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Mathematics

Consistent Criterion for Hypothesis Testing and Consistent Estimator of Parameters for Stationary Gaussian Structures in the Radon Space

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Statistical methods can be used to determine probabilistic characteristics. Among the problems of statistics there is a class of problems in which the number of observations is unique. Despite the uniqueness of the observations, in many cases it is possible to reliably determine the values of the parameters of the unknown distribution or to reliably choose one from an infinite number of competing hypotheses about the exact shape of the distribution. When a parameter is reliably determined by single observation we say that there exists a consistent estimator of parameters and consistent criterion for hypothesis testing. Stationary Gaussian statistical structures are defined in the paper. Necessary and sufficient conditions for the existence of consistent estimators of parameters, consistent estimators of any parametric function, and unbiased estimators of any parametric function are given. Necessary and sufficient conditions for the existence of consistent criterion for testing hypothesis, consistent criterion for testing hypothesis of any parametric function and unbiased criterion of any parametric function have been established. Moreover, we consider the case where statistical observation space is a Radon separable complete metric space or Radon complete non-separable metric space. *© 2024 Bull. Georg. Natl. Acad. Sci.*

statistical structure, stationary Gaussian structures, consistent estimators, consistent criteria, orthogonal structure, weakly separable structure, strongly separable structure, Radon space

Let (E,S) be a statistical observation space and $\{\mu_i, i \in I\}$ be a given family of probability measures on (E,S). Let I be the set of parameters. The following definitions are taken from [1-16].

Definition 1. An object $\{E,S,\mu_i, i \in I\}$ is called statistical structure.

Definition 2. Statistical structure $\{E,S,\mu_i, i \in I\}$ is called orthogonal (singular) (O) if the family of probability measures $\{\mu_i, i \in I\}$ consists of pairwise singular measures (i.e. $\mu_i \perp \mu_j$, $\forall i \neq j$).

Remark 1. Let $E=[0,1]$ and S be a Borel σ-algebra of subsets of [0,1]. Let $\mu_1(B)=2l(B\cap[0,1/2])$; $\mu_2(B)=2l(B\cap(1/2,1])$ and $\mu_3(B)=3l(B\cap[0,1/3])$ $(B\in S)$, where l is the Lebesgue measure on S. Then $\mu_1\perp\mu_2$, $\mu_2\perp\mu_3$, but μ_1 is not orthogonal to μ_3.

Definition 3. Statistical structure $\{E,S,\mu_i,i\in I\}$ is called weakly separable (WS) if there exists a family of S-measurable sets $\{X_i,i\in I\}$ such that

$$\mu_i(X_j)=\begin{cases}1, & if\ i=j;\\ 0, & if\ i\neq j\end{cases}\quad (i,j\in I).$$

Definition 4. Statistical structure $\{E,S,\mu_i,i\in I\}$ is called separable (S) if there exists a family of S-measurable sets $\{X_i,i\in I\}$ such that

$$1)\ \mu_i(X_j)=\begin{cases}1, & if\ i=j;\\ 0, & if\ i\neq j\end{cases}\quad (i,j\in I);$$

$$2)\ \forall i,j\in I:\ card(X_i\cap X_j)<c,\ if\ i\neq j,$$

where c denotes the power of continuum.

Definition 5. Statistical structure $\{E,S,\mu_i,i\in I\}$ is called strongly separable (SS) if there exists a disjoint family of S-measurable sets $\{X_i,i\in I\}$ such that the relations are fulfilled:

$$\$\ \mu_i(X_i)=1,\ \forall i\in I.$$

Example 1. Let E=[0,1]×[0,1] and S be a Borel σ-algebra of subsets of E. Let us take S-measurable sets

$$X_i=\begin{cases}0\le x\le 1,\ y=i, if\ \ i\in(0,1];\\ 0\le x\le 1,\ 0\le y\le 1, if\ \ i=0\end{cases}$$

and assume that l_i, $i\in(0,1]$, are linear Lebesgue measures on X_i, and l_0 is a plane Lebesgue measure on $[0,1]\times[0,1]$. Then the statistical structure $\{[0,1]\times[0,1],S,l_i,i\in[0,1]\}$ is orthogonal, but not weakly separable.

Let I be a set of hypotheses and let $B(I)$ be σ-algebra of subsets of I which contains all finite subsets of I.

