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Hydrology

Assessment Methods of Risk Factors Reservoirs Pollution

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The quality of water in reservoirs is determined by the presence of different chemical and biological elements in the water, the excessive concentration of which is caused by various anthropogenic factors. Among these factors are surface and subsurface runoffs from agricultural land areas, as well as water erosion. A complete statistical analysis of different water elements was carried out for reservoirs of Eastern Georgia (Samgori, Sioni, Tsalka), on basis of which statistical parameters of the observed series were obtained. Reliability characteristics have been established for these reservoirs. *© 2024 Bull. Georg. Natl. Acad. Sci.*

water quality, parametric assessment, water quality reliability

The quality of water in reservoirs is determined by the presence of different chemical and biological elements in the water, excessive concentration of which is caused by various anthropogenic factors. Methods for identification of these factors are not uniquely defined, and due to this, the assessment of reservoir water quality is ambivalent as well. Therefore, the assessment of water quality state needs distinctly different approach. For quantitative determination of reservoir water composition and properties the methods of reliability theory, which are successfully applied for solution of different problems of hydraulics and hydro-amelioration will be used [1-6].

Parametric assessment of the reliability of water quality in reservoirs. The reliability of water quality in reservoirs should be understood as the ability of the water in a reservoir to maintain the required quality within certain spatial boundaries, under specified conditions and for a certain period of time, i.e. such a probable state of water quality in which the concentration of various chemical and biological ingredients dissolved in water does not exceed the maximum permissible concentrations.

As far as water quality is determined by the concentration of each dissolved ingredient, the reliability of water quality has to be interpreted as parametric reliability. Parametric reliability is the probability that the concentrations of water quality ingredients will not exceed their maximum permissible concentrations during the service life.

$$P = p\left(Z \subset \vartheta\right) = \left\{R_{11} < Z_1 < R_{12}; R_{12} < Z_2 < R_{22}; ..., R_{i_1} < Z_i < R_{i_2} / t \le T\right\}, \quad (1)$$

where R_{11}, R_{12}, ... , R_{i_n} are maximum permissible concentrations of i-ingredient.

When the concentrations of different water quality ingredients exceed their maximum permissible concentrations in a given length of time, one may consider that this ingredient failure takes place in this moment. Let us assume that i-ingredient is subject to failure at moment $t = t_0$. It is thought. In reliability theory it is considered that t_0 is a random value, so it can be most fully characterized by the distribution function. It expresses the probability of the fact that i-ingredient experiences failure prior to moment *t*. It is frequently assumed in the theory of reliability that $q(t)$ function is continuous, as well as its first derivative, which is a probability density of a failure. The $q(t)$ function is frequently used along with function $P(t) = 1 - q(t) = P(t_0 \ge t)$, which expresses the probability of no failure (reliability) of i-ingredient in the t time interval. It is frequently called reliability function, as well.

It is necessary to establish the distribution law of i-ingredient reliability for determination of reliability function. For these purposes it is needed to determine the failure intensity (failure hazard) function, which is expressed by the following relation: $a(t) = P'(t) / P(t)$. From this relation the link between i-ingredient probability of no failure and failure intensity (failure hazard) follows:

$$P(t) = \exp\left\{-\int_0^t \alpha(x)\,dx\right\}. \quad (2)$$

For definition of reliability function it is necessary to establish the law of reliability distribution. It is known from the literary sources [1,3] that for analysis of such elements, which have passed a treatment period, the exponential distribution law is generally used, according to which mathematical expectation coincides with its mean square deviation. This fact is frequently used for checking the hypothesis on exponential distribution of reliability.

The normal distribution law is used in case of gradual changes of element parameters, or else when sudden failures constitute a small part of total number of observations. In addition, different branches of science use log-normal, Weibull, gamma-distribution, etc. as a reliability distribution.

