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Biomechanics

Determination of Analytical Relationship between Peak Loads on Foot's Plantar Part, Foot Support Area and Patient's Weight in Children with CP

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(Presented by Academy Member Temur Naneishvili)

This paper presents pedographic studies of children with cerebral palsy. As a result of this research, peak pressure on the plantar part of the foot, the supporting area of the foot and the patient's body weight have been determined. Analytical relationship of these parameters is established and regression equation is derived which shows the connection between the relative deformation of insole materials and experimental factors. The proposed mathematical model makes it possible to select in advance the materials of various strength necessary in the production of individual orthopedic insoles for a concrete patient, depending on the numerical values of the factors. This is especially important in the process of designing and manufacturing individual orthopedic insoles. The paper also presents geometric interpretation of the regression equation in the Cartesian right-handed coordinate system. Research in this area is particularly significant for children with cerebral palsy. *© 2025 Bull. Georg. Natl. Acad. Sci.*

orthopedic insoles material, children cerebral palsy, pedograph method

Cerebral palsy (CP) comprises a group of clinical syndromes that affect an individual's ability to move and coordinate effectively throughout their life. In children, CP develops as a result of irreversible brain damage affecting their ability to control muscles. This is often manifested through motor disorders, namely, paralysis and abnormal muscle tone, either decreased or increased. When gross motor skills are impaired, the activities involving large muscles, such as sitting, standing, climbing, maintaining balance, running, etc., become extremely difficult [1-5].

According to the Center for Disease Control and Prevention (CDC), cerebral palsy is the most common motor disability in children. Recent studies indicate that the incidence rate varies from 1 to 4 per 1000 children, with approximately 18 million individuals of all ages affected worldwide.

Determining the specific form of cerebral palsy is extremely important in terms of prognosis and development of a treatment program. The approach to the issue is individual and includes the development of an effective combination of pharmacological treatments, rehabilitation and orthopedic interventions. Early and targeted intervention in the treatment process ensures partial prevention of muscle, tendon and bone deformations and contractures.

In the treatment of children with cerebral palsy, it is very important to provide patients with individual orthopedic insoles (in order to maintain balance and redistribute local loads on the plantar part of the ankle). Orthoses, which are highly recommended, also play a vital role in managing the condition effectively.

In Georgia, the demand for individual orthopedic devices is a pressing issue raised by young patients and their parents, as well as pediatricians and orthopedic traumatologists. Research in this area is particularly significant, as orthoses currently manufactured in the country rely on outdated technologies, without insoles, and they do not meet individual needs of a concrete patient. As a result, orthoses manufactured in this way in most cases fail to ensure the correct position of the lower limb and effective management of the treatment.

In Western countries, lower limb orthoses are mainly made from polymer materials, in which the foot is placed on the non-relief and rigid surface of the orthosis, which often causes pain and discomfort. Children often refuse wearing such orthoses. To address this issue, it is essential to manufacture orthoses equipped with individual orthopedic insoles according to individual data taken on a pedograph. A multifunctional orthosis made using in tailored to individual measurements obtained through pedographic analysis. A multifunctional orthosis made using innovative technologies, in combination with an insole, can significantly improve the quality of treatment and comfort for patients with cerebral palsy (CP).

Research Methods and Materials

When designing individual orthopedic insoles, it is essential to take into account individual patient data, particularly, peak loads on the plantar part of the ankle, the supporting area of the foot and patient's weight. The above data were collected using a pedograph Emed -25 at/D manufactured by the Novel company (Munich, Germany). Based on the relevance of the research, a fundamental requirement in the design of personalized orthopedic insoles is to establish an analytical correlation between the mentioned factors in order to select appropriate materials [6, 7].

Mathematical research method. Due to the specifics of the research object, the initial task was divided into two subtasks: the development of a mathematical model of the research object and study of the extrema of the mathematical model.

