Informatics

Application of Stochastic Optimization Method for Solving Linear and Nonlinear Mathematical Programming Problems

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Abstract. This study discusses the possibility of using a novel stochastic optimization method for solving linear and nonlinear mathematical programming problems. The mathematical model of optimization problems of many technical objects, including building structures, can be represented as a linear and nonlinear mathematical programming problem. In most cases, the so-called classical method, known as the "simplex method", is used to solve linear mathematical programming problems. However, this approach has notable drawbacks, particularly its computational complexity and routine nature of the calculations. There is no single, general method for solving nonlinear mathematical programming, and all of the existing methods are individual and require careful consideration of the target function, which significantly limits their applicability. In the case of a complex configuration of target function, these difficulties may be multiple parameters and multiple extrema of target function, including the presence of a global extremum, discontinuity of the target function, the presence of "saddle" and "ravine" type points and many other types of difficulties that necessitate additional measures. Theoretical justification for creating a single general method that would allow us to solve the above problems using one general method is logical, but an unambiguous answer for such cases has not yet been found. However, an attempt is made to overcome these difficulties relatively easily and to use the stochastic optimization method to solve both linear and nonlinear mathematical programming problems. The paper presents mathematical models of two real objects for which a set of computer programs has been developed to obtain numerical solutions. © *2025 Bull. Georg. Natl. Acad. Sci.*

Keywords: target function, linear nonlinear mathematical programming, stochastic optimization method

Introduction

Let us formulate a general optimization problem. Assume we have a target function of a certain object expressed in the following form:

$$F(x) = F(x_1, x_2, ..., x_n),$$

where $x_1, x_2, ..., x_n$ are the parameters of this function and there are restrictions on them, which can be presented in the form of both equations and inequalities:

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n &= b_2 \\ &\dots\dots\dots\dots\dots\dots \\ a_{n1}x_1 + a_{n2}x_2 + ... + a_{nn}x_n &= b_n \end{aligned}.$$

The task is to find the extreme value of the target function and those values of the corresponding parameters that ensure extreme value of this function. To achieve this, the following steps are undertaken:

1. Initially, all the necessary data required for the operation of the method are defined including the target function $F(x) = F(x_1, x_2, ..., x_n)$; all existing restrictions $G_j \;\; j = 1,2,...,M$ and their total number (*M*); the optimization parameters $x_i, \;\; i = 1,2,3,...,V$ and their number (*V*); the lower and upper bounds of the possible change of parameters; the number of statistical trials (*S*); an extremely small or large value of *R* (if needed).
2. The process of conducting statistical trials begins.
3. The random numbers generator Rnd( ) will cyclically start producing random numbers, which subsequently (using the appropriate algorithm) are transformed into search parameters;
4. The values of all restrictions will be calculated and checked on the cyclically required parameter, respectively;
5. If all restriction conditions are simultaneously satisfied, the value of the target function *F* will be calculated and tested on the condition $F > R$ (or $F < R$). If this condition is fulfilled, then R is given the value of *F*, which will be stored in the computer memory along with other search options to use the next cycle as a new value of the *R*-variable. Otherwise, if the condition $F > R$ or $F < R$ is not fulfilled, the program returns to step 2 to continue the search process.

The above process will be repeated until the number of statistical trials exceeds *S*. The final result of the program will be the minimum (or maximum) value of the target function along with the coordinates of the extremum point, i.e., the values of the search parameters.

To clearly illustrate the essence of the method, it is helpful to consider specific examples (linear and nonlinear) and solve them using the optimization method by the appropriate computer programs.

Let us determine the optimal amount of water purified by the water cleaning station applying the method of linear mathematical programming. Task 1. Suppose we have a water cleaning station with a specified product, from which we need to get two different qualities of water for drinking and technical purposes. The complete description of the object is given in [1]. Here, we present only its mathematical model which is as follows:

$$\begin{aligned} &x_1 + x_2 \le 3500 \\ &0.17 \cdot x_1 + 0.07 \cdot x_2 \le 395 \\ &0 \le x_1 \le 2000 \\ &0 \le x_2 \le 2500, \end{aligned} \qquad (1)$$

where x_1 and x_2 denote the amounts of purified drinking water and purified technical water, respectively, which should not be negative values. The target function, which reflects the income from the water supply is to be maximized, and is written as

$$F = 0.037x_1 + 0.028x_2 \to \max. \qquad (2)$$

Formulas (1), (2) represent a linear mathematical programming problem in this particular case. This problem can be solved graphically, analytically, or by applying the simplex method. It can also be easily solved using WOLFRAM programming.

