საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, ტ. 18, №3, 2024
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 18, no. 3, 2024

Mechanics

Determination of the Impact Interaction Parameters in the Elastic-Plastic Impact Process

Tamaz Natriashvili*, Slava Mebonia, Michael Ben Khaim§**

* *Academy Member, LEPL Rafiel Dvali Institute of Machine Mechanics, Tbilisi, Georgia*
** *LEPL Rafiel Dvali Institute of Machine Mechanics, Tbilisi, Georgia*
§ *Ariel University Center, Ariel, Israel*

The paper presents a method for calculating the impact parameters in the case of elastic-plastic impact interaction. The mechanical model of impact interaction is given, which takes into account the characteristic properties of the impacting bodies and represents the process of interaction of the forces of the colliding bodies. According to this method, the parameters of the strength function are determined by processing the deformation-stress diagrams obtained during the mechanical tests of metals and alloys, which greatly simplifies the solution of problem and does not require expensive experimental work to determine the parameters of the strength function. *© 2024 Bull. Georg. Natl. Acad. Sci.*

impact parameters, mechanical model, strength function

The issues of the impact interaction of solids are addressed in fundamental works of many authors [1-14]. However, these publications mainly deal with the elastic domain of loading impacting bodies, and despite great generality of these studies, they cannot be used automatically for design of metal-forming machines, owing to the need to take due account for plastic deformations. The work of the rolling mills, as well as the forging and metal-forming equipment, has specific features, that are supported by interaction between the working members of the machines and the overheated metal. This complicates the calculations of the impact loads in view of the need to take due account for the plastic properties of metal at high temperatures.

Defing the parameters of impact interaction of parts is critical for calculation and design of the forging and metal-forming equipment, as well as other heavy-duty machines, particularly the rolling mills. When calculating the impact loads, there must be defined the parameters, such as the impact pulse shape, the maximum value of the impact force, the maximum deformation, the duration of impact and other values.

The paper describes the methodology for calculating the parameters of impact interaction, taking into account the main provisions of the phenomenological theory of impact. The described methodology for calculating the parameters of impact interaction applies not only to the mechanisms and units of the above-mentioned machines, but also to other heavy-duty machines.

The parameters of the impact pulse are defined by patterns of the impact interaction of bodies. The mathematical model of the impact process of bodies has the following form:

$$M\ddot{\delta} + P(\delta,\dot{\delta}) = 0,$$

where $M = \dfrac{m_1 m_2}{m_1 + m_2}$ – the effective mass of impacting bodies; $P(\delta,\dot{\delta})$ – a force feedback function, which is defined by the material of bodies, their shapes and the type of the used model of environment [15].

As a result of the solution to this equation, it is possible to obtain all the above-mentioned parameters of impact interaction. However, it is first necessary to determine the force feedback function, that is, the dependency of local plastic deformation on a contact force.

In this respect, let us note that static dependency of the value of local compression on a contact force as an equation $\delta = bP^{2/3}$, which is known in H. Hertz's theory is also used upon collision of elastic bodies [16]. In the case of calculating the impact interaction forces in the machines for metal treatment under pressure, this equation cannot be used, since this dependency neglects the plastic deformation of metal, which occurs while rolling, forging, stamping and in other processes of metal treatment under pressure.

The major difficulty in the study and calculation of the impact processes is that the impact forces and stresses can be determined only in relation to the study of dynamic deformation of impacting bodies. Since the strains and stresses are not propagated instantaneously from the contact zone, but at the finite speeds, provided that the propagation speed varies for the particular types of stresses, then the complex stress-strain state appears in the bodies. If in addition to elastic deformations, we also take into account plastic strains, which is necessary when designing the impact processes using the hot-forming method, difficulties multiply beyond measure.