The first subtask can be solved using mathematical methods of experiment design, while the second subtask – by finding the extrema of the function.

Based on the study of the research object, it is determined that the output parameter is the relative deformation of the supinator (y, %). This parameter is crucial and central in the process in the design of insoles as it directly influences the selection of materials for the insole and the process of treating patients. Given the characteristics of the research object, the first subtask aims to determine the analytical relationship between the following factors, namely: the peak load on the plantar part of the ankle – x_1 (kPa), ankle support area – x_2 (cm^2) and x_3 – patient weight (kg),

$$y = f\left(x_1,\ x_2,\ x_3\right). \tag{1}$$

To solve the first subtask in the study, a mathematical research method was used, namely the second-order rotational planning method [8-12]. According to experimental studies, the dependence of the relative deformation of the insoles on the above factors is nonlinear. Therefore, the research process can be represented as a second-order polynomial:

$$y = b_0 + \sum_{i=1}^{n} b_i x_i + \sum_{i,j=1}^{n} b_{ij} x_i x_j + \sum_{i=1}^{n} b_{ii} x_i^2, \tag{2}$$

where b_0 is the free term of the regression equation; b_i, b_{ij} and b_{ii} are single, double and quadratic coefficients of the interaction factors; x_1, x_2 and x_3 represent the factors influencing the deformation process of the insole materials. The coefficients of the regression equation were calculated using the following formulas:

$$b_0 = a_1 \sum_{i=1}^{N} y_i - a_2 \sum_{i=1}^{N} \sum_{j=1}^{k} x_{ij}^2 y_i, \tag{3}$$

$$b_i = a_3 \sum_{i=1}^{N} x_i y_i, \tag{4}$$

$$b_{ij} = a_4 \sum_{i=1}^{N} \sum_{j=1}^{k} x_i x_j y_i, \tag{5}$$

$$b_{ii} = a_5 \sum_{i=1}^{N} x_{ii}^2 y_i + a_6 \sum_{i=1}^{N} \sum_{j=1}^{k} x_{ij}^2 y_i - a_7 \sum_{i=1}^{N} y_i. \tag{6}$$

The values α_1, α_2, α_3 ... α_7 in the formulas for calculating the coefficients of the regression equation are taken from special tables [11, 12] depending on the number of factors and the number of tests. At the first stage, we made an experimental plan based on a number of factors.

Ethylene vinyl acetate (EVA) polymer materials of various strengths were used in the research process.

Research Results

The experiment involved 60 patients with CP. The peak load on the plantar part of foot in this population was in the following range $-x_1 = 153 - 187$ kPa (kPa), the supporting area of the foot $-x_2 = 82 - 98$ cm^2 and the weight of the patients $-x_3 = 22 - 38$ kg in the interval.

Table. Indicators of experimental factors (x_1, x_2 and x_3) in coded and natural units

Factors	The level of factors variation					Variation interval
In coded units	-1.68	-1	0	+1	+1.68	
In natural units Peak load on the plantar part of the foot x_1, kPa.	153	160	170	180	187	10
The supporting area of the foot x_2, kPa.	82	85	90	95	98	5
The weight of the patients x_3, cm^2.	22	25	30	35	38	5

The experiment matrix was composed of 20 trial data and 8 parallel trials. The number of factors was k=3, with a total number of trials N=20, the number of zero points of the plan n_0=6, the core points of the plan n_b=8 and the end points of the plan n_k=6.