To solve the system of equations (1) the solution of the first two equations can be expressed as follows:

```
In[1]:= Solve[{ x1 + x2 == 3500, 0.17*x1 + 0.07*x2 == 395}, {x1, x2}]
Out[1]=
        {{x1 → 1500., x2 → 2000.}}
```

This solution with account of other conditions creates a controlled polygon. Any point in this polygon represents a set of permissible solutions as it satisfies all four conditions of the expression (1). As to the values x_1 and x_2, they correspond to those values of the permissible solutions that give the maximum value to the target function.

If we put the values of x_1 and x_2 in the formula (2) and calculate the values of the target function, we have:

```
In[1]:=
        f = 0.037 * 1500 + 0.028 * 2000
Out[1]=
        111.5
```

The value of the obtained F is an optimal solution to the problem. Figure 1 illustrates a graphical interpretation of the restrictions.

Table 1. The results obtained by stochastic optimization method

x_1	x_2	Target function value F
1411.095	1333.56	89.551
747.072	2404.883	94.978
1899.113	910.0467	95.748
1832.328	1075.653	97.914
1192.183	2081.825	102.402
1095.337	2307.386	105.134
1492.709	1880.777	107.892
1476.247	1950.139	109.225
1401.784	2077.095	110.024
1542.084	1896.43	110.157
1436.817	2056.5	110.744
1469.45	2020.489.	110.943
1508.457	1917.976	111.196
1487.318	2012.078	111.338
1494.93	2004.592	111.441
1497.999	2000.712	111.445
1495.756	2003.936	111.453
1496.169	2003.608	111.459
Optimum		
1496.169	2003.608	111.459

```
Plot[{3500 - x1, (395 - 0.17 * x1) / 0.07, (111.5 - 0.037 * x1) / 0.028,
  2500}, {x1, 0, 2000}]
```

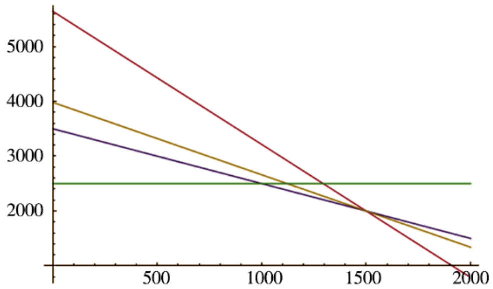

Fig. 1. Graphical interpretation of the restrictions and target function.

Application of stochastic optimization method to solve nonlinear mathematical programming problems. Task 2. A cylindrical transverse carving is shown in Fig. 2. It is required that the cylinder bears the load P acting on it.

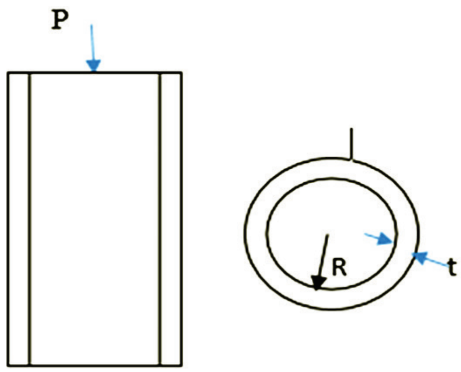

Fig. 2. A cylindrical transverse carving

The purpose of optimization is to determine the values of the R and t parameters that ensure the minimum weight $G_{\min}$ of the cylinder. In case of tensions, sustainability, Euler power of losing sustainability, as well as restrictions on local outbreaks transverse carving options are:

$$A = \pi(R+t)^2 - \pi R^2 = \pi(2Rt + t^2),$$

where A is an area of transverse intersection of an object.