Definition 6. We will say that the statistical structure $\{E,S,\mu_i,i\in I\}$ admits a consistent estimator of parameters $i\in I$ (CE) if there exists at least one measurable mapping $f:(E,S)\to(I,B(I))$, such that

$$\mu_i(\{x: f(x)=i\})=1,\ \forall\ i\in I.$$

Let H be the set of hypotheses and $B(H)$ be σ-algebra of subsets of H which contains all finite subsets of H. Let $\{\mu_h,h\in H\}$ be the family of probability measures on (E,S).

Definition 7. We will say that the statistical structure $\{E,S,\mu_h,h\in H\}$ admits a consistent criterion (CC) for hypothesis testing if there exists at least one measurable mapping $\delta:(E,S)\to(H,B(H))$, such that

$$\mu_h(\{x:\delta(x)=h\})=1,\ \forall\ h\in H.$$

Definition 8. Any measurable mapping $\delta:(E,S)\to(H,B(H))$ is called a statistical criterion.

Definition 9. The probability

$$\alpha_h(\delta)=\mu_h(\{x:\delta(x)\neq h\})$$

is called the probability of error of the h -th for a given criterion δ .

Definition 10. We will say that the statistical structure $\{E,S,\mu_i,i\in I\}$ admits consistent estimator of any parametric function (CEF) if for any real bounded function $g:(I,B(I))\to(R,B(R))$ there exists at least one measurable function $f:(E,S)\to(R,B(R))$, such that

$$\mu_i(\{x:f(x)=g(i)\})=1,\ \forall\, i\in I.$$

Definition 11. We will say that the statistical structure $\{E,S,\mu_h,h\in H\}$ admits consistent criterion for hypothesis testing of any parametric function (CCF) if for any real bounded function $g:(H,B(H))\to(R,B(R))$ there exists at least one measurable function $f:(E,S)\to(R,B(R))$, such that

$$\mu_h(\{x:f(x)=g(h)\})=1,\ \forall\, h\in H.$$

Definition 12. We will say that the statistical structure $\{E,S,\mu_i,i\in I\}$ admits unbiased estimator of any parametric function (UCE) if for any real bounded measurable function $g:(I,B(I))\to(R,B(R))$ there exists at least one measurable function $f:(E,S)\to(R,B(R))$ such that

$$\int_E f(x)\mu_i(dx)=g(i),\ \forall\, i\in I.$$

Definition 13. We will say that the statistical structure $\{E,S,\mu_h,h\in H\}$ admits unbiased criterion of any parametric function (UCC) if for any real bounded measurable function $g:(H,B(H))\to(R,B(R))$ there exists at least one measurable function $f:(E,S)\to(R,B(R))$

such that

$$\int_E f(x)\mu_h(dx)=g(h),\ \forall\, h\in H.$$

By (ZFC) we denote the formal system of Zermelo-Fraenkel with the addition of the axiom of choice (AC), i.e. (ZFC)=(ZF) & (AC). By (ZFC) & (CH) we denote the theory with the addition of a continuum hypothesis (CH): $2^{\chi_0}=\chi_1$, where χ_1 is the first uncountable cardinal number, and by (ZFC) & (MA) we denote the theory with the addition of Martin's axiom (MA). It is known that in the theory (ZFC) & (CH) Martin's axiom (MA) is automatically satisfied. It is well known that Martin's axiom (MA) is much weaker than the continuum hypothesis (CH). Moreover, the negation of the continuum hypothesis ($\neg$ CH) is compatible with Martin's axiom [1-4].

A. Skorokhod defined a weakly and strongly separable statistical structure. A. Kharazishvili defined and studied a separable statistical structure [1-4, 15]. Z. Zerakidze defined and studied consistent criteria for hypothesis testing [7, 12-14, 16].

A. Skorokhod proved that in the theory (ZFC) & (CH) an arbitrary weakly separable statistical structure, whose cardinality is not greater than the cardinality of the continuum, is strongly separable (see [1]). G. Pantsulaia proved that if an arbitrary weakly separable statistical structure, whose cardinality is continuum is strongly separable then the continuum hypothesis (CH): $2^{\chi_0}=\chi_1$ is true [15].

