საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, ტ. 18, №2, 2024
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 18, no. 2, 2024

Mathematics

Numerical Analysis of the Stress State of a Layered Orthotropic Truncated Paraboloid Rotating Shell in Case of Combined Loading

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This paper considers the problem of axisymmetric nonlinear deformation by the surface and contoured force of orthotropic paraboloidal truncated rotational layered shell. For the numerical analysis of the problem, it is applied a variant of the nonlinear theory of shells, which is elaborated on a base of hypothesis problem of broken lines. A particular example of a paraboloidal rotational layered shell deformation is given. Numerical realization of this particular example is presented. The obtained numerical results are compared with the results obtained by means of linear theory.

layered shell, deformation, nonlinear theory

The present paper is devoted to an investigation of the deformation problem of layered shells composed of significantly different mechanical characteristics layers. To study the deformed-stress state of this class of layered shells, it is important to consider the non-uniformity of shear deformations along the shell thickness.

For layered shells as a whole package, the theories built considering the hypothesis of lines or Kirchhoff-Liav do not allow to take into account the non-uniformity of transverse shear deformations. Along the thickness of each layer of the layered shell, if you allow the change of the tangential displacements according to the linear law, then it will be possible to take into account the non-uniformity of the shear deformations along the thickness of the layered shell. In this case, the order of the system of differential equations describing the deformed-stress state of the layered shell will depend on the number of layers. The theories and methods built with such an approach complicate the creation of mathematical models for layered membranes and their numerical realization [1-5].

To overcome the above difficulties one variant of the specified theory is proposed. This version of the refined theory is based on the assumption of a local angular rotation of the linear element along the thickness in the shell layers, which is due to transverse shear deformations. This option also takes into account the conditions of continuity of stresses and displacements on the contact surfaces of the layered shell layers [6, 7]. Such an approach has the possibility that the order of the system of differential equations describing the deformed-stress state of layered shells will no longer depend on the number of layers of the shell. Such an approach in the case of linear deformations of layered shells is proposed in [8]. This approach was generalized in the case of geometrically nonlinear deformations of layered shells with layers of constant thickness in [7].

In this paper, the problem of geometrically nonlinear deformation of an orthotropic truncated rotating paraboloid with layers of constant thickness is considered.

In the case of the proposed refined theory, let's quote the main equations and ratios of the theory of geometrically nonlinear deformation of orthotropic layered shells [3, 7, 8].

In the case of non-linear deformation of layered membranes, tangential displacements have the following form [6, 9-11]

$$u_\alpha^{(i)} = u + a_1^{(i)}\gamma_\alpha^{(0)} + \gamma\left(\Psi_\alpha + a_2^{(i)}\gamma_\alpha^{(0)}\right),$$
$$u_\beta^{(i)} = v + b_1^{(i)}\gamma_\beta^{(0)} + \gamma\left(\Psi_\beta + b_2^{(i)}\gamma_\beta^{(0)}\right), \qquad (1)$$

where u and v are displacements of the coordinate surface, Ψ_α and Ψ_β are the complete turning angles of the normal, $\gamma_\alpha^{(0)}$ and $\gamma_\beta^{(0)}$ are transverse shear deformations of that layer in which the coordinate surface passes, α and β are the orthogonal curvilinear coordinates of the coordinate surface. The images of the quantities $a_1^{(i)}, a_2^{(i)}, b_1^{(i)}, b_2^{(i)}$ contained in the formulas of displacements (1) are presented in the papers [3,6,7].

Taking into account the images of displacements (1), the deformation components can be imagined as follows

$$\varepsilon_{\alpha\alpha}^{(\gamma)} = \varepsilon_{\alpha\alpha}^{(i)} + \gamma\,\kappa_{\alpha\alpha}^{(i)},\ \ \varepsilon_{\alpha\beta}^{(\gamma)} = \varepsilon_{\alpha\beta}^{(i)} + \gamma\,2\kappa_{\alpha\beta}^{(i)},$$
$$\varepsilon_{\beta\beta}^{(\gamma)} = \varepsilon_{\beta\beta}^{(i)} + \gamma\,\kappa_{\beta\beta}^{(i)},\ \ \varepsilon_{\alpha\gamma}^{(\gamma)} = \gamma_\alpha^{(i)},\ \ \varepsilon_{\gamma\gamma}^{(\gamma)} = 0,\ \ \varepsilon_{\beta\gamma}^{(\gamma)} = \gamma_\beta^{(i)}. \qquad (2)$$