Thus, the problem of impact becomes much more complicated when considering elastic-plastic interactions. At this stage, two approaches to solving the problems of elastic-plastic impact are known:

1) When describing elastic-plastic collisions of material particles, the Gerstner's hypothesis is accepted [17], according to which, when going beyond the elastic limit, local deformation δ consists of the elastic δ_e and plastic δ_p components, which develop independently of each other when subjected to loadings, that is,

$$\delta = \delta_e + \delta_p = bP^{2/3} + \lambda P, \text{ when } \dot{\delta} \geq 0;$$

$$\delta = \delta_e + \delta_p = bP^{2/3} + \lambda P_{\max}, \text{ when } \dot{\delta} < 0.$$

The linear dependency $\delta(P)$ in a plastic zone is confirmed by the experiments conducted by N. N. Davidenkov and the works of A. Yu. Ishlinsky [18]. The disadvantage of this approach is the lack of sound recommendations on the choice of a function of the force feedback in the plastic domain.

2) A different approach to the problem of elastic-plastic impact is outlined in the work of G. S. Batuev [17]. In the study of elastic-plastic collisions of material particles, the experimental curves of static deformation are used, which are described by the functions of the type of $P = b\delta^n$, where P is a compressive force; δ – local bearing failure; b, n – constants, depending on the material and shape of the bodies.

In work [17], the parameters b, n are determined by two-fold numerical integration of oscillogram of accelerations $(\ddot{\delta})$ of the impact process. It is advisable to construct the dynamic dependency of the magnitude of local bearing failure on a contact force in the logarithmic coordinates, in which this dependency will be linear:

$$\lg\delta = \lg b + n\lg P.$$

Figure 1 illustrates in the logarithmic coordinates the dynamic power characteristics $\delta(P)$ of deformable conic elements.

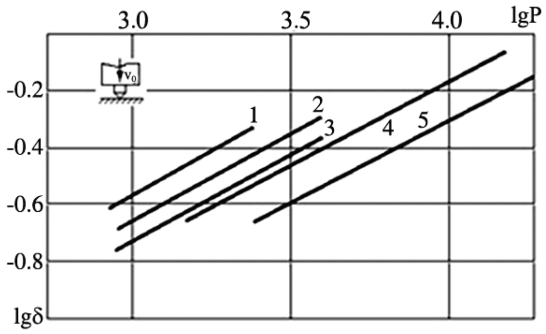

Fig. 1. The power characteristics of deformable conic elements.

This work also established the permissibility of using the experimental static characteristics obtained during compression of the investigated deformable elements in the impact problems with low velocities. At the impact velocities $v_0 \le 100$ m/sec, the mechanical characteristics of materials change insignificantly, so it can be assumed that the static power characteristic of deformable element practically does not differ from the dynamic one and can be used to calculate the impact force. The static power characteristic is very convenient in use when solving the impact problems, because this allows us to by pass the complex and expensive dynamic tests of the material. The theoretical works of Kh. A. Rakhmatulin, T. Karman and D. Taylor [14] proved that the mechanical behavior of materials depends on the deformation velocity only at the critical points, for example, upon reaching the yield stress, and is basically similar to the material when subjected to static loading.

The identity of static and dynamic equations of state offers a way of using the static stress-strain curves $(\sigma - \varepsilon)$ in the impact problems. Mathematically, this is reduced to the joint solution of the main impact equation $M\ddot{\delta} + P = 0,$ where $M = \dfrac{m_1 m_2}{m_1 + m_2}$ is the reduced mass of the impacting bodies and various expressions for the function of the force feedback $P(\delta, \dot{\delta}),$ determined by the condition of the material.

The disadvantage of the above method for solving problems of elastic-plastic impact is the complexity of the experimental determination of the parameters b and n.

The phenomenological theory of impact developed in the works [15, 19-21] allows to calculate the impact parameters not only in the case of elastic, but also for viscous-elastic, elastic-plastic and plastic impact interactions, i.e., with a different nature of the impact processes. The main issue of the phenomenological approach when calculating the impact processes is the construction of a mechanical model that takes into account the characteristic properties of the impacting bodies.