Let $\xi_i(t,\omega)=\theta_i(t)+\Delta(t,\omega)$, $t\in T\subset R$, $\forall i\in I$ be Gaussian real stationary processes, where T is a closed bounded subset of R with zero means $E(\Delta(t,\omega))=0$, $E\xi_i(t,\omega)=\theta_i(t)$, $t\in T$, $\forall i\in I$ and the correlation function $E(\Delta(t,\omega)\Delta(s,\omega))=R(t-s)$. Let $\{\mu_{\theta_i},i\in I\}$ be the corresponding probability measures on S and $f_i(\lambda)$, let $\lambda \in R$, $\forall i \in I$ be spectral densities such that the following relations are fulfilled

$$(1+\lambda^2)^{-N} \cdot K_i \le f_i(\lambda) \le C_i(1-\lambda^2)^{-N}, \ i \in I,$$

where K_i and C_i, $i \in I$ are positive constants.

We will assume that the functions $\theta_i(t)$ themselves or their derivatives satisfy the conditions:

$$\int_{-\infty}^{\infty} [\Theta_i^{(m)}(t)]^2 \, dt = +\infty, \ \forall i \in I, \ m = 0,1,...,n.$$

Then the corresponding probability measures μ_{θ_i} and μ_{θ_j} are pairwise orthogonal $\forall i, j \in I$ (see [1,5]) and $\{E, S, \mu_{\theta_i}, i \in I\}$ are Gaussian orthogonal stationary statistical structures.

Next we study countable Gaussian stationary statistical structures which admit consistent criteria for hypothesis testing as well as consistent estimators of the parameters. Then we study continuum Gaussian stationary statistical structures which admit consistent criteria for hypothesis testing as well as a consistent estimators of the parameters.

The Case of a Countable Gaussian Statistical Structure

Theorem 1. A countable Gaussian stationary statistical structure $\{E, S, \mu_{\theta_i}, i \in I\}$, $card I = \chi_0$ admits a consistent estimator of parameters if and only if the functions $\theta_i(t)$ itself or its derivatives satisfies conditions:

$$\int_{-\infty}^{\infty} [\Theta_i^{(m)}(t)]^2 \, dt = +\infty, \ \forall i \in I, \ m = 0,1,...,n.$$

Proof. Necessity. Since a countable statistical structure $\{E, S, \mu_{\theta_i}, i \in I\}$, $card I = \chi_0$ admits a consistent estimator of the parameter, there exists a measurable mapping

$\delta : (E, S) \to (I, B(I)),$

such that

$$\mu_{\theta_i}(\{x : \delta(x) = \theta_i\}) = 1, \ \forall i \in I.$$

Let $X_{\theta_i} = \{x : \delta(x) = \theta_i\}$, then it is evident that $X_{\theta_i} \cap X_{\theta_j} = \varnothing$, $\forall i \neq j$ and $\mu_{\theta_i}(X_{\theta_i}) = 1$, $\forall i \in I$. Therefore, $\mu_{\theta_i}(E \setminus X_{\theta_i}) = 0$ and $\mu_{\theta_j}(X_{\theta_i}) = 0$, $\forall j \neq i$. Hence, the measures μ_{θ_i} and μ_{θ_j} are orthogonal and the functions $\theta_i(t)$ themselves or their derivatives satisfy the conditions:

$$\int_{-\infty}^{\infty} [\Theta_i^{(m)}(t)]^2 \, dt = +\infty, \ \forall i \in I, \ m = 0,1,...,n.$$

So the necessity is proved.

Sufficiency. Let the functions $\theta_i(t)$ themselves or their derivatives satisfy the conditions:

$$\int_{-\infty}^{\infty} [\Theta_i^{(m)}(t)]^2 \, dt = +\infty, \ \forall i \in I, \ m = 0,1,...,n,$$

