

# Nonlinear Analysis of the Enguri Dam Geodynamical Datasets

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The aim of our study was the investigation of the dynamics of time distribution characteristics of the Enguri dam foundation displacement. The analysis was carried out in the period 2020-2022. Different modern methods of nonlinear analysis: DFA (detrended fluctuation analysis) and MF-DFA (multifractal detrended fluctuation analysis) were used. The results obtained in this work are important for the investigation of the Enguri dam dynamic structures. The analysis of the dynamics of the displacement time series of the Enguri dam allows us to establish the pattern of nonlinear dynamics in the normal regime and detect significant deviations from it. The outcomes of this work will become the basis for further research of the dam behaviour in order to avoid catastrophe caused by damage of dam and foundation displacement. © 2024 Bull. Georg. Natl. Acad. Sci.

Enguri dam, displacement, nonlinear analysis

The Enguri high dam is one of the highest active dams in the world, which was built in the 1970s in the canyon of the Enguri River (West Georgia). The dam was built in the seismic active region. From the results of observation, we can conclude that the filling-discharging the reservoir causes deformation processes in the dam foundation. The Enguri dam presents important and interesting object for scientific research. The observation of different processes at the base of the Enguri dam started in 1974. One of the most interesting subjects of research is the observed movement (strain) at the base during filling-unloading the reservoir [1-6]. Modern methods of nonlinear dynamics are used

for the dam strain time distribution analysis, during change of the water level in reservoir.

## Methods

For estimating long-term correlations of the dam strain time series during load-unload of reservoir we used the methods of DFA (detrended fluctuation analysis), and MF-DFA (multifractal detrended fluctuation analysis).

In the time series analysis, the DFA is a method for determining the statistical self-similarity of the parts of the system. The DFA scaling parameter includes full information about the correlation of time series and determines long-term correlations in the non-stationary time series [7, 8]. The DFA is

used in many research fields, for example: geophysics, geodynamics, meteorology, biology, bioinformatics, economics, etc. This scaling analysis method provides a simple quantitative parameter representing the correlation properties of a signal. Compared to various well-known methods, the DFA has an advantage. It reveals long-range correlations embedded in the non-stationary time series [9].

The DFA consists of two steps:

(1) the data series  $B(k)$  are shifted by the mean  $B$  and integrated (cumulatively summed),  $y(k) = \sum_{i=1}^k [B(i) - B]$ , then segmented into windows of various sizes  $\Delta n$ ;

(2) in each segmentation the integrated data is locally fit to a polynomial  $y_{\Delta n}(k)$  (originally, and typically, linear) and the mean squared residual  $F(\Delta n)$  (fluctuations):

$$F(\Delta n) = \sqrt{\frac{1}{N} \sum_{k=1}^N (y(k) - y_{\Delta n}(k))^2},$$

where  $N$  is the total number of data points. Note that  $F(\Delta n)$  can be considered as the average of the summed squares of the residual found in the windows. The  $n$ -th order polynomial regressor in the DFA family is typically denoted as DFA $n$ , with unlabeled DFA often referring to DFA1.

Multifractal detrended fluctuation analysis (MF-DFA) is used to characterize the variability and uncertainty in empirical time series data. The MF-DFA has become the central method to characterize the variability and uncertainty in empirical time series data.

The MF-DFA is the strongest technique for detecting multifractality in a time series. It takes the average volatility of the time series in each interval as a statistical point that is subsequently used to calculate volatility functions. It then determines generalized Hurst exponents based on the power law of volatility functions. The important advantage of the MF-DFA over other approaches is its ability to detect long-term correlations in non-

stationary time series. Below we outline the key steps and formulas underlying the analysis [10].

The first step of the MF-DFA is to construct the "profile",  $Y(j)$  by integration after subtracting from the time series,  $R(i)$  its average,  $\bar{R}$ :

$$Y(j) = \sum_{i=1}^j (R(i) - \bar{R}), i = 1, \dots, N.$$

The second step of the MF-DFA is to divide the profile  $Y(j)$  into  $N_s = \text{int}\left(\frac{N}{s}\right)$  non-overlapping segments of equal length  $s$ .

In the third step of the MF-DFA, we compute the local trend for each of the  $2N_s$  segments by a least-squares fit of the series.

The fourth step of the MF-DFA involves averaging over all segments  $v$  from the second step to obtain the  $q^{\text{th}}$ -order fluctuation functions (order Hurst exponent):

$$F_q(s) = \left\{ \frac{1}{2N_s} \sum_v^{2N_s} [F^2(s, v)]^{\frac{q}{2}} \right\}^{\frac{1}{q}} \text{ if } q \neq 0$$

and

$$F_q(s) = \left\{ \frac{1}{4N_s} \sum_v^{2N_s} \ln[F^2(s, v)] \right\} \text{ if } q = 0,$$

$$F_q(s) \propto s^{h(q)}.$$

The exponent  $h(q)$  is called a generalized multifractal Hurst exponent and is related to the classical monofractal Hurst exponent  $H$ .

