Informatics

Planning of Sustainable Urban System Based on Modified Voronoi Diagram

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The paper presents mathematical models and algorithms for sustainable planning of urban systems, economic spaces and regions based on a modified Voronoi diagram. The obtained algorithms make it possible to create comfortable living space for the population and organize the infrastructure and road communications of the region at the lowest cost. *© 2024 Bull. Georg. Natl. Acad. Sci.*

urban system, sustainable planning, Voronoi diagram

Planning of the urban system is generally based on the principles of sustainable development, ensuring the creation and maintenance of decent living conditions for the population, harmonization of the use and development of the relevant territory, economic and social prerequisites and the preservation of long-term development potential; effective coordination of transport/engineering infrastructure with the global urban system; access to communications, information and development of social infrastructure [1]. Such planning requirements and principles correspond to the fractal principle of urban planning, which includes the Voronoi diagram method [2,3]. The scope of application of the Voronoi diagram is very diverse: geology, archeology, biology, cartography, marketing, astronomy, urban systems, computer science and others.

Classical Voronoi diagram method assumes the homogeneity of objects, which is far from reality and. Therefore, the scope and possibilities of its use are limited. By adding the potential capacity of objects modified Voronoi method additionally acquires the function of sustainable planning and zoning.

Suppose that in the considered area E, on the coordinate plane there are N material points (objects) $A_i(x_i;y_i)$, $i\in\left[\overline{1;N}\right]$. According to the definition of Voronoi diagram, for each $i\in\left[\overline{1;N}\right]$, the „coverage zone“ of point $A_j(x_j;y_j)$, is the set of all points E, from which the distance to the point $A_j(x_j;y_j)$, is less than the distance of the other $A_j(x_j;y_j)$, (for each $j\in\left[\overline{1;N}\right]$, $j\neq i$).

In this case, the allocation of „coverage zones“ is not difficult. The „coverage zone“ of point $A_j(x_j;y_j)$, is the intersection $E\bigcap\left(\bigcup_{\substack{j=1\\ j\neq i}}^{n} S_{ij}\right)$, where the set S_{ij} is defined as: median of the segment connecting point

$A_j(x_j;y_j)$, and $A_j(x_j;y_j)$, divides the plane into two half-planes. S_{ij} denotes one of these two half-planes, in which point $A_j(x_j;y_j)$, is located.

As mentioned above, in the general case, different points (objects) have different potentials and weights (for example: consumed electricity, gas, water, number of visitors, etc.).

In this case, constructing Voronoi diagram divides the area E under consideration into given material points (objects) with different potentials, preferential "attractions," "encompassing" areas.

The following Theorem is valid [4]:

Theorem. If the distance between two points A and B of the plane is equal to a and $k>1$ is equal to any number, then the set of all points of the plane that are at a distance k from point A is a circle, whose radius is $\frac{ka}{k^2-1}$ and whose center is located on the $(A;B)$ ray at a distance $\frac{k^2a}{k^2-1}$, from point A.

The obtained result allows us to construct a Voronoi diagram for the case when the potentials of points (objects) are different. The application of the modified Voronoi model for planning an urban system, economic space, regions, districts, sustainable transport and social infrastructure are shown below.

The methodology of the urban system and urban planning is based on the creation of certain regional economic spaces. Let the economic space E comprises N regions. Each one includes M_i quantity A_{ij} districts. The total economic potential of each $i \in [1,N]$ region (industrial, economic, markets, etc.), $P_i = \oplus p_i^j; i=1,2,...,N; j=1,...,M_i$; where p_i^j - economic potential j - area, in the region - i : $\oplus$ - additive sum sign.

Let us say we know the number of objects (points) $\{A_{i1};A_{i2};...;A_{iM_i}\}$ that make up the region, and their potentials - E_{ij} "zones of attraction". regions $\{E_1;E_2;...;E_N\}$ To determine the optimal boundaries (for the comfort of the population, for the arrangement of communications, the road network with minimal costs, etc.), based on the above theorem, we will distinguish for each $E_i, i \in \overline{[1;N]}$ points of the set belonging to A_i the "region of attraction". In Fig. 1 division of space is shown E to regions, for each specific value P_i. Regions will be divided into districts using a similar algorithm.

As in Fig. 1, it can be seen that the planning of the urban system is fractal, which allows for optimal organization of infrastructure. The zones of optimal location of the transport and communication network are the dividing lines of neighboring regions - l_{ik} (see Fig. 1).

To determine the potential of the region area, it is possible to use q-analysis method [5] . Each object in the economic space E can be considered as a simple complex, and a set of objects as a simple complex [6]. On the basis of the q analysis method [4,6], the relationship between the objects of the considered region is determined, which allows optimally planning the economic space and determining the potential of the region. Let's consider a simple example.

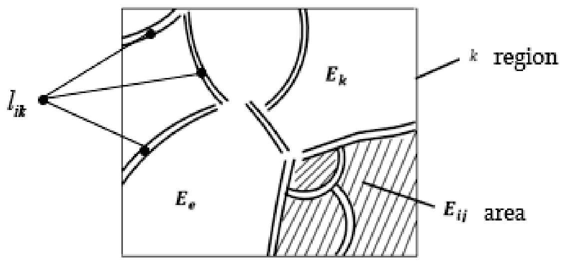

Fig. 1. Division of economic space into regions and districts based on modified Voronoi diagram

Allowed $A=\{A_1,A_2,...,A_5\}$ multiple objects in the considered region, $P=\{P_1,P_2,...,P_8\}$ parameters of objects (water, electricity, gas, parking area, quantity of visitors and others).