The moment of inertia is equal to :

$$I = \pi R^3 t.$$

The Euler critical load P_k of the loss of stability is equal to:

$$P_k = \frac{\pi^2 E.I}{4L^2} = \frac{\pi^3 ER^3 t}{4L^2},$$

where L is the height of the column; E-Young's modulus. The weight of the column is calculated by the formula:

$$G_{\min} = \pi(2Rt + t^2)L\rho g,$$

where pg is specific weight of the material.

The axial tension of a σ_c column is defined by the expression:

$$\sigma_c = \frac{P}{A} = \frac{P}{\pi\left(2R + t^2\right)}.$$

The axial critical tension σ_{ck} of losing sustainability is equal

$$\sigma_{ck} = kEt / R,$$

where k coefficient, the steel approximately equals 0.6, the local sustainability restriction will be expressed as the following inequality:

$$\sigma_c \leq \sigma_{ck} \text{ or } P - 2\pi kEt^2 \leq 0.$$

Thus, the optimization problem is formulated as a single-criterion optimality, nonlinear mathematical programming problem, which can eventually be stated as follows: it is necessary to select R and t settings in such a way that the weight be minimal

$$G_{\min} = \pi(2Rt + t^2)L\rho g,$$

and complete the restrictions below:

$$R_1 \leq R \leq R_2,$$

$$t_1 \leq t \leq t_2,$$

$$\sigma_{ck} = kEt / R,$$

$$P - 2\pi kEt^2 \leq 0,$$

where R_1 and R_2 denote the lower and upper edge. The numeric values of the problem-2 edge of the inner and outer diameters of the cylinder, and the lower t_1 and upper t_2 edge of the wall thickness.

The numeric values of the task-2 are
$E = 3.10^7 \, pound / inch^2$, $k = 0.6$,
$p.g = 0.283 \, pound / inch^3$
$L = 144$ inch, $P = 25.10^3 \, pound / inch^2$
$R_1 = 4$ inch, $R_2 = 5$ inch, $t_1 = 2$ inch, $t_2 = 3$ inch.

The results of the optimization stochastic method for the problem 2 are given in Table 2.

Table 2. The results of the optimization stochastic method for the task 2

R	t	Target function Gmin
4.705	2.533	336.421
4.579	2.289	279.472
4.871	2.056	235.209
4.676	2.015	224.772
4.402	2.016	221.41
4.347	2.002	218.083
4.066	2.014	216.649
4.131	2.002	215.351
4.104	2.003	215.22
4.076	2.002	214.58
4.069	2	214.118
4.005	2.002	213.811
4.008	2	213.524
4.006	2	213.434
Optimum		
4.006	2	213.434

The stochastic optimization method has the following advantages over other determined methods:

1. Stochastic optimization methods are applied when the target function involves one or multiple parameters as well as many extrema, including global ones. Their application is particularly advisable even when the target function is non-differentiated or its analytical form is unknown but calculated. The advantage of the above method can be considered an important feature such as the sustainability of the "ravine type", as well as finding the extreme importance of "saddle type".

2. Programming of stochastic optimization methods is significantly simple compared to deterministic methods.

3. The methods for solving nonlinear problems vary and often require an individual approach in each case depending on the characteristics of the target function. This complexity was caused by the insufficient computational power of earlier computers, and the lack of appropriate mathematical techniques capable of finding global extrema in complex functions. Stochastic optimization offers a promising direction for the development of a unified solution method. However, for this it is

essential to provide easy-to-date on initial data, restrictions and check-in, as well as the purpose of the image function on the computer in the form of standard functions or sub-programs, similar to the implementation used in software WOLFRAM.

4. An important advantage of the proposed stochastic optimization method is that it is implemented in Visual Basic, making it compatible with widely used software platforms like Excel and Visual Studio. This compatibility significantly enhances its accessibility and ease of use, especially for students and users with diverse levels of technical expertise.