The components $\varepsilon_{\alpha\alpha}^{(i)}, \varepsilon_{\alpha\beta}^{(i)}, \varepsilon_{\beta\beta}^{(i)}, \kappa_{\alpha\alpha}^{(i)}, \kappa_{\alpha\beta}^{(i)}, \kappa_{\beta\beta}^{(i)}$ of the deformation (2) are given in [6]. Quantities that characterize the deformation of the coordinate surface have the following form:

$$\varepsilon_{\alpha\alpha} = \varepsilon_\alpha + \frac{1}{2}\theta_\alpha^2,\ \ \varepsilon_{\beta\beta} = \varepsilon_\beta + \frac{1}{2}\theta_\beta^2,\ \ \overset{*}{\varepsilon}_{\alpha\beta} = \varepsilon_{\alpha\beta} + \theta_\alpha\theta_\beta,$$
$$\theta_\alpha = -\frac{1}{A}\frac{\partial\omega}{\partial\alpha} + k_1 u,\ \ \theta_\beta = -\frac{1}{B}\frac{\partial\omega}{\partial\beta} + k_2 v,\ \ \gamma_\alpha^{(0)} = \Psi_\alpha - \theta_\alpha,\ \ \gamma_\beta^{(0)} = \Psi_\beta - \theta_\beta, \qquad (3)$$

where ω is the normal displacement, assumed to be constant over the thickness, A and B are Lame parameters, $\varepsilon_\alpha, \varepsilon_\beta, \varepsilon_{\alpha\beta}$ are given in [3, 6].

Considering the generalized Hooke’s law in the case of orthotropic material and the deformation components presented in the form of images (2), in the case of shells, Hooke's law is written in a different form, which is called the elasticity ratio. Elasticity ratios play the same role in the shell theory as Hooke's law does in elasticity theory. They have the following form [3, 6]:

$$N_\alpha = C_{11}\,\varepsilon_{\alpha\alpha} + C_{12}\varepsilon_{\beta\beta} + K_{11}\kappa_\alpha + K_{12}\kappa_\beta + A_{11}\frac{\partial\gamma_\alpha^{(0)}}{\partial\alpha} + A_{12}\gamma_\alpha^{(0)} + B_{11}\frac{\partial\gamma_\beta^{(0)}}{\partial\beta} + B_{12}\gamma_\beta^{(0)},$$

$$N_\beta = C_{12}\,\varepsilon_{\alpha\alpha} + C_{22}\varepsilon_{\beta\beta} + K_{12}\kappa_\alpha + K_{22}\kappa_\beta + A_{21}\frac{\partial\gamma_\alpha^{(0)}}{\partial\alpha} + A_{22}\gamma_\alpha^{(0)} + B_{21}\frac{\partial\gamma_\beta^{(0)}}{\partial\beta} + B_{22}\gamma_\beta^{(0)},$$