Matrix incidence $\wedge$ between multiples A and P has view:

$$\wedge = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 0 & 1 & 1 & 1 \end{pmatrix}. \quad (2)$$

The matrix $\wedge \cdot \wedge^T$ element c_{ii} shows the number of characteristic parameters A_i of the matrix element A. Element – $c_{ij} (i \neq j)$ shows how many common characteristic parameters A_i and A_j have.

We are considering $\wedge \wedge^T - \Omega$ matrix, where Ω – matrix, has dimension $n \times n$, all elements of which are equal to - 1. Since $\wedge \wedge^T - \Omega$ this is a symmetric matrix, we can write:

$$\wedge \wedge^T - \Omega = \begin{pmatrix} 3 & 2 & 2 & 1 & 2 \\ 2 & 4 & 3 & 2 & 2 \\ 2 & 3 & 4 & 2 & 2 \\ 1 & 2 & 2 & 5 & 3 \\ 2 & 2 & 2 & 3 & 5 \end{pmatrix}.$$

As we mentioned above, each element (an object) A_i sets A we can consider as simplex which vertices are the elements of the set P related to:

$$A_1 = \{P_1; P_5; P_6; P_7\}$$

$$A_1 = \{P_1; P_2; P_5; P_6; P_8\}$$

The diagonal elements of the matrix $\wedge \wedge^T - \Omega$ show the dimension of the elements $A_1, A_2, ..., A_n$ as simplexes, and the element standing at the intersection of the row i and column j shows the dimension of the common face of the simplexes A_i and A_j [5].

Suppose, q denotes the dimension of the complex λ, corresponding to the ratio, the largest among dimensions $A_1, A_2, ..., A_n$ simplexes). For each k, $0 \leq k \leq q$, we consider those simplexes A_i, which dimension is greater or equal to k. Let us divide the considered set of simplexes into subsets (classes equivalence) [5, 6].

Vector $Q = (Q_q, Q_{q-1}, ..., Q_2; Q_1; Q_0)$ is called the structural vector of a given relationship (a given complex) [7].

The structure vector corresponding to this relationship has the form: $Q = (2, 4, 3, 1, 1, 1)$. For example, when $Q_3 = 3$, this number sets of simplexes whose dimension is greater or equal to 3, i.e. simplicial complex is divided into three equivalence classes (in the example we are considering this is each simplex): $\{A_1\}; \{A_2, A_3\}; \{A_4, A_5\}$; when, $Q_2 = Q_1 = Q_0 = 1$; then any two simplexes are related at this level, and we only have one equivalence class.

In the economic space, region, area where objects should be located $A_1, A_2, ..., A_n$, V depending on the structure of the vector Q, it is advisable to place objects A_2 and A_3, and also objects A_4 and A_5, next to each other. This means that depending on the coordinates of the structural vector, we can design the region in such a way that infrastructure and road communications are implemented sustainably and at the lowest cost.

Now consider the case when the economic space E comprises I region, and every region includes M_i area. Without limiting the generality, we mean that regions and districts are characterized by a single indicator, respectively, S_i and S_{ij}. With this indicator may be the population size, the level of population satisfaction with the service sector, etc. There is the following balance relationship between regions and districts.

$$S_i = \sum_{j=1}^{M_i} S_{ij}, \quad i = 1, 2, ..., I . \tag{3}$$

Let us produce r - type of product (or resource) in each region, respectively in each district [7]. We mean that the production or resources of certain products depend on the corresponding indicator of other regions. Let us denote by $Q_{ik}(S)$, and $q_{ijk}(\overline{S})$ with $i = 1, 2, ..., I$, $k = 1, 2, ..., r$ $j = 1, 2, ..., M_i$ respectively, regional and district production functions, where $\overline{S} = \left\{S_{11}, ..., S_{M_1}; ...; S_{I,1}, ..., S_{I,M_I}\right\}$

$$\sum_{j=1}^{M_i} q_{ijk}(\overline{S}) = Q_{ik}(S); \quad i = 1, ..., I, \; k = 1, ..., r . \tag{4}$$

Let us assume that the economic indicator of each region is known, but the regional indicators are unknown. Since balance relations (4) are satisfied by many sets of regional indicators, the question obviously arises as which set of regional parameters is to be choosen from the allowed sets, i.e., by what criteria the objects should be distributed among districts in order to achieve the planned regional indicators.

As a criterion for the distribution of objects across regions, as a measure of uncertainty, one can use generalized information of Boltzmann entropy [6, 7]

$$H(\overline{S}) = -\sum_{i=1, j=1}^{I, M_i} S_{ij} \ln \frac{S_{ij}}{a_{ij} e},$$

where $\overline{S}$ is a matrix with elements S_{ij}, characterizes the distribution of regional indicators, e – number Nepera. Parameter a_{ij} characterizes the connectivity degree between i and j regions. Redistribution of regional economic indicators between regions is the solution to the following problem

$$H(\overline{S}) \Rightarrow \max, \quad \overline{S} \in D,$$

where D characterizes the balance relationships between the indicators of regions and districts.

$$D = \left\{ S_{ij} \Big/ S_i = \sum_{j=1}^{M_i} q_{ijk}; \sum_{j=1}^{M_i} q_{ijk}(\overline{S}) = Q_{ik}(S); i = 1, ..., I, k = 1, ..., r \right\}.$$

At the next stage, the weights are determined p_i^j, on the basis of which a modified Voronoi diagram is constructed.

ინფორმატიკა

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