Conclusion

A modern version of the stochastic optimization method for solving both linear and nonlinear mathematical programming problem has been applied. This method does not require precise knowledge of the target function's characteristics. Computer experiments show that the results calculated by the stochastic optimization method are consistent with those obtained using well-known approaches, including those by Zoutendijk, the Frank-Wolfe algorithm, Kuhn-Tucker conditions, and various graphical methods. The comparison confirms that the outcomes closely align with an acceptable level of accuracy typically required in engineering practice. The above approach is implemented using the algorithmic language Visual Basic. Compared to other deterministic methods, it is relatively easy to program. Its integration with Excel and Visual Studio is simple and convenient for both students and other interested users.

ინფორმატიკა

ოპტიმიზაციის სტოქასტური მეთოდის გამოყენება წრფივი და არაწრფივი მათემატიკური დაპროგრამების ამოცანების ამოსახსნელად

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ნაშრომში განხილულია ოპტიმიზაციის უახლესი სტოქასტური მეთოდის გამოყენების შესაძლებლობა წრფივი და არაწრფივი მათემატიკური დაპროგრამების ამოცანების ამოსახსნელად. მრავალი ტექნიკური ობიექტის და, მათ შორის, სამშენებლო კონსტრუქციების ოპტიმიზაციის ამოცანების მათემატიკური მოდელი შეიძლება წარმოვადგინოთ, როგორც წრფივი და არაწრფივი მათემატიკური დაპროგრამების ამოცანა. წრფივი მათემატიკური დაპროგრამების ამოცანების ამოსახსნელად, უმეტეს შემთხვევაში, იყენებენ ე.წ. კლასიკურ მეთოდს,

რომელიც ცნობილია, „სიმპლექს" მეთოდის სახელწოდებით. ამ მეთოდის მნიშვნელოვანი ნაკლი მდგომარეობს გამოთვლების სირთულესა და რუტინულობაში. არაწრფივი მათემატიკური დაპროგრამების ამოცანების ამოხსნისათვის ერთი, ზოგადი მეთოდი არ არსებობს, ხოლო არსებული მეთოდებიდან ყველა ინდივიდუალურია და მოითხოვს მიზნის ფუნქციისადმი ფრთხილ მიდგომას, რაც მნიშვნელოვნად ზღუდავს მათი გამოყენების შესაძლებლობას. რთული კონფიგურაციის მიზნის ფუნქციის შემთხვევაში, კერძოდ, ეს სირთულეები შეიძლება იყოს: მიზნის ფუნქციის მრავალპარამეტრიანობა და მრავალექსტრემუმიანობა, მათ შორის გლობალური ექსტრემუმის არსებობა, მიზნის ფუნქციის წყვეტადობა, „უნაგირა" და „ხევის" ტიპის წერტილების არსებობა და სხვა მრავალი სახის სირთულე, რაც ქმნის დამატებითი ღონისძიებების გატარების აუცილებლობას. ერთიანი ზოგადი მეთოდის შექმნის თეორიული დასაბუთება, რომელიც შესაძლებლობას მოგვცემს ამოვხსნათ ზემოთ მითითებული ამოცანები, ერთი ზოგადი მეთოდის გამოყენებით ლოგიკურია, მაგრამ ერთმნიშვნელოვანი პასუხი ასეთი შემთხვევებისათვის ჯერჯერობით ნაპოვნი არ არის, მაგრამ არის მცდელობა შედარებით მარტივად იქნეს დაძლეული ზემოთ აღნიშნული სიძნელეები და ამისათვის გამოყენებულ იქნეს, ნაშრომში შემოთავაზებული, ოპტიმიზაციის სტოქასტური მეთოდი, როგორც წრფივი, ისე არაწრფივი მათემატიკური დაპროგრამების ამოცანების ამოსახსნელად. ნაშრომში განხილულია ორი რეალური ობიექტის მათემატიკური მოდელი, რომელთა რიცხვითი ამონახსნების მისაღებად დამუშავებულია კომპიუტერული პროგრამების კომპლექსი, ნაშრომში მოყვანილია შესაბამისი შედეგები.

REFERENCES

1. Kikacheishvili C., Klimiashvili L., Botsvadze N. (2001) Methods for determining the optimal parameters of water supply systems, 240p. Tbilisi (in Georgian).
2. Hog E., Arora J. (1983) Applied optimal design. Mechanical and structural systems, 470p. The University of Iowa.

Received March, 2025