For the MF-DFA analysis, we use the generalized Hurst exponent, which has no upper limit and expressed as:

$$H = \begin{cases} h(q) & \text{for stationary time series} \\ h(q) - 1 & \text{for non-stationary time series} \end{cases}$$

The estimation of  $H$  represents fundamental base, as we want to know the long-term dependence of a time series.

## Results and Discussion

Nonlinear DFA analysis of the Enguri dam foundation displacement data in 2020-2022 was carried out. The results of the DFA analysis of displacement show the long-range correlation of

scaling features, changes in dynamical structures, and the regularity of the system. DFA analysis was carried out for polynomial fitting for polynomials of the order  $p$ :  $p = 2, 3, 4, 5$  (Fig. 1).

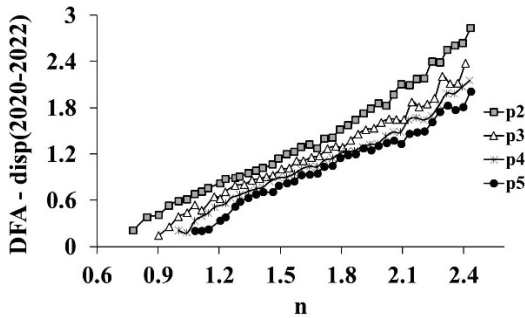


Fig. 1. The DFA analysis of the Enguri dam foundation displacement in 2020-2022 for various values of polynomial ( $p$ ).

From the DFA analysis of the Enguri dam data sets, we can see how the structure of the dynamics changes by increasing the polynomial degree.

Multifractal detrended fluctuation analysis (MF-DFA) of long-term correlations of the power law of non-stationary Enguri dam foundation displacement data in 2020-2022 was carried out. The variation of the multifractal characteristics was carried out for polynomial fitting with  $p=3$  (Fig. 2 – Fig. 5).

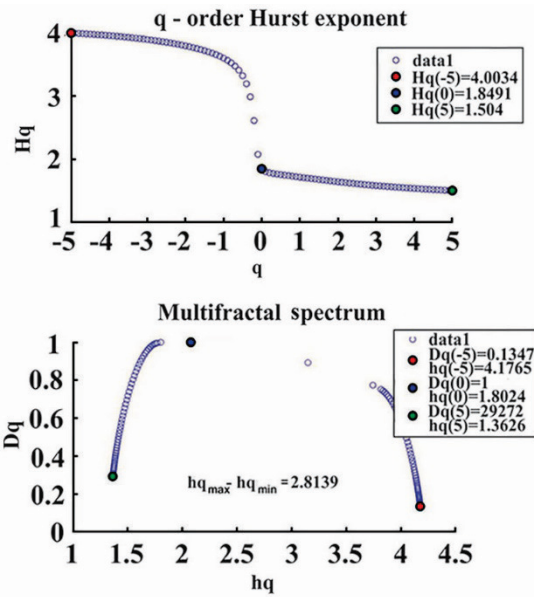


Fig. 2. Regression slopes  $Hq$  and multifractal spectrum  $Dq$ , which depend on the  $q$ -order Hurst exponent for the interval:  $-5 < q < 5$ .

Multifractal spectrum  $Dq$  is calculated by:

$$D(q) = h(q) - \tau(q),$$

where the multifractal scaling exponent  $\tau(q)$  is:

$$\tau(q) = qh(q) - 1.$$

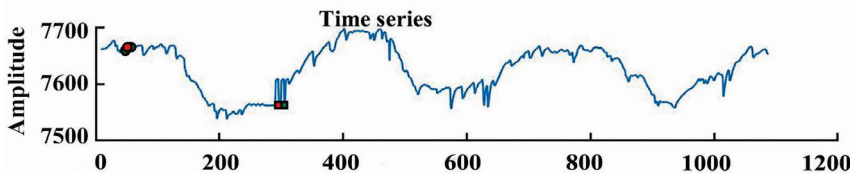


Fig. 3. The Enguri dam foundation displacement time series in 2020-2022.