Then a statistical structure $\{E,S,\mu_{\theta_i},i\in I\}$, $cardI=\chi_0$ is orthogonal, i. e. the singularity of probability measures $\{\mu_{\theta_i},i\in I\}$ implies the existence of a family of S-measurable sets X_{ij} such that for any $i\neq j$ we have $\mu_{\theta_j}(X_{ij})=0$ and $\mu_{\theta_i}(E\setminus X_{ij})=0.$ If now we consider the sets $X_{\theta_i}=\cup_{j\neq i}(E\setminus X_{ij})$, we will see that $\mu_{\theta_i}(X_{\theta_i})=0$ and $\mu_{\theta_j}(E\setminus X_{\theta_i})=0$, $\forall j\neq i.$ It means that the statistical structure $\{E,S,\mu_{\theta_i},i\in I\}$ is weakly separable and there exists a family of S-measurable sets $\{\widetilde{X}_{\theta_i},i\in I\}$ such that

$$\mu_{\theta_j}(\widetilde{X}_{\theta_i})=\begin{cases}1, & if \;\; i=j;\\ 0, & if \;\; i\neq j.\end{cases}$$

Let us consider the sets

$$\overline{X}_{\theta_i}=\widetilde{X}_{\theta_i}\setminus(\widetilde{X}_{\theta_i}\cap(\cup_{j\neq i}\widetilde{X}_{\theta_i})),\;\; i\in I.$$

It is clear that $\overline{X}_{\theta_i}\cap\overline{X}_{\theta_j}=\varnothing$, $\forall i\neq j$ and $\mu_{\theta_i}(\overline{X}_{\theta_i})=1,$ $\forall i\in I.$ Define the mapping $f:(E,S)\to(I,B(I))$ as follows $f(\overline{X}_{\theta_i})=\theta_i,$ $i\in I.$ Then we have $\mu_{\theta_i}(\{x:f(x)=\theta_i\})=1,$ $\forall i\in I,$ i.e. the statistical structure $\{E,S,\mu_{\theta_i},i\in I\}$ admits a consistent estimator of parameters. ■

It's easy to prove that:

$$(SS)\Rightarrow(S)\Rightarrow(WS)\Rightarrow(O)\Rightarrow(CE)\Rightarrow(CEF)\Rightarrow(UCE).$$

The following theorem is proven similarly to Theorem 1.

Theorem 2. A countable Gaussian stationary statistical structure $\{E,S,\mu_{\theta_h},h\in H\}$, $cardH=\chi_0$ admits a consistent criterion for hypothesis testing if and only if the functions $\theta_i(t)$ themselves or their derivatives satisfy the conditions:

$$\int_{-\infty}^{\infty}[\Theta_h^{(m)}(t)]^2\,dt=+\infty,\;\forall h\in H,\; m=0,1,...,n.$$

Theorem 3. A countable Gaussian stationary statistical structure $\{E,S,\mu_{\theta_i},i\in I\}$, $cardI=\chi_0$ admits an unbiased estimator of any parametric function if and only if the functions $\theta_i(t)$ themselves or their derivatives satisfy the conditions:

$$\int_{-\infty}^{\infty}[\Theta_i^{(m)}(t)]^2\,dt=+\infty,\;\forall i\in I,\; m=0,1,...,n.$$

Proof. Necessity. Since a countable statistical structure $\{E,S,\mu_{\theta_i},i\in I\}$, $cardI=\chi_0$ admits an unbiased estimator of any parametric function, then for any real bounded measurable function $g:(I,B(I))\to(R,B(R))$ there exists at least one measurable function $f:(E,S)\to(R,B(R))$ such that

$$\int_E f(x)\mu_{\theta_i}(dx)=g(i),\;\forall\, i\in I.$$

It follows that the statistical structure $\{E,S,\mu_{\theta_i},i\in I\}$, $cardI=\chi_0$ is weakly separable. From weakly separability there follows orthogonality and from orthogonality the functions $\theta_i(t)$ themselves or their derivatives satisfy the conditions:

$$\int_{-\infty}^{\infty} [\Theta_i^{(m)}(t)]^2 dt = +\infty, \ \forall i \in I, \ m = 0,1,...,n$$

and the necessity is proved.