$$N_{\alpha\beta} = C_{66}\,\varepsilon^*_{\alpha\beta} + 2K_{66}\kappa_{\alpha\beta} + k_2(K_{66}\varepsilon^*_{\alpha\beta} + 2D_{66}\kappa_{\alpha\beta}) + (A_{16} + k_2 E_{16})\frac{\partial\gamma^{(0)}_\alpha}{\partial\beta} +$$
$$+(A_{26} + k_2 E_{26})\gamma^{(0)}_\alpha + (B_{16} + k_2 F_{16})\frac{\partial\gamma^{(0)}_\beta}{\partial\alpha} + (B_{26} + k_2 F_{26})\gamma^{(0)}_\beta,$$
$$N_{\beta\alpha} = C_{66}\,\varepsilon^*_{\alpha\beta} + 2K_{66}\kappa_{\alpha\beta} + k_1(K_{66}\varepsilon^*_{\alpha\beta} + 2D_{66}\kappa_{\alpha\beta}) + (A_{16} + k_1 E_{16})\frac{\partial\gamma^{(0)}_\alpha}{\partial\beta} + \tag{4}$$
$$+(A_{26} + k_1 E_{26})\gamma^{(0)}_\alpha + (B_{16} + k_1 F_{16})\frac{\partial\gamma^{(0)}_\beta}{\partial\alpha} + (B_{26} + k_1 F_{26})\gamma^{(0)}_\beta,$$
$$M_\alpha = K_{11}\,\varepsilon_{\alpha\alpha} + K_{12}\,\varepsilon_{\beta\beta} + D_{11}\kappa_\alpha + D_{12}\kappa_\beta + E_{11}\frac{\partial\gamma^{(0)}_\alpha}{\partial\alpha} + E_{12}\gamma^{(0)}_\alpha + F_{11}\frac{\partial\gamma^{(0)}_\beta}{\partial\beta} + F_{12}\gamma^{(0)}_\beta,$$
$$M_\beta = K_{12}\,\varepsilon_{\alpha\alpha} + K_{22}\,\varepsilon_{\beta\beta} + D_{12}\kappa_\alpha + D_{22}\kappa_\beta + E_{21}\frac{\partial\gamma^{(0)}_\alpha}{\partial\alpha} + E_{22}\gamma^{(0)}_\alpha + F_{21}\frac{\partial\gamma^{(0)}_\beta}{\partial\beta} + F_{22}\gamma^{(0)}_\beta,$$
$$M_{\alpha\beta} = M_{\beta\alpha} = K_{66}\,\varepsilon^*_{\alpha\beta} + 2D_{66}\kappa_{\alpha\beta} + E_{16}\frac{\partial\gamma^{(0)}_\alpha}{\partial\beta} + E_{26}\gamma^{(0)}_\alpha + F_{16}\frac{\partial\gamma^{(0)}_\beta}{\partial\alpha} + F_{26}\gamma^{(0)}_\beta,$$
$$Q_\alpha = K_1\gamma^{(0)}_\alpha,\;\; Q_\beta = K_2\gamma^{(0)}_\beta,$$

where $N_\alpha, N_\beta, N_{\alpha\beta}, N_{\beta\alpha}$ are tangential forces, Q_α, Q_β are decisive forces, M_α, M_β are turning moments, $M_{\alpha\beta}, M_{\beta\alpha}$ are twisting moments, $C_{ij}, K_{ij}, D_{ij}, K_1, K_2$ are the stiffness characteristics of the shell, which are determined by the elasticity parameters and thicknesses of the shell layers, $A_{11}, A_{12}, ..., F_{16}, F_{26}$ are the values depended on the mechanical and geometrical parameters of the shell, k_1, k_2 are the coefficients of curvature [3, 6].

The equilibrium equations describing the deformed-stress state of the shells have the following form [3,6]

$$\frac{\partial BN_\alpha}{\partial\alpha} + \frac{\partial AN_{\beta\alpha}}{\partial\beta} + \frac{\partial A}{\partial\beta}N_{\alpha\beta} - \frac{\partial B}{\partial\alpha}N_\beta + ABk_1Q^*_\alpha + ABq_1 = 0,$$
$$\frac{\partial AN_\beta}{\partial\beta} + \frac{\partial BN_{\alpha\beta}}{\partial\alpha} + \frac{\partial B}{\partial\alpha}N_{\beta\alpha} - \frac{\partial A}{\partial\beta}N_\alpha + ABk_2Q^*_\beta + ABq_2 = 0,$$
$$\frac{\partial BQ^*_\alpha}{\partial\alpha} + \frac{\partial AQ^*_\beta}{\partial\beta} - ABk_1N_\alpha - ABk_2N_\beta + ABq_3 = 0, \tag{5}$$
$$\frac{\partial BM_\alpha}{\partial\alpha} + \frac{\partial AM_{\beta\alpha}}{\partial\beta} + \frac{\partial A}{\partial\beta}M_{\alpha\beta} - \frac{\partial B}{\partial\alpha}M_\beta - ABQ_\alpha = 0,$$
$$\frac{\partial AM_\beta}{\partial\beta} + \frac{\partial AM_{\alpha\beta}}{\partial\alpha} + \frac{\partial B}{\partial\beta}M_{\beta\alpha} - \frac{\partial A}{\partial\beta}M_\alpha - ABQ_\beta = 0,$$