When the concentrations of different ingredients of water quality exceed maximum permissible concentrations in a given length of time, it may be considered that at this moment failures of these ingredients take place. Failures of various ingredients can be taken as random values independent of each other. In such cases, approximate reliability of reservoir water quality, or probability of no failure in the t time interval can be expressed as a product of random and independent events:

$$P(t) = \prod_{i=1}^{n}\left(P_i(t)\right), \quad (3)$$

where $P_i(t)$ is probability of different ingredients' event; $P(t)$ is a total probability, which allows us to establish a failure-free operation of reservoir water quality prior to first failure. Failure intensities are distributed by any law in this case.

Using this relation, there has been calculated the probability of no failure of nitrogen compounds (NO_2, NH_4) in waters of two – Samgori and Sioni – reservoirs. Let us remark here that it is seen from the results that probability of no failure is subject to exponential distribution. Therefore, NO_2 and NH_4 reliability function is expressed as follows:

$$P = \exp(-\alpha t). \tag{4}$$

Reliability characteristics of NO_2 and NH_4 calculated by means of the relation (4) for Samgori and Sioni reservoirs are given in Table 1.

Table 1. Reliability characteristics for Samgori and Sioni reservoirs

		Reliability probability, %	Approximate reliability, %
Samgori reservoir	NO_2 NH_4	78.9 86.6	68.3
Sioni reservoir	NO_2 NH_4	76.6 89.6	68.6

As one can see, nitrogen compounds reliability in both reservoirs is very low, so it is necessary to take all possible actions to increase reliability of each of elements.

When calculating parametric reliability, one cannot neglect time-to-time variability of parameters. In this case, it is possible to use the theory of random function outliers for calculation of the reliability of reservoir water quality [4].

Applying the theory of outliers requires fulfillment of definite conditions. In particular, i-ingredient variability has to be subject to normal distribution law. The process has to be continuous and differentiable. It is necessary to establish a predicted level, to relation of which one has to search the outliers of random function $K(t)$. It is possible to take maximum permissible concentration (MPC) of i-ingredient as such level.

Outlier probability, i.e. the chance of the fact that $K(t)$ function will cross MPC within dt time interval, may be expressed as follows:

$$P\left[K(t) < K_{zdk}; K(t+dt) > K_{zdk}\right]. \tag{5}$$

As far as i-ingredient variability is a differentiable process, then

$$K(t+dt) = K(t) + K'(t)dt, \tag{6}$$

where $K'(t)$ is a rate of i-ingredient concentration variability, therefore

$$K'(t) = \frac{dK}{dt}, \tag{7}$$

from where $K(t+dt) > K_{zdk}$. This expression is equivalent to the following equation: $K_{zdk} - K'(t)dt < K(t)$. Instead of these two inequations we can write down one two-sided inequality:

$$K_{zdk} - K'(t)dt < K(t) < K_{zdk}. \tag{8}$$

In order to calculate the two-sided inequality (8), it is necessary to know two-dimensional distribution function $f(K, K')$. If $f(K, K')$ function is known, then probability of outliers may be written down in the form of the following expression:

$$P\left[K_{zdk}-K'(t)dt<K(t)<K_{zdk}\right]=\int_0^{\infty}\int_{K_{zdk}-K'(t)}^{K_{zdk}} f(K,K')\,dK\,dK'. \quad (9)$$

Using the mean-value theorem and performing a simple mathematical transformation, we obtain a mean number of outliers per unit of time

$$n_{K_z}=\int_0^{\infty}K'f\left(K_{zdk},K'\right)dK'. \quad (10)$$

In case of a stationary differential normal process, K_{zdk} and K' values don't depend to each other, therefore

$$f\left(K_{zdk},K'\right)=\frac{1}{\sigma_K\sqrt{2\pi}}\exp\frac{-\left(K_{zdk}-M_K\right)^2}{2\sigma_K^2}\frac{1}{\sigma_{K'}\sqrt{2\pi}}\exp\frac{-K^2}{2\sigma_K^2}, \quad (11)$$

where M_K is a mathematical expectation of i-ingredient, while σ_K and $\sigma_{K'}$ are mean square deviation of the random $K(t)$ process and its first derivative.