Using the calculation formulas (3-6) for the coefficients of the regression equation along with a program compiled on the computer, the coefficients with the following values were computed:

$b_0 = a_1 \cdot 138.6 - a_2 \cdot (77.97 + 74.3 + 74.1) = 10.19$

$b_1 = a_3 \cdot 2.5715 = 0.19$

$$b_2 = a_1 \cdot 2.609 = 0.191$$
$$b_3 = a_1 \cdot 22.6 = 1.65$$
$$b_{12} = a_4 \cdot (-0.2875) = -0,04$$
$$b_{13} = a_4 \cdot (-0.6625) = -0.08$$
$$b_{23} = a_4 \cdot (-0.6875) = -0.09$$
$$b_{11} = a_5 \cdot 77.97 + a_6 \cdot (77.97 + 74.3 + 74.1) - a_7 \cdot 138.6 = -1.44$$
$$b_{22} = a_5 \cdot 74.3 + a_6 \cdot (77.97 + 74.3 + 74.1) - a_7 \cdot 138.6 = -0.72$$
$$b_{33} = a_5 \cdot 74.1 + a_6 \cdot (77.97 + 74.3 + 74.1) - a_7 \cdot 138.6 = -1.68\,.$$

Based on the coefficients, the regression equation is derived in the following form:

$$y(x_1, x_2, x_3) = 10.19 + 0.19x_1 + 0.191x_2 + 1.65x_3 - 0.04x_1x_2 - 0.08x_1x_3 - 0.09x_2x_3 - 1.44x_1^2 - 0.72x_2^2 - 1.68x_3^2. \quad (7)$$

The coefficients of the regression equation were estimated using the Student's criterion. All coefficients are within the range of $b_i - \Delta b_i$ and $b_i + \Delta b_i$. Therefore, all coefficients are significant and cannot be excluded from the equation. The homogeneity of the dispersion of the data obtained as a result of the tests was checked using the Cochrane coefficient and the adequacy of the regression equation using the Fisher coefficient. The calculated values of the Cochrane and Fisher criteria are given below:

$$K_k = 0.14 \quad \mathrm{K}_F = 0.38.$$

The Cochrane and Fisher criteria do not exceed 1, and the reliability probability is 95%. Based on this, we can conclude that the resulting mathematical model is adequate to the experiment. All factors of the experimental space are important, which is confirmed by the presence of factors x_1, x_2 and x_3 in the regression equation.

Based on the obtained results (regression equation is meant), we conducted the following studies to determine the optimal values of the factors – addressing the second subtask: finding the extrema of the obtained function. We solved a second-order differential equation (mathematical model) derived from a system of partial differential equations, which expressed in the following form:

$$\begin{cases} \dfrac{\partial y}{\partial x_1} = 0.19 - 0.04x_2 - 0.08x_3 - 2.88x_1 = 0 \\ \dfrac{\partial y}{\partial x_2} = 0.191 - 0.04x_1 - 0.09x_3 - 1.44x_2 = 0 \\ \dfrac{\partial y}{\partial x_3} = 1.65 - 0.08x_1 - 0.09x_2 - 3.36x_3 = 0 \end{cases} . \quad (8)$$

The solutions of the system of equations in coded units have the following values: $x_1 = 0.051$, $x_2 = 0.101$ and $x_3 = 0.487$, and in natural units: $x_1 = 169.5$ kPa (kPa), $x_2 = 89.5$ cm^2 and $x_3 = 27.6$ kg.

Figures 1-3 show the geometric interpretation of the mathematical model (7) in the Cartesian right-hand coordinate system. Figure 1 illustrates the dependence of the relative deformation of the insole material between the peak load on the plantar part of the foot and the supporting area of the foot. Figure 2 shows the dependence of the relative deformation of the insole material between the peak load on the plantar part of the foot and the patient's weight. Figure 3 shows the dependence of the relative deformation of the insole material between the supporting area of the foot and the patient's weight.

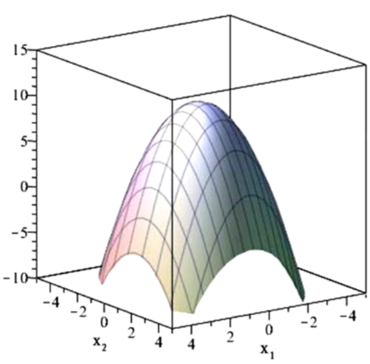

Fig. 1. Dependence of the relative deformation of the insole material between the peak load on the plantar part of the foot and supporting area of the foot.