where

$$Q^*_\alpha = Q_\alpha - (N_\alpha + k_1M_\alpha)\theta_\alpha - (N_{\alpha\beta} + k_1M_{\alpha\beta})\theta_\beta,$$
$$Q^*_\beta = Q_\beta - (N_{\beta\alpha} + k_2M_{\beta\alpha})\theta_\alpha - (N_\beta + k_2M_\beta)\theta_\beta. \tag{6}$$

The quantities q_1, q_2, q_3 included in the system of equations (5) represent the projections of the shell's surface charge on the α, β, γ coordinate axes, respectively.

Let us dwell on the study of problems of deformation of class shells with layered rotational form. In the case of rotating shells, $\alpha = s$ represents the arc length of the meridian of the coordinate surface, $\beta = \theta$ is the central angle in a parallel circle.

In the case of rotary shells, the following system of nonlinear differential equations for solving the problem of axisymmetric deformation of a layered rotary shell is obtained from equations (2)-(6) presented in the above general form:

$$\frac{d\overline{Y}}{ds}=A^{*}(s)\overline{Y}+\overline{F}\left(s,\overline{Y}\right)+\overline{f}_{(s)}\,;\qquad \overline{Y}=\left\{N_s,Q_s^{*},M_s,u,\omega,\Psi_s\right\}^{T}. \tag{7}$$

Here, the matrix elements $A^{*}\left(s\right)$ included in the system of differential equations (7) are the components of the nonlinear vector-function $\overline{F}\left(s,\overline{Y}\right)$ and the components of the vector $\overline{f}_{(s)}$ are defined as in the paper [7].

If we add boundary conditions to the system of differential equations (7), we get a nonlinear boundary problem.

To solve the nonlinear boundary value problem, we use the linearization method and the discrete orthogonalization method [4, 12].

On the basis of the proposed refined theory, let us consider as an example the problem of deformation of a three-layer orthotropic truncated rotating paraboloidal shell.

The left contour of the truncated paraboloid shell is subjected to a compressive force *P* parallel to the axis of rotation, while the right contour is rigidly fixed, and the normal surface force q_3 acts on the surface of the shell. When solving the problem, we mean that the coordinate surface passes through the middle of the middle layer of the shell Fig. 1.

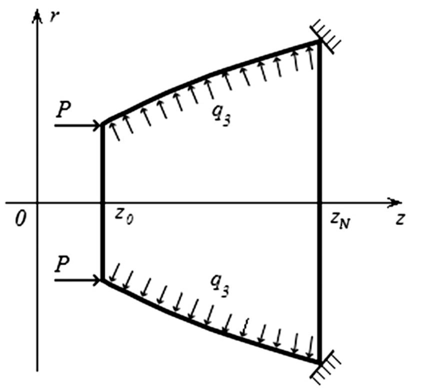

Fig. 1. The three-layer orthotropic truncated rotating paraboloidal shell.

The parametric equation of the meridian of the coordinate surface of the paraboloidal rotating shell has the following form:

$$r=a\sqrt{z}\,,\quad z=z_0,\quad z_0\le z\le z_N\,.$$

Here, r is the distance from the coordinate surface to the point of rotation.

The geometric characteristics of the shell are given by the following formulas:

$$\sin\varphi=\frac{1}{\gamma},\qquad \cos\varphi=\frac{a}{2\gamma\sqrt{z}},$$

$$\frac{1}{R_s}=-\frac{a}{4\left(\gamma\sqrt{z}\right)^{3}},\qquad \gamma=\sqrt{1+\frac{a^{2}}{4z}},$$

where φ is the angle between the coordinate surface normal and the axis of rotation.