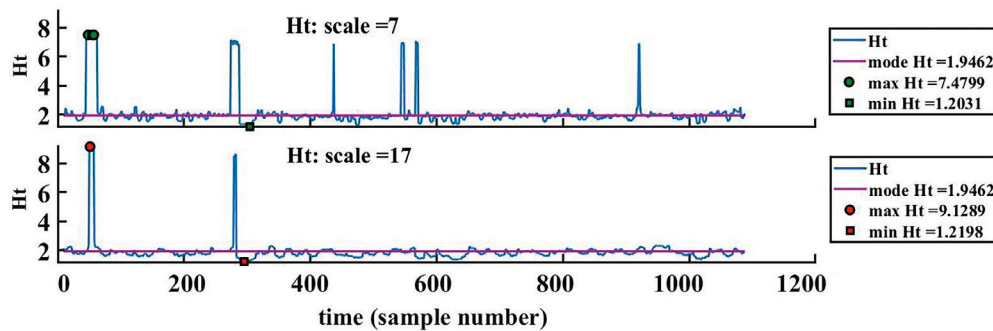
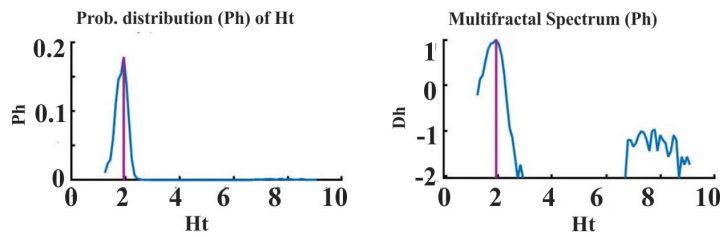


Fig. 4. The MF-DFA analysis of the Enguri dam foundation displacement in 2020-2022:  $Ht$ :  $q$  – generalized multifractal Hurst exponent time signal.



**Fig. 5.** The MF-DFA analysis of the Enguri dam foundation displacement in 2020-2022. Percentage of output variable: Ph – probability distribution of Ht, Dh- multifractal spectrum.

We can see optimal value for the inflection point value:  $q = 0 - Hq = 1.8491$  and  $Dq$  is given by parabola function with a maximum at generalized multifractal Hurst exponent  $-hq(0)$ , multifractal spectrum  $-Dq(0)$  and mean value = 2.8139. The generalized Hurst exponent  $h(q)$  is not constant over the interval of  $q$  and consequently the multifractal scaling exponent  $\tau(q)$  is not linear, indicating clearly that the time series is multifractal.

The values of Ht, the  $q$ -order of generalized multifractal Hurst exponent time signal were also calculated. The local generalized multifractal exponent (Ht) can now be computed from the local fluctuation of real time series signal (Fig. 3) estimated as well as the logarithmic function (Ht) (Fig. 4). In Fig. 4 we can see a non-stability, that under variation at the orders of scale  $s = 7$  and  $s = 17$  changed maximum and minimum of Ht, but Ht mode is constant (mode  $Ht \approx 2$ ). These changes in dynamic structure of time series clearly observed in Fig. 5, where the plot of Ph-probability distribution of Ht and Dh- multifractal spectrum represents the relationship in the form of parabola and shows an increase in the thresholds at the *mode*  $Ht \approx 2$ .

## Conclusions

The time series of the Enguri dam foundation displacement has been analyzed and the results were obtained in 2020-2022 period. The methods of nonlinear analysis: DFA and MF-DFA, help us to reveal the dynamics of dam strains. The results obtained by our analysis are important for the investigation of the Enguri dam behavior. The analysis of the dynamics of the displacement time series of the Enguri dam allows us to establish the pattern of nonlinear dynamics in the normal regime. We suppose that significant deviations from the above obtained values of multifractal characteristics should be analyzed in detail to decide whether the anomaly is important for dam stability.

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გეოფიზიკა

## ენგურის კაშხლის გეოდინამიკურ მონაცემთა მასივის არაწრფივი ანალიზი

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კვლევის მიზანი იყო ენგურის კაშხლის საძირკვლის გადაადგილების დროითი განაწილების მახასიათებლების დინამიკის შესწავლა. ანალიზი ჩატარდა 2020-2022 წლებში. გამოყენებული იყო არაწრფივი ანალიზის სხვადასხვა თანამედროვე მეთოდი: ტრენდმოცილებული ფლუქტუაციის ანალიზი და მულტიფრაქტალური ტრენდმოცილებული ფლუქტუაციის ანალიზი. ნაშრომში მიღებული შედეგები მნიშვნელოვანია ენგურის კაშხლის დინამიკური სტრუქტურის კვლევისთვის. კაშხლის გადაადგილების დროითი სერიების დინამიკის ანალიზი საშუალებას გვაძლევს განვსაზღვროთ არაწრფივი დინამიკის გამოსახულება ნორმალურ რეჟიმის პირობებში და გამოვავლინოთ მისგან მნიშვნელოვანი გადახრები. ნაშრომში განხილული ანალიზის შედეგები გახდება საფუძველი კაშხლის დინამიკის შემდგომი კვლევისთვის და დაეხმარება მეცნიერებს თავიდან აიცილონ გლობალური კატასტროფა, რომელიც შეიძლება გამოწვეული იყოს კაშხლის საძირკვლის გადაადგილებით, გარე ზემოქმედების გამო.

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