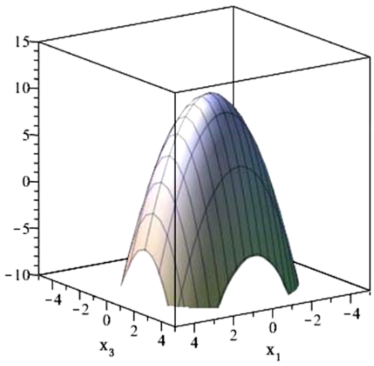

Fig. 2. Dependence of the relative deformation of the insole material between the peak load on the plantar part of the foot and patient's weight.

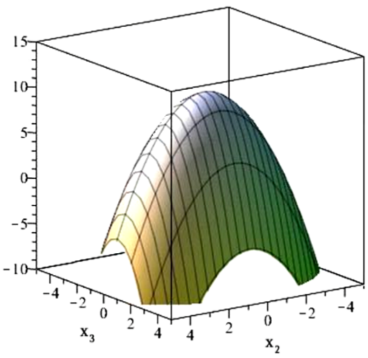

Fig. 3. Dependence of the relative deformation of the insole material between supporting area of the foot and patient's weight.

Discussion

Thus, as a result of the pedographic study of patients with CP, the peak loads on the plantar part of the foot, the dimensions of the supporting area of the foot and patients's weight were determined. Based on the use of mathematical research methods, an analytical relationship was established between the peak loads on the plantar part of the foot, the support area of the foot and the patient's weight. The regression equation is an analytical relationship between the relative deformations of the material – y and the factors – x_1, x_2, x_3. The obtained mathematical model is adequate to the experiment, as evidenced by the values of the Student, Cochran and Fisher coefficients. On the basis of the conducted experiment, it was determind that the optimal values of the coefficients $x_1 = 169.5$ kPa, $x_2 = 89.5$ cm^2 and $x_3 = 27.6$ kg correspond to a relative deformation of the materials y=7-8%. Based on the numerical values of the factors, relative deformation of materials, is optimal, which is confirmed by the regression equation.

Conclusion

The mathematical model applied in this experiment makes it possible to determine in advance relative deformations and bending resistance of the instep materials based on the numerical values of the factors. This is especially important in the process of designing and manufacturing insteps. Such research is particularly significant when it comes to regulating health problems in the treatment of patients with cerebral palsy.

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ბიომექანიკა

ანალიზური კავშირის დადგენა ტერფის პლანტარულ ნაწილზე მოსულ პიკურ დატვირთვებს, ტერფის საყრდენ ფართსა და პაციენტის წონას შორის ცერებრული დამბლით დაავადებულ ბავშვებში

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(წარმოდგენილია აკადემიის წევრის თ. ნანეიშვილის მიერ)

ნაშრომში წარმოდგენილია ცერებრული დამბლით დაავადებული ბავშვების პედოგრაფიული კვლევები. კვლევის შედეგად დადგენილია ტერფის პლანტარულ ნაწილზე მოსული პიკური დატვირთვები, ტერფის საყრდენი ფართი და პაციენტის წონა. დადგენილია ანალიზური კავშირი ტერფის პლანტარულ ნაწილზე მოსულ პიკურ დატვირთვებს, ტერფის საყრდენ ფართსა და პაციენტის წონას შორის. მიღებულია რეგრესის განტოლება, რომელშიც ასახულია ანალიზური კავშირი სუპინატორების მასალების ფარდობით დეფორმაციასა და ექსპერიმენტის ფაქტორებს შორის. მიღებული მათემატიკური მოდელი საშუალებას გვაძლევს წინასწარ შევარჩიოთ კონკრეტული პაციენტისათვის ინდივიდუალური ორთოპედიული სუპინატორებისთვის საჭირო სხვადასხვა სიმაგრის მასალა ფაქტორების რიცხობრივ სიდიდეებზე დამოკიდებულებით, რაც განსაკუთრებით მნიშვნელოვანია ინდივიდუალური ორთოპედიული სუპინატორების პროექტირებისა და დამზადების პროცესში. ნაშრომში წარმოდგენილია ასევე რეგრესის განტოლების გეომეტრიული ინტერპრეტაცია დეკარტეს მარჯვენა კოორდინატთა სისტემაში. აღნიშნული მიმართულებით კვლევა განსაკუთრებით აქტუალურია, როდესაც საქმე ეხება ცერებრული დამბლით დაავადებულ ბავშვებს.