Boundary conditions in the case of the proposed problem have the following form:

$$\begin{aligned}&N_s\cos\varphi_0+Q_s\sin\varphi_0=0,\\&M_s=0,\\&N_s\sin\varphi_0-Q_s\cos\varphi_0=N_z^{0}=P=-mq_3,\quad z=z_0\\&u=W=\Psi_s=0,\quad z=z_N.\end{aligned}$$

Let us denote h_1,h_2,h_3 thicknesses of the outer middle and inner layers of the shell, respectively, E_1^i,E_2^i represent the modulus of elasticity in the directions of the s and θ coordinate axes, respectively, ν_{12}^i,ν_{21}^i

are Poisson's coefficients, G_{13}^{i} is the shear modulus on the $\theta = const$ plane, where $i = 1,2,3$ is the number of the shell layer.

Numerical realization of the proposed problem is carried out for the following values of shell parameters:

$$a = 20;\ z_0 = 2.25;\ z_N = 6.25;\ h_1 = 0.15;\ h_2 = 1;\ h_3 = 0.15;\ E_1^1 = 1.5\cdot10^5;\ E_2^1 = 3\cdot10^5;$$
$$E_1^2 = 2\cdot10^4;\ E_2^2 = 3\cdot10^4;\ E_1^3 = 1.5\cdot10^5;\ E_2^3 = 3\cdot10^5;\ \gamma_{12}^1 = 0.2;\ \gamma_{21}^1 = 0.34;\ \gamma_{12}^2 = 0.1;$$
$$\gamma_{21}^2 = 0.15;\ \gamma_{12}^3 = 0.2;\ \gamma_{21}^3 = 0.34;\ G_{13}^1 = 0.15\cdot10^5;\ G_{13}^2 = 0.15\cdot10^4;\ G_{13}^3 = 0.15\cdot10^5.$$

Table 1 shows the numerical data of the bending function W obtained as a result of solving the proposed problem, when $q_3 = 50$ and the compressive force P takes different values, namely $m = 4,5,6$ during compression. In the Table here and later in the self-aligned triangle, the upper numerical data represent the values of the bending function W obtained based on the application of linear theory, while the lower data are obtained based on the proposed refined nonlinear theory.

Table 1. The numerical data of the bending function, when $q_3 = 50$

z	$q_3 = 50$		
	$N_z = -4q_3$	$N_z = -5q_3$	$N_z = -6q_3$
	W		
2.25	6.2874 2.3914	4.3657 1.5780	2.4439 0.61862
2.75	6.2448 2.8628	4.9215 2.3813	3.5982 1.7936
3.25	5.8369 2.9368	4.9694 2.6663	4.1018 2.3268
3.75	5.0806 2.7326	4.5465 2.5847	4.0124 2.3964
4.25	4.0677 2.3494	3.7678 2.2686	3.4679 2.1663
4.75	2.9184 1.8480	2.7740 1.8024	2.6295 1.7467
5.25	1.7652 1.2659	1.7141 1.2391	1.6630 1.2085
5.75	0.74565 0.63549	0.73911 0.62133	0.73257 0.60694
6.25	0.0 0.0	0.0 0.0	0.0 0.0

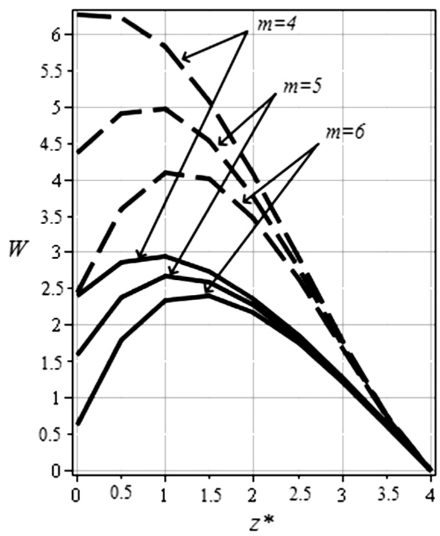

Fig. 2. The regularity of the distribution of the bending function W along the meridian of the paraboloid shell.