The nature of interaction of bodies on impact depends on the relative approach velocity of bodies on impact, their configuration in the contact zone and the ratio of the rheological characteristics of the material (elasticity, viscosity and plasticity). By choosing the ratio of the rheological characteristics, it is possible to build the model of the medium that sufficiently accurately describes the process of material deformation on impact. In this case, for each model, a corresponding relationship is established between the value of local bearing failure δ and the force P Under static loading, this relationship has the form $P = P(\delta),$ while in the case of dynamic loading – it has the form $P = P(\delta, \dot{\delta}),$ since the force in this case depends also on the approach velocity of bodies. This relationship, which is also called the function of the force feedback, can be derived in the form of a functional connection between the elastic-plastic deformation δ and the magnitude of the force P .

By placing between discrete masses the rheological models that replicate the relationship between the loading force and deformation, it is possible to replicate with some accuracy the process of force interaction of bodies on impact. Herein lies the essence of phenomenological method – description of phenomena occurring during collisions of various bodies.

In rheological models, which are a continuous medium endowed with properties of the real objects, it is assumed that a solid is simultaneously characterized by the properties of elasticity, viscosity and plasticity, the mechanical analogs of which are a spring, viscous friction damper and a dry friction element. The combination of these elements offers a means of obtaining the mechanical analogs of the real media. To each of these real media corresponds a certain equation of state that connects the magnitudes of stresses, strains and the deformation velocities $(\sigma, \varepsilon, \dot{\varepsilon})$. The generation of these equations is the ultimate goal of phenomenological method for studying impact interaction of the real objects.

The phenomenological approach to solving the problems of impact interaction involves the determination of the parameters of the strength function not experimentally, but on the basis of the conclusions obtained when studying the rheological models.

Let us consider the model of the elastic-plastic impact, for which the strength function $(P\alpha)$ is represented by the analytical expression:

$$P = b\delta^n.$$

Let us first determine a proportionality factor b in the deformation equation, for which we turn to the graph of the strength function shown in Fig. 2.

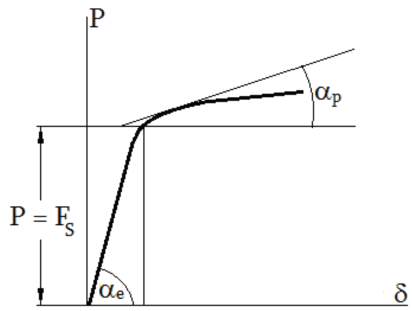

Fig. 2. The graph of the strength function.

From the formula of the strength equation $P = b\delta^n$ and the graph shown in Fig. 1, we have:

$$b = \frac{P}{\delta^n} = \frac{F_S}{\delta^n}.$$

In the elastic range of stress, the strength function is linear, that is, $F_S = \delta_S \cdot tg\alpha_e = \delta_S \cdot c_L$, from which $\delta_S = \dfrac{F_S}{c_L}$, where c_L is a linear contact stiffness; F_S – the deformation force corresponding to the beginning of flowage.

By inserting this value of δ into the expression for b we shall obtain:

$$b = \frac{F_S}{\left(\dfrac{F_S}{c_L}\right)^n} = F_S \cdot \left(\frac{c_L}{F_S}\right)^n = c_L^n \cdot F_S^{1-n}.$$

Let us determine the parameter n. From the graph shown in Fig. 1, we have: $\mathrm{tg}\alpha_e = k_e = = \dfrac{P}{\delta} = \dfrac{b\delta^n}{\delta} = b\delta^{n-1}$. For the zone of flowage according to the graph shown in Fig. 1, there can be set: $\mathrm{tg}\alpha_p = k_p = \dfrac{dP}{d\delta} = nb\delta^{n-1}$. Dividing this equality by the previous one, we obtain the expression for determining the parameter n : $n = \dfrac{\mathrm{tg}\alpha_p}{\mathrm{tg}\alpha_e} = \dfrac{E_p}{E}$.

By this means, the parameter n – the strainhardening exponent – is equal to the ratio of the tangents of the angles of the strength function of the graph in the elastic and flowage zones, or the ratio of the modulus of plasticity to the elastic modulus of the material.