REFERENCES

1. Cerebral palsy. National clinical practice recommendation (Guideline). 2008 https://www.moh.gov.ge/uploads/guidelines/2017/06/08/2a8eb834ae63b2d457b5646187a3ba69.pdf
2. Leonard R., Sweeney J., Damiano D., Bjornson K., Ries J. (2021) Effects of orthoses on standing postural control and muscle activity in children with cerebral palsy. Pediatric physical therapy: The Official Publication of the Section on Pediatrics of the American Physical Therapy Association, **33**(3): 129.
3. Yates H. (ed.) (2014) Handbook on cerebral palsy: risk factors, therapeutic management and long-term prognosis, 285, Nova Biomedical.
4. Kane K.J., Lanovaz J.L. and Musselman K.E. (2019) Physical therapists' use of evaluation measures to inform the prescription of ankle-foot orthoses for children with cerebral palsy. *Physical & Occupational Therapy in Pediatrics*, **39**(3): 237-253.
5. Galli M., Cimolin V., Pau M., Leban B., Brunner R., Albertini G. (2015) Foot pressure distribution in children with cerebral palsy while standing. *Research in Developmental Disabilities*, 41: 52-57. doi: 10.1016/j.ridd.2015.05.006.
6. Shalamberidze M., Sokhadze Z., Tatvidze M. (2021) Projection of orthopedic supinators considering the loads on locally over-pressure areas. *Bull. Georg. Natl. Acad. Sci*., **15**(2): 74-81.
7. Shalamberidze M., Tatvidze M. (2023) Pedographic studies of foot deformities in children with cerebral palsy and establishment of the analytical relationship between the foot support area and peak pressure. *Bull. Georg. Natl. Acad. Sci.,* **18**(1): 136-141.
8. Ivanovsky R.I. (2008) Teoria veroiatnosti i matematicheskaia statistika. Osnovy, prikladnye aspekty primerami i zadachami v srede (Probability theory and mathematical statistics. Basics, applied aspects with examples and problems in the mathcad), 332. M. (in Russian).
9. Nivorozhkina L.I., Morozova Z.A. (2005) Matematicheskaia statistika s elementami teorii veroiatnosti v zadachakh s resheniiami (Mathematical statistics with elements of probability theory in problems with solutions), 193. M. (in Russian).
10. Spirina M.S., Spirin P.A. (2007) Teoria veroiatnosti i matematicheskaia statistika (Probability theory and mathematical statistics), 127. M. (in Russian).
11. Spirin N.A., Lavrov V.V., Zajnullin L.A., Bondin A.R., Burykin A.A. (2015) Metody planirovaniia i obrabotki rezul'tatov inzhenernogo eksperimenta: Uchebnoe posobie / N.A. Spirin, V.V. Lavrov, L.A. Zajnullin, A.R. Bondin, A.A. Burykin; Pod obshch. red. N.A. Spirina. – Ekaterinburg: OOO «UINC», 290 s. ISBN 978-5-9904848-4-9.
12. Akhnazarova S.L., Kafarov V.V. (1985) Optimizatsia eksperimenta v khimii i khimicheskoi tekhnologii (Optimization of experiment in chemistry and chemical technology), 327. M.: Vysshaia shkola (in Russian).

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