\sigma = \frac{\rho_n^* - \rho}{\rho_m^* - \rho} = \delta_{\rho,b}$ and whereas $\sigma_v^2 = \delta_n$; where $\delta_n$ is a geometric linear scale multiplier, finally we will get [1, 3]:

$$\delta_n = \delta_{\rho,b} \cdot \delta_c^2 \cdot \delta_{w_0T}^2. \tag{6}$$

The equation (6) establishes the linkage between geometric scale and river bed material particles scale. In order to express this linkage more clearly, let us present the equation (6) in other form, for which purpose we can use for Chezy coefficient the following expression: $C_s = K\left(\frac{H}{d}\right)^{\frac{1}{6}}$,

then

$$\delta_n = \left(\frac{\rho_n^* - \rho}{\rho_m^* - \rho}\right)^{\frac{3}{2}} \left(\frac{d_n}{d_m}\right)^{\frac{1}{2}} \cdot \left[\frac{\left(w_0^2 + T\rho^{-1}\right)_n}{\left(w_0^2 + T\rho^{-1}\right)_m}\right]. \tag{7}$$

The expression (7) makes it possible to select linear coefficient of the model material according to its thickness and density.

When establishing the model material diameter according to preselected linear scale of the model, due to the fact that a functional linkage between particle diameter and hydraulic thickness is unclear, calculation has to be made in two stages: first, the model material diameter has to be preliminary determined according to the linear scale, through $d_m = \frac{d_n}{\delta_n}$ dependence, by means of which the $w_0 = ad^n$ link (when exponent power $n$ = 0.5÷2.0, and coefficient $a$ is determined by the area, in which a particle is situated) and numerical value of adhesion $T$ are established, and afterwards, the model material diameter is finally specified by trial and error method using the dependence (7).

The equation (7) shows that a scale multiplier of the river bed model material will not coincide with the value of linear scale multiplier. This circumstance can be explained by the fact that due to quite big thickness of natural sand, there are usually no adhesion forces between its particles, whereas they have a significant value in the model. Let us show it on the example. Let us assume that for the model the same density material is selected as for nature. We express the hydraulic thickness of this model sand as $w_{0,m} = a_1 d_m^n$, and for natural sand as $w_{0,n} = a_2 d_n^{n_2}$. Let us express adhesion stress via sand particle diameter using the dependence $T_m = \frac{a}{d_m \rho}$.

Plugging this value in the expression (7), we get:

$$\delta_n = \sigma_d \left( \frac{a_2^2 d_n^{2n_2-1} d_m^2}{a_1^2 d_m^{2n_2+1} + a_0} \right). \tag{8}$$

For model sand thickness $d$ =0.1 mm and natural sand thickness $d$ >2 mm we get $n_1$ =1.0; $n_2$ =0.5. $a_1 = \left( \frac{g}{11.2\sqrt{v}} \sigma \right)^{\frac{2}{3}}$ =128 m/sec; $a_2 = 1.2\sqrt{g\sigma}$ =4.8 m$^{0.5}$/sec; $a_0 = \sigma \cdot 10^{-8}$ m$^3$/sec$^2$. Then $\delta_n = \delta_d \cdot 2.3^{1.5}$, i.e. in this case $\delta_d = \frac{\delta_n}{3.5}$.

When there are no adhesion forces between the model sand particles, i.e. when $a = a_2$ and $n_1 = n_2$ =0.5, the expression (8) gives us $\delta_n = \delta_d \left( \frac{a_1^2 d_n^0 d_m^2}{a_2^2 a_m^2} \right)^{\frac{3}{2}}$ and $\delta_n = \delta_d$.

The use of the theoretical formula (3) of non-scouring velocity as the modeling criterion makes it possible to select for the model a sand with relatively coarse fraction regardless of the value of linear scale multiplier, with no adhesion forces between its particles, so the modeling conditions are substantially simplified.

The analysis of formula (3) shows that $V_0^*$ value is changed according to hyperbolic law depending on particle diameter [4]. On both sides of the curve there are situated similar points of the same $V_0^*$ accordance and different diameters. Adhesion forces have decisive importance for establishment of non-scouring velocity of particles located at the left branch of a curve, while for particles of the right branch they simply don't exist.

This circumstance shows that the main modeling condition (1) is fulfilled for two diameters of the model material, since a necessary $V_0^*$ value always answers these two $d_1$ and $d_2$ diameters.

Otherwise: $V_{0,1}^{*}=\left[\frac{\sigma}{9g}C_{s,1}^{2}\left(w_0^2+T\rho^{-1}\right)\right]^{0.5}$ and $V_{0,2}^{*}=\left[\frac{\sigma}{9g}C_{s,2}^{2}\left(w_0^2+T\rho^{-1}\right)\right]^{0.5}$, since $V_{0,1}^{*}=V_{0,2}^{*}$ and $\left(\frac{T}{\rho}\right)=0$, we get $\frac{C_{s,1}^2}{C_{s,2}^2}\frac{w_{0,1}^2+T\rho^{-1}}{w_0^2}=1$.