Sufficiency. Let the functions $\theta_i(t)$ themselves or their derivatives satisfy the conditions:

$$\int_{-\infty}^{\infty} [\Theta_i^{(m)}(t)]^2 dt = +\infty, \ \forall i \in I, \ m = 0,1,...,n,$$

Then a countable Gaussian stationary statistical structure is orthogonal. The singularity of probability measures $\{\mu_{\theta_i}, i \in I\}$, $cardI = \chi_0$ implies that the statistical structure $\{E, S, \mu_{\theta_i}, i \in I\}$, $cardI = \chi_0$ is strongly separable, i. e. $\mu_{\theta_i}(X_{\theta_i}) = 1$, $\forall i \in I$ and $X_{\theta_i} \cap X_{\theta_j} = \varnothing$, $\forall i \neq j.$ Define the mapping $f : (E, S)) \to (I, B(I))$ as follows $f(X_{\theta_i}) = \theta_i$, $\forall i \in I.$ Then we have

$$\mu_{\theta_i}(\{x : f(x) = \theta_i\}) = 1, \ \forall\, i \in I.$$

■

From Theorems 1, 2 and 3 we obtain the following results.

Theorem 4. For a countable Gaussian stationary statistical structure $\{E, S, \mu_{\theta_i}, i \in I\}$, $cardI = \chi_0$ we have

$$(SS) \Leftrightarrow (S) \Leftrightarrow (WS) \Leftrightarrow (O) \Leftrightarrow (CE) \Leftrightarrow (CEF) \Leftrightarrow (UCE);$$
$$(SS) \Leftrightarrow (S) \Leftrightarrow (WS) \Leftrightarrow (O) \Leftrightarrow (CC) \Leftrightarrow (CCF) \Leftrightarrow (UCC).$$

The Case of a Continuum Gaussian Statistical Structure

Let $\{\mu_h, h \in H\}$ be a Gaussian probability measures defined on the measurable space (E, S). For each $h \in H$ denote by $\overline{\mu}_h$ the completion of the measure μ_h, and denote by $dom(\overline{\mu}_h)$ the σ-algebra of all $\overline{\mu}_h$-measurable subsets of E. Let $S_1 = \cap_{h \in H} dom(\overline{\mu}_h).$

Definition 14. A Gaussian statistical structure $\{E, S_1, \overline{\mu}_h, h \in H\}$ is called strongly separable if there exists the family of S_1-measurable sets $\{Z_h, h \in H\}$ such that the relations are fulfilled:

1) $\overline{\mu}_h(Z_h) = 1, \ \forall h \in H;$

2) $Z_{h_1} \cap Z_{h_2} = \varnothing \ \ \forall h_1 \neq h_2; \ h_1, h_2 \in H;$

3) $\cup_{h \in H} Z_h = E.$

Definition 15. We will say that the orthogonal Gaussian statistical structure $\{E, S_1, \overline{\mu}_h, h \in H\}$ admits a consistent criterion for testing hypothesis if there exists at least one measurable mapping $\delta : (E, S_1) \to (H, B(H)),$ such that

$$\overline{\mu}_h(\{x : \delta(x) = h\}) = 1, \ \forall\, h \in H.$$

Let E be an arbitrary Hausdorff topological space and μ be a Borel measure on E.

Definition 16. We will say that the measure μ is a Radon measure if for each set $X \in B(E)$ we have

$\mu(X) = sup\{\mu(K) : \ K \ is\ compact\ in\ E\ and\ K \subset X\}.$

Definition 17. We will say that the Hausdorff topological space E is a Radon space if every Borel probability measure on E is a Radon measure.

Let E be a Radon separable complete metric space.

Theorem 5. In order for the Borel orthogonal Gaussian statistical structure $\{E, S_1, \overline{\mu}_h, h \in H\}$, $cardH = c$ to admit a consistent criterion for hypotheses testing in the theory of (ZFC) & (MA), it is necessary and sufficient that this statistical structure be strongly separable.

Proof. Necessity. The existence of a consistent criterion for hypotheses testing $\delta : (E, S_1) \to (H, B(H))$ implies that $\overline{\mu}_h(\{x : \delta(x) = h\}) = 1, \ \forall\, h \in H$. Setting $X_h = \{x : \delta(x) = h\}$ for $h \in H$ we get:

1) $\overline{\mu}_h(X_h) = 1, \ \forall h \in H$;
2) $X_{h'} \cap X_{h''} = \varnothing$ for all different parameters h' and h'' from H;
3) $\cup_{h \in H} X_h = E.$

Hence, the statistical structure $\{E, S_1, \overline{\mu}_h, h \in H\}$ is strongly separable.