In practical problems, the strain-hardening exponent n can be found from the stress-strain diagram of the material (Fig. 3).

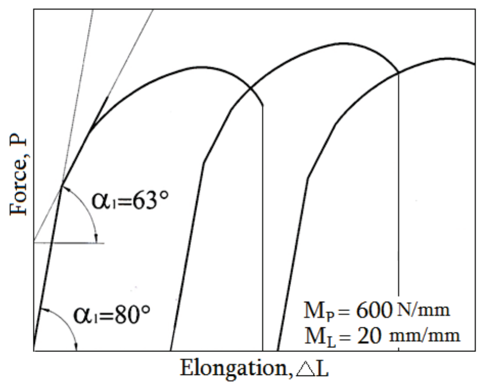

Fig. 3. The stress-strain diagram for alloy steel $(t = 20^0\mathrm{C})$.

$$n = \frac{\mathrm{tg}\alpha_2}{\mathrm{tg}\alpha_1} = \frac{\mathrm{tg}63^0}{\mathrm{tg}80^0} = \frac{1.96}{5.61} = 0.35,$$

where α_1, α_2 are the diagram tilting angles in the elastic and plastic domains.

Correct determination of the stiffness parameter is of great importance in solving the impact problems using the phenomenological models of the inelastic media. In a number of works [20, 21], the value of this parameter was established experimentally, while with a phenomenological approach, the linear parameter of stiffness is determined analytically, which greatly facilitates the subsequent calculations.

The dependence of a contact force on the approach of bodies in the first phase of elastic impact is determined by the Hertz formula: $P = k\delta^{\frac{3}{2}}$. The nonlinearity of this dependence complicates the calculations, and it therefore needs to be linearized. It is most appropriate to find the contact stiffness c_L, which simulates the elastic properties of the material, from the condition of equality of potential energies of deformations at the moment of maximum compression for the linear $(P = c_L\delta)$ and nonlinear systems:

$$\int_0^{\max\delta} c_L\delta \cdot d\delta = \int_0^{\max\delta} k\delta^{\frac{3}{2}} d\delta.$$

Upon integration, we obtain the following equality: $\frac{c_L\delta_{\max}^2}{2} = \frac{2}{5}k\delta_{\max}^{\frac{5}{2}}$, from which the linear contact stiffness is equal to: $c_L = \frac{4}{5}k\delta_{\max}^{\frac{1}{2}}$.

By inserting in this expression the value of maximum compression $\delta_{\max}$ from work [11], that is, $\delta_{\max} = \left(\frac{5}{4}\frac{mv_0^2}{k}\right)^{0.4}$, we obtain the following formula to determine the linear contact stiffness during the elastic deformation:

$$c_L = 0.84k^{0.8}m^{0.2}v_0^{0.4},$$

where k is a proportionality factor in the Hertz's formula; this factor can be determined by the well-known formula: $k = \frac{2E}{3\left(1-\mu^2\right)}\sqrt{\frac{R_1R_2}{R_1+R_2}}$, where E is the elastic modulus; μ – the Poisson ratio; R_1, R_2 – the radii of curvature of the impacting bodies; m – reduced mass of the impacting bodies, v_0 – the impact velocity.

An important parameter of the plastic models is the force F_S, at which the permanent strains appear in the material. When bodies interact with linear contact stiffness, the yielding force can be calculated with sufficient accuracy by the formula: $F_S = v_S\sqrt{c_Lm}$, where v_S is the impact velocity at which the plastic appear in the material of bodies.

The use of a nonlinear dependence $P = b\delta^n$, where b and n are the parameters depending on the physical and mechanical properties of metal at a given temperature and the shapes of bodies interacting in the contact zone, greatly complicates the problem of determining the impact force.

Let's linearize the dependence $P = b\delta^n$, from the condition of equality of potential energies of deformations for the nonlinear and linear systems (Fig. 4) at the moment of maximum compression [22], that is: $\int_0^{\max\delta} b_1\delta^q d\delta = \frac{b_1\delta_{\max}^{q+1}}{q+1} = \frac{\overline{c}\delta_{\max}^2}{2}$, from which the stiffness of a linear system will equal to: $\overline{c} = \frac{2b_1\delta_{\max}^{q-1}}{q+1}$.