Let us express here hydraulic thicknesses using $w_{0,1}=a_1d_1^{n_1}$ and $w_{0,2}=a_2d_2^{n_2}$ dependencies again, while Chezy coefficient as $C_{s,1}=K\left(\frac{H}{d_1}\right)^{\frac{1}{6}}$ and $C_{s,2}=K\left(\frac{H}{d_2}\right)^{\frac{1}{6}}$.

We will get:

$$\left(\frac{H}{d_1}\frac{d_2}{H}\right)^{\frac{1}{3}}=\frac{a_1d_1^{2n_1}+\frac{a_0}{d_1}}{a_2^2\cdot d_2^{2n_2}}\text{, from here }\left(d_2^{2n_2}\right)^{-\frac{1}{3}}=\frac{a_1d_1^{2n_1+1}+a_0}{a_2^2\cdot d_1^{\frac{4}{3}}}. \qquad (9)$$

The equation (9) allows us to replace the thin model material with a sand of bigger diameter, but the velocity similarity is disturbed in this case.

For observance of hydrodynamic similarity it is necessary to distort a bottom slope. The distortion degree is taken from the following equality: $v=C_{s,1}\sqrt{R\cdot i_1}=C_{s,2}\sqrt{R\cdot i_2}$, from where

$$\frac{C_{s,1}}{C_{s,2}}=\sqrt{\frac{i_2}{i_1}}\text{; }\sqrt[3]{\frac{d_2}{d_1}}=\frac{i_2}{i_1}\text{, i.e. }i_2=i_1\cdot\sqrt[3]{\frac{d_2}{d_1}}. \qquad (10)$$

Based on the above mentioned, as a conclusion it may be said that numerical values of $a_1$ and $a_2$ coefficients, as well as $n_1$ and $n_2$ exponent power have to be selected according to initial thickness of the model material and corresponding area of its similar $d_2$ particle.

There are the following four areas: The first area, where $d$ <0.1 (mm), $a=\frac{g}{18v}\sigma$, $n$ =2; the second area, where $d$ varies between 0.1-0.6 mm, $a=\left(\frac{g}{11.2\sqrt{v}}\sigma\right)^{\frac{2}{3}}$, $n$ =1; the third area, where $d$ varies within limits of 0.6-2.0 mm, $a=\left(\frac{g}{4.4v^{0.3}}\sigma\right)^{0.55}$, $n$ =0,67; and the fourth area, where $d$ >2.0, $a=1.2\sqrt{g\sigma}$, $n$ =0,5; as far as such particles replace the particles with $d$ <0.2 mm, one may presuppose that particles similar to those with $d$ =0.2-0.05 mm are located in the second area, according to which the values of $a_2$ and $n_2$ are established. The same is for $d$ = 0.05 - 0.02 mm particles in the third area, and for $d$ <0.02 mm particles in the fourth one.

Such approach takes into account peculiar types of sediment movement and therefore, peculiarities of river bed evolution processes.

*ჰიდროლოგია*

# წყალსადინარებში კალაპოტური პროცესების მოდელირება

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**მდინარის მყარი ნატანის, მისი ცალკეული ნაწილაკებისა და ზოგადად ნატანისა და კალაპოტის ურთიერთქმედება ხასიათდება დიდი მრავალფეროვნებითა და სირთულით. მიუხედავად ამისა, ფართო ლაბორატორიული და ნატურალური დაკვირვებების შედეგად შესაძლებელი გახდა საერთო მოდელის შემუშავება, რომელიც დაფუძნებულია წყლის ნაკადის საშუალო და არაგამრეცხი სიჩქარის თანაფარდობაზე. მოდელის საშუალებით შესაძლებელია მასალის სიმსხოსა და სიმკვრივის მიხედვით შეირჩეს მისი ხაზოვანი მასშტაბი, ასევე სპეციალური დაშვებების საშუალებით მოდელის წვრილი მასალა შეიძლება შეიცვალოს უფრო მსხვილი დიამეტრის ქვიშით. ამგვარად, მიიღება კოეფიციენტები და ხარისხის მაჩვენებლები, რომლებიც განსხვავებულია ნაწილაკის დიამეტრის სხვადასხვა ზომისათვის.**

## REFERENCES

1. Voinich-Sianozhentski T. G., Kereselidze N. B. (1977) Criteria of similarity and of bed load modeling. *17th Congress of the International Association for Hydraulic Research,* **1:** 198-205.
2. Mirtskhulava Ts. (1984) Osnovy fiziki i dinamiki rechnykh rusel. p. 204. Kolos, M. (in Russian).
3. Znamenskaia N. S. (1976) Donnye nanosy i ruslovye processy. p. 189. Gidrometeoizdat, Leningrad (in Russian).
4. Voinich-Sianozhentski T. G. (1974) Gidrodinamika ustevikh uchastkov rek. Trudy ZakNIIGMI, vip. 46(52), Gidrometeoizdat, Leningrad (in Russian).

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