Plugging (11) in (9), we obtain:

$$n_{K_z}=\frac{1}{2\pi}\frac{\sigma_{K'}}{\sigma_K}\exp\frac{-\left(K_{zdk}-M_K\right)^2}{2\sigma_{K^2}} \quad (12)$$

or

$$n_{K_z}=\overline{n}_{\overline{K}}\exp\frac{-\left(K_{zdk}-M_K\right)^2}{2\sigma_{K^2}}. \quad (13)$$

$$\overline{n}_{\overline{K}}=\frac{N_0}{t_0}, \quad (14)$$

where N_0 is an average number of zeroes of the random $K(t)$ process in t_0 time. For its establishment it is necessary to count the number of K_{zdk} level intersections by the $K(t)$ process curve.

Formulas (12) and (13) are the equations describing variability of average number of outliers, which were derived for the first time by Rice [1,2,6]. At the same time, they are the fundamental equations of statistical mechanics.

The first derivative n'_K of mean square deviation of $K(t)$ process may be determined using the following relation [4,5]:

$$\sigma'_K=\frac{\sqrt{2}}{\Delta}\sigma_K\sqrt{1-R_\tau}, \quad (15)$$

where R_τ is a normal correlation function of $K(t)$ process for $\tau\approx\Delta$ value; τ – interval duration. $R_\tau=K(\tau)/\sigma^2k$. $K(\tau)$ is an average value of correlation function for $\tau=1$ value.

Average duration of outliers τ_k and average period between outliers $\overline{\tau}$ are the important characteristics of a random function. One can use the following formulas for establishment of these characteristics:

$$\overline{\tau}_K=\pi\frac{\sigma_K}{\sigma_{K'}}\left[1-\Phi\left(\frac{K_z-M_K}{\sigma_K}\right)\exp\frac{-\left(K_z-M_K\right)^2}{2\sigma_K^2}\right], \quad (16)$$

$$\overline{\tau}=2\pi\frac{\sigma_K}{\sigma_{K'}}\Phi\left(\frac{K_z-M_K}{\sigma_K}\right)\exp\frac{\left(K_z-M_K\right)^2}{2\sigma_K^2}. \quad (17)$$

Excess of i-ingredient concentration over maximum permissible concentration is of rare occurrence and one can use Poisson law for its forecasting, according to which exceedance of i-ingredient concentration over maximum permissible concentration may be written down as follows:

$$P = \frac{n_{K_z}^m \exp\left(-n_{K_z} t\right)}{m}, \tag{18}$$

where $n_{K_z} - K(t)$ is the average number of function outliers, and m us a number of $K(t)$ excesses during t time.

In order to exclude i-ingredient concentration excess over maximum permissible concentration, i.e. not to allow water pollution with i-ingredient, it is necessary that $m = 0$. Therefore, the expression (18) will be as follows:

$$P = \exp\left(-n_{K_z} \cdot t\right). \tag{19}$$

If we solve equations (9) and (19) simultaneously for $\bar{m}_K$, we obtain

$$\bar{M}_K = \frac{K_{zdk}}{1 + \left[\dfrac{-2\sigma_K^2 \ln\left(\dfrac{-\ln P}{n_z \cdot t}\right)}{M_K^2}\right]^{1/2}}. \tag{20}$$

Based on this:

$$\eta_0 = 1 + \left[\frac{-2\sigma_K^2 \ln\left(\dfrac{-\ln P}{n_z \cdot t}\right)}{m_K^2}\right]^{1/2}. \tag{21}$$

η_0 is the characteristic parameter of reliability, which was obtained for the first time by Ts. Mirtskhulava with the purpose of forecasting of erosion processes and deformations [1,2].

For its calculation it is necessary to establish statistical parameters $\left(M_K, \sigma_K, \sigma_{K'}\right)$ of i-ingredient set of variate series and average number of outliers $\left(n_K\right)$.