Sufficiency. Since the Gaussian statistical structure $\{E, S_1, \overline{\mu}_h, h \in H\}$, $cardH = c,$ is strongly separable, there exists a family $\{Z_h, h \in H\}$ of elements of σ -algebra $S_1 = \cap_{h \in H} dom(\overline{\mu}_h)$ such that:

1) $\overline{\mu}_h(Z_h) = 1, \ \forall h \in H$;
2) $Z_h \cap Z_{h'} = \varnothing$ for all different h and h' from H;
3) $\cup_{h \in H} Z_h = E.$

For $x \in E,$ we put $\delta(x) = h,$ where h is the unique hypothesis from the set H for which $x \in Z_h$. The existence of such a unique hypothesis from H can be proved using conditions 2), 3).

Take now $Y \in B(H).$ Then $\{x : \delta(x) \in Y\} = \cup_{h \in Y} Z_h.$ We must show that $\{x : \delta(x) \in Y\} \in dom(\overline{\mu}_h)$ for each $h \in H$.

If $h_0 \in Y,$ then

$$\{x : \delta(x) \in Y\} = \cup_{h \in Y} Z_h = Z_{h_0} \cup (\cup_{h \in Y \setminus \{h_0\}} Z_h).$$

On the other hand, from the validity of conditions 1), 2), 3) it follows that

$$Z_{h_0} \in S_1 = \cap_{h \in H} dom(\overline{\mu}_h) \subseteq dom(\overline{\mu}_{h_0}).$$

On the other hand, the inclusion

$$\cup_{h \in Y \setminus \{h_0\}} Z_h \subseteq (E \setminus Z_{h_0})$$

implies that $\overline{\mu}_{h_0}(\cup_{h \in Y \setminus \{h_0\}} Z_h) = 0,$ and hence, $\cup_{h \in Y \setminus \{h_0\}} Z_h \in dom(\overline{\mu}_{h_0}).$ Since $dom(\overline{\mu}_{h_0})$ is a σ -algebra, we conclude that

$$\{x : \delta(x) \in Y\} = Z_{h_0} \cup (\cup_{h \in Y \setminus \{h_0\}} Z_h) \in dom(\overline{\mu}_{h_0}).$$

If $h_0 \notin Y,$ then $\{x : \delta(x) \in Y\} = \cup_{h \in Y} Z_h \subseteq (E \setminus Z_{h_0})$ and we conclude that $\overline{\mu}_{h_0}\{x : \delta(x) \in Y\} = 0.$ The last relation implies that $\{x : \delta(x) \in Y\} \in dom(\overline{\mu}_{h_0}), \ \forall Y \in B(H).$ Hence,

$$\{x : \delta(x) \in Y\} \in \cap_{h \in H} dom(\overline{\mu}_h) = S_1.$$

Therefore, the mapping $\delta : (E, S_1) \to (H, B(H))$ is a measurable mapping. Since $B(H)$ contains all finite subsets of $H,$ we ascertain that

$$\overline{\mu}_h(\{x : \delta(x) = h\}) = \overline{\mu}_h(Z_h) = 1, \ \forall h \in H,$$

i.e. this statistical structure admits a consistent criterion for hypothesis testing. ■

The following theorem is proven similarly to Theorem 5.

Theorem 6. In order for the Borel orthogonal Gaussian statistical structure $\{E, S_1, \overline{\mu}_i, i \in I\}$, $cardI = c,$ to admit a consistent estimators of parameter in the theory of (ZFC) & (MA) it is necessary and sufficient that this statistical structure be strongly separable.

Let E be Radon complete non-separable metric space, whose topological weight is not a real-valued measurable cardinal. Let S_1 be a Borel σ-algebra on E.

Theorem 7. In order for the Borel orthogonal Gaussian statistical structure $\{E, S_1, \overline{\mu}_h, h \in H\}$, $cardH = c,$ to admit a consistent criterion for hypotheses testing in the theory of (ZFC) & (MA) it is necessary and sufficient that this statistical structure be strongly separable.

Proof. Sufficiency. Recall now the following lemmas.