The graphs in Fig. 2 show the regularity of the distribution of the bending function W along the meridian of the paraboloid shell $\left(z^* = z - z_0\right)$ obtained as a result of the impact of the normal surface force $q_3 = 50$ and the compressive force P $(m = 4,5,6)$ acting on it . The solutions of the given problem are represented by continuous lines using the obtained non-linear theory, and by intermittent lines the solution is presented by the linear theory.

Table 2 shows the numerical values of the bending function W obtained as a result of the numerical realization of the proposed problem, respectively, in the case of different values of the normal surface force q_3 and in the case of the parameter $m = 4$ included in the image of the compressive force *P*. The upper numerical data presented in the row of the Table represent the results obtained using the linear theory, and the lower numerical data represent the results obtained using the non-linear theory.

Table 2. The numerical values of the bending function

z	q_3				
	10	20	30	40	50
	$m=4$				
	W				
2.25	1.2575 0.89521	2.5151 1.4379	3.7725 1.8293	5.0299 2.1368	6.2874 2.3914
2.75	1.2490 0.94635	2.4979 1.5862	3.7469 2.0853	4.9959 2.5016	6.2448 2.8628
3.25	1.1674 0.92012	2.3348 1.5768	3.5021 2.1031	4.6695 2.5482	5.8369 2.9368
3.75	1.0161 0.82572	2.0323 1.4376	3.0484 1.9361	4.0645 2.3608	5.0806 2.7326
4.25	0.81353 0.68112	1.6271 1.2055	2.4406 1.6415	3.2541 2.0175	4.0677 2.3494
4.75	0.58368 0.50564	1.1674 0.91378	1.7511 1.2633	2.3347 1.5714	2.9184 1.8480
5.25	0.35304 0.31907	0.70608 0.59289	1.0591 0.83742	1.4122 1.0602	1.7652 1.2659
5.75	0.14913 0.14262	0.29826 0.27536	0.44739 0.40086	0.59652 0.52059	0.74565 0.63549
6.25	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0	0.0 0.0

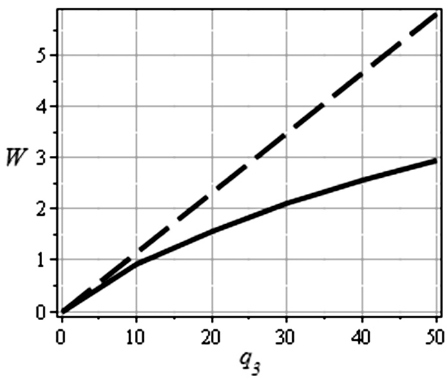

Fig. 3. The relationship between the bending function and the normal surface load.

The graphs in Fig. 3 show the relationship between the bending function W at the point $z=3.25$ and the normal surface load q_3. The continuous line represents the result obtained by the nonlinear theory, and the dashed line represents the result obtained by the linear theory.

მათემატიკა

კომბინირებული დატვირთვის შემთხვევაში ფენოვანი ორთოტროპული წაკვეთილი პარაბოლოიდური ბრუნვითი გარსის დაძაბული მდგომარეობის რიცხვითი ანალიზი

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ნაშრომში განხილულია ზედაპირული და კონტურული ძალების კომბინირებული ზემოქმედებით ფენოვანი ორთოტროპული წაკვეთილი პარაბოლოიდური ბრუნვითი გარსის ღერძსიმეტრიული არაწრფივი დეფორმაციის ამოცანა. შემოთავაზებული გარსის დეფორმაციის პროცესის რიცხვითი ანალიზის მიზნით გამოყენებულია გარსთა არაწრფივი თეორიის ერთ-ერთი ვარიანტი, რომელიც აგებულია ტეხილთა ჰიპოთეზის გათვალისწინების საფუძველზე. მოყვანილია ფენოვანი პარაბოლოიდური ბრუნვითი გარსის დეფორმაციის კერძო მაგალითი. შემოთავაზებული არაწრფივი თეორიის საფუძველზე ჩატარებულია კერძო მაგალითის რიცხვითი რეალიზაცია. მიღებული რიცხვითი შედეგები შედარებულია წრფივი თეორიით მიღებულ შედეგებთან.

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Received April, 2024