Results

For calculation of the first derivative $\left(\sigma_{K'}\right)$ of mean square deviation of some polluting elements of three reservoirs of Eastern Georgia (Samgori, Sioni and Tsalka reservoirs), the correlation functions of these elements have been determined. The values for $\tau = 1$ step are given in Table 2, which shows an increase in the probability of no failure leads to the improvement of reliability characteristics.

In the end, we can state that a reliability characteristic η_0, at the same time is a function of reliability, so we can forecast the probability of i-ingredient concentration excess over its maximum permissible norm.

The given method has to be considered as a first step on the way, by means of which we have to develop a new method of reservoirs water pollution forecasting, which is based on reasonable risk-factors and some issues of theory of reliability and statistical mechanics have been used for its creation.

Table 2. Parameters of reliability characteristic some Georgia's reservoirs

Element	M_K	σ_K	$\sigma_{K'}$	n_K	P			K_{zdk}
					0.1	0.5	0.9	
					η_0	η_0	η_0	
Samgori reservoir								
Bco	1.88	1.36	0.15	0.012	1.76	1.88	2.01	3.0
NH_4	0.43	1.41	0.014	0.0053	1.15	1.27	1.40	0.40
NO_2	0.019	0.024	0.0018	0.011	1.53	1.65	1.78	0.02
Cu	0.0049	0.0052	0.0002	0.006	1.02	1.12	1.25	0.001
Oil	0.039	0.060	0.0049	0.0128	1.43	1.54	1.66	0.05
Sioni reservoir								
Bco	1.81	1.10	0.029	0.0023	1.72	1.83	1.95	3.0
NH_4	0.59	0.60	0.040	0.0099	1.09	1.20	1.33	0.40
NO_2	0.017	0.019	0.014	0.0388	1.59	1.70	1.83	0.02
Cu	0.006	0.009	0.0005	0.0071	1.01	1.10	1.19	0.001
Oil	0.037	0.042	0.0033	0.0001	1.37	1.49	1.61	0.005
Tsalka reservoir								
Bco	2.43	0.98	0.053	0.0073	1.34	1.47	1.59	3.0
NH_4	1.03	0.42	0.023	0.0028	1.24	1.35	1.46	0.40
NO_2	0.026	0.015	0.0007	0.0069	1.23	1.34	1.46	0.02
Cu	0.011	0.012	0.00009	0.00084	1.06	1.17	1.29	0.001
Oil	0.078	0.072	0.0038	$0.4 \cdot 10^{-4}$	1.05	1.16	1.28	0.05

Conclusion

A complete statistical analysis of different water elements was conducted for reservoirs of Eastern Georgia (Samgori, Sioni, Tsalka) on basis of which statistical parameters of the observed series have been obtained. The issues of reliability theory were used for quantitative determination of reservoir water composition and properties. Such probable state of water quality determining chemical elements is taken as the water quality reliability, when their concentrations do not exceed the maximum permissible concentrations. The reliability of water quality is assessed as a parametric one using the theory of outliers of random function and reliability characteristics are established for each reservoir.

ჰიდროლოგია

წყალსატევების გაჭუჭყიანების რისკ-ფაქტორების შეფასების მეთოდები

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წყალსატევების წყლის ხარისხი განპირობებულია წყალში არსებული სხვადასხვა ქიმიური და ბიოლოგიური ელემენტებით, რომელთა მოჭარბებული კონცენტრაცია გამოწვეულია სხვადასხვა ანთროპოგენური ფაქტორით. ერთ-ერთი ასეთი ფაქტორია სასოფლო-სამეურნეო სავარგული მიწებიდან, როგორც ზედაპირული, ასევე მიწისქვეშა ჩამონადენი და წყლისმიერი ეროზია. აღმოსავლეთ საქართველოს წყალსაცავებისათვის (სამგორის, სიონის, წალკის) გაკეთდა სრული სტატისტიკური ანალიზი წყლის სხვადასხვა ელემენტისათვის, რომლის საფუძველზეც მიღებულ იქნა დაკვირვებული რიგების სტატისტიკური პარამეტრები. წყალსაცავებისთვის დადგინდა საიმედოობის მახასიათებლები.

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