Lemma 1. (see [15]). Let (E, ϱ) be a complete separable metric space and let μ be a Borel probability measure defined on $(E, B(E))$. Let $\{X_h\}_{h\in H}$, $cardH \le c,$ be a family of $B(E)$-measurable sets and $\mu(X_h) = 0,\ \forall h \in H.$ Then in the theory (ZFC) & (MA): $\mu^*(\cup_{h\in H} X_h) = 0.$

Lemma 2. (see [3]). Let (E, ϱ) be a complete non-separable metric space, whose topological weight is not a real-valued measurable cardinal than in the theory (ZFC) & (MA) and let μ be an arbitrary Borel σ-finite measure defined on $(E, B(E))$. Then there exists a closed separable subspace $E(\mu) \subset E,$ such that $\mu(E(\mu)) = 1$ and $\mu(E \setminus E(\mu)) = 0.$

Therefore, we easily ascertain that the following relations are true

$$(\forall h)\,(\forall \{X_h\}_{h\in H})\,(cardH \le c)\ \&\ \forall h\,(h \in H \Rightarrow \mu(X_h) = 0) \Rightarrow$$

$$\Rightarrow \mu^*(\cup_{h\in H} X_h) = \mu^*[(\cup_{h\in H} X_h) \cap (E(\mu))] + \mu^*[(\cup_{h\in H} X_h) \cap (E \setminus E(\mu))] = 0.$$

The statistical structure $\{E, S_1, \overline{\mu}_h, h \in H\}$, $cardH = c,$ is strongly separable because there exists a family $\{Z_h, h \in H\}$ of elements of the σ-algebra $S_1 = \cap_{h\in H} dom(\overline{\mu}_h)$ such that: 1) $\overline{\mu}_h(Z_h) = 1,\ \forall h \in H;$ 2) $Z_{h_1} \cap Z_{h_2} = \varnothing\ \ \forall h_1 \ne h_2;\ h_1, h_2 \in H;$ 3) $\cup_{h\in H} Z_h = E.$

In what follows, the proof of the theorem can easily be completed similarly to the proof of the sufficient part of Theorem 1.

Necessity. The necessary part is proved similarly to the proof of the necessary part of Theorem 1. ■

მათემატიკა

ჰიპოთეზათა შემოწმების ძალდებული კრიტერიუმები და პარამეტრების ძალდებული შეფასებები სტაციონარული გაუსის სტატისტიკური სტრუქტურებისათვის რადონის სივრცეში

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ალბათური მახასიათებლების დასადგენად შეიძლება გამოვიყენოთ სტატისტიკური მეთოდები. სტატისტიკის პრობლემებს შორის არის პრობლემების კლასი, რომლებშიც დაკვირვებების რაოდენობა უნიკალურია. მიუხედავად დაკვირვებების შეზღუდულობისა, ხშირ შემთხვევაში, შესაძლებელია საიმედოდ განისაზღვროს უცნობი განაწილების პარამეტრების მნიშვნელობები ან საიმედოდ აირჩეს ერთი უსასრულო რაოდენობის კონკურენტი ჰიპოთეზებიდან განაწილების ზუსტი ფორმის შესახებ. როდესაც პარამეტრი საიმედოდ განისაზღვრება ერთი დაკვირვებით, ჩვენ ვამბობთ, რომ არსებობს პარამეტრის ძალდებული შეფასება და, ასევე, ჰიპოთეზის ტესტირების ძალდებული კრიტერიუმი. ნაშრომში განსაზღვრულია სტაციონარული გაუსის სტატისტიკური სტრუქტურები. მოცემულია პარამეტრების ძალდებული შეფასების, ნებისმიერი პარამეტრული ფუნქციის ძალდებული შეფასების და ნებისმიერი პარამეტრული ფუნქციის გადაუადგილებადი შეფასების არსებობისთვის აუცილებელი და საკმარისი პირობები. დადგენილია ჰიპოთეზის შესამოწმებლად ძალდებული კრიტერიუმის, რომელიმე პარამეტრული ფუნქციის ჰიპოთეზის შესამოწმებლად ძალდებული კრიტერიუმის და რომელიმე პარამეტრული ფუნქციის გადაუადგილებადი კრიტერიუმის არსებობისთვის აუცილებელი და საკმარისი პირობები. უფრო მეტიც, ჩვენ განვიხილავთ შემთხვევას, როდესაც სტატისტიკური დაკვირვების სივრცე არის რადონის სეპარაბელური სრული მეტრული სივრცე ან რადონის სრული არასეპარაბელური მეტრული სივრცე.

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