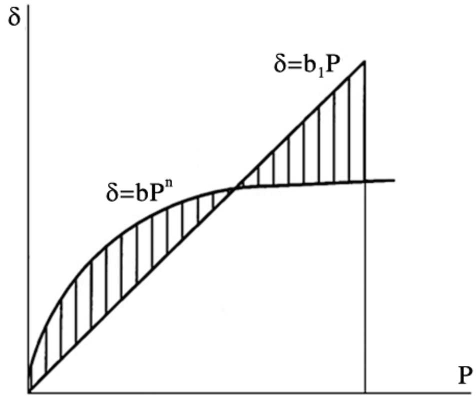

Fig. 4. Dependence of plastic deformation on the force.

In order to determine the stiffness of a linearized system, it is necessary to calculate the value of maximum bearing failure of the edges of the impacting bodies. The calculation can be carried out according to the formula given in work [20]:

$$\delta_{\max} = 0.7575\left(\frac{mv_0^2}{k}\right)^{0.4}\frac{Q}{q},$$

where m is reduced mass of the impacting bodies; v_0 – initial impact velocity; k – a proportionality factor in the Hertz's formula; q – the parameter equal to, $q = \frac{v_0}{v_S}$; v_S – minimum velocity, at which the first flow shears appear; Q – the parameter equal to $Q = \left[\frac{q}{2}(1+n)\right]^{\frac{1}{1+n}}$.

By inserting the value of maximum bearing failure of the edges of bodies into the stiffness formula, and calculating the stiffness and then compliance of a non-linearized system, we obtain the linear dependency $\delta = \lambda P$, the use of which greatly simplifies the calculations.

Thus, based on the above, we can conclude the following:

1. The phenomenological approach to solving the problems of impact interaction does not require expensive experimental work to determine the parameters of the strength function. 2. The parameters of the strength function can be determined by processing the stress-strain diagrams obtained during the mechanical tests of metals and alloys, which greatly facilitates the solution of the problem set. 3. After determining the strength function, it becomes possible to solve the basic differential equation of impact, which allows to determine the major parameters of the process of impact interaction of solids.

მექანიკა

დარტყმითი ურთიერთქმედების პარამეტრების განსაზღვრა დრეკად-პლასტიკური დარტყმის პროცესში

თ. ნატრიაშვილი*, ს. მებონია, მ. ბენ ხაიმი§**

** აკადემიის წევრი, სსიპ რაფიელ დვალის მანქანათა მექანიკის ინსტიტუტი, თბილისი, საქართველო*
*** სსიპ რაფიელ დვალის მანქანათა მექანიკის ინსტიტუტი, თბილისი, საქართველო*
§ არიელის საუნივერსიტეტო ცენტრი, არიელი, ისრაელი

ნაშრომში წარმოდგენილია დარტყმის პარამეტრების გამოთვლის მეთოდი დრეკად-პლასტიკური დარტყმითი ურთიერთქმედების შემთხვევაში. მოცემულია დარტყმითი ურთიერთქმედების მექანიკური მოდელი, რომელიც ითვალისწინებს დარტყმის სხეულების დამახასიათებელ თვისებებს და განასახიერებს შეჯახებადი სხეულების ძალთა ურთიერთქმედების პროცესს. ამ მეთოდის მიხედვით ძალოვანი ფუნქციის პარამეტრები განისაზღვრება ლითონებისა და შენადნობების მექანიკური გამოცდის დროს მიღებული დეფორმაცია-ძაბვის დიაგრამების დამუშავებით, რაც მნიშვნელოვნად ამარტივებს პრობლემის გადაჭრას და არ საჭიროებს ძვირადღირებული ექსპერიმენტული სამუშაოს ჩატარებას ძალოვანი ფუნქციის პარამეტრების დასადგენად.

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Received June, 2024