საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, ტ. 19(193), №3, 2025
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 19(193), no. 3, 2025

Informatics

Parameter Identification of One Class of Nonlinear Systems with Positive Feedback

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Abstract. Building a mathematical model using system identification methods requires solving various problems depending on the a priori information about the processes occurring in the system. The successful solution of the parameter identification problem at the established model structure significantly determines the building of an adequate system model. In this work, the parameter identification problem of nonlinear dynamic systems operating with positive feedback is considered in the frequency domain when harmonic signals act on the input of the research system. The mentioned identification problem is considered in the condition when the research system is represented by a Wiener model with unit feedback. The nonlinear ordinary differential equation describing this model is obtained considering the peculiarities of the functioning of the systems operating under such manufacturing conditions. Based on these characteristics, the method of small parameters can be applied to solve differential equations. The problem of parameter identification is reduced to solving algebraic equations using the Fourier approximation for the periodic signals received at the output of the research system in the steady-state regime. The estimation of parameters is carried out using the least squares method. The developed parameter identification method has been investigated in terms of the accuracy of the obtained results. *© 2025 Bull. Georg. Natl. Acad. Sci.*

Keywords: identification, nonlinear system, block-oriented model, feedback, parameter

Introduction

Most processes in real manufacturing systems are nonlinear and have many characteristics that make them difficult to control. An important class of nonlinear systems includes those operating with positive feedback. In such systems, which are widely used in manufacturing, part of the output material that remains unprocessed in the required state is returned to the input of this system via a feedback loop for reprocessing. The systems functioning with positive feedback are characterized by relatively high efficiency and maximum utilization of raw materials (Nagiev, 1962).

System control using technical means requires formalizing the processes occurring in it as a mathematical model. System processes can be formalized using system identification methods. In this case, it becomes necessary to solve various problems that arise depending on the a priori information about the processes occurring in the system and the observability of the signals acting on it (Eykhoff, 1974). The construction of an adequate system model significantly depends on the successful solution of the parameter identification problem for the given model structure.

When modeling practical manufacturing processes with nonlinear models, block-oriented models are usually used, which are composed of various modifications of the Hammerstein and Wiener models.

The main difficulty in identifying systems with block-oriented models is that most of these models are nonlinear in terms of the search parameters. The main results in the area of parameter identification are obtained for open systems by representing them with simple and cascade Hammerstein and Wiener models (e.g., Brouri, 2022; Brouri et al., 2023; Giri, & Bai, 2010; Jin et al., 2021; Jing et al., 2021; Li et al., 2022; Liu et al., 2024; Shanshiashvili, & Rigishvili, 2020).

In the field of system identification operating with positive feedback, when identifying with block-oriented models, the success is much less compared to open systems. This is due to the fact that during conducting an active experiment, it is impossible to sequentially calculate the signals received at the outputs of linear dynamic and nonlinear static elements, as is feasible in open systems. In this case, it is necessary to solve nonlinear differential equations that are associated with mathematical difficulties.

The parameter identification method of nonlinear systems operating with positive feedback, when block-oriented models represent them, was developed in the works of Salukvadze, & Shanshiashvili, 2013; Prangishvili et al., 2016. In these studies, when estimating dynamic parameters in the transient regime during an active experiment, it is necessary to calculate derivatives based on experimental data. This approach can lead to erroneous results under conditions of incorrect measurements. It should be also noted that there are other approaches related to parameter identification of the closed-loop systems (Burghi et al., 2019; Oualla et al., 2022; Shakib et al., 2020).

In this paper, the parameter identification problem of nonlinear systems operating with positive feedback is considered in their representation by the Wiener model with unit feedback. When composing the corresponding differential equation of the model, the features of the operation of manufacturing processes with positive feedback are taken into account, and the characteristics of such systems are used. When forming the input harmonic signals for frequency-domain identification, the fact is taken into account that due to the peculiarities of the operation of systems with positive feedback, the output signal cannot be obtained at a zero input signal. The developed method allows the estimation of the dynamic parameters using the least squares method. It relies on the Fourier approximation of periodic oscillations obtained at the output of the research system and the known static parameter estimates obtained in the stationary mode of system operation.

Mathematical Equations of the Wiener Model with Unit Feedback

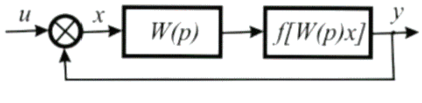

Fig. 1. Wiener model with unit feedback.

Block-oriented models are defined via various combinations of connections between nonlinear static and linear dynamic elements. In the Wiener model with unity feedback, a nonlinear static element is connected sequentially after the linear dynamic element (Fig. 1).

We assume that the nonlinear element represented by a polynomial function and the dynamic element is described by a linear ordinary differential equation.

The following equation describes the Wiener model with unit feedback:

$$y(t) = W^{-1}(p) f^{-1}\left[y(t)\right] - u(t), \tag{1}$$

where $W(p)$ is the transfer function of the linear dynamic element in the operational form, i.e., $p \equiv d/dt$, $f(\bullet)$ is a polynomial function.

In order to obtain the differential equation describing the Wiener model, let's use the characteristic features of practical manufacturing processes operating with positive feedback.

In some manufacturing processes, such as ore processing, systems with positive feedback are widely used. In such systems, it is possible to conditionally to separate the direct and feedback channels (Arefiev, 1969). The initial material is moved through a direct channel and processed into a finished product, and the unprocessed material to the required condition is moved through the feedback channel. Through determining a mathematical model of the feedback channel, it is possible to build a model of the entire system.

The static characteristic of the feedback channel of the manufacturing processes mentioned above is approximated by a second-degree polynomial function (Arefiev, 1969):

$$f(x) = c_1 x + c_2 x^2, \tag{2}$$

where there is no free term since the output product cannot be obtained at zero supply of the initial material.

The transfer function of the dynamic part of the object has the following form (Arefiev, 1969):

$$W(p) = \frac{1}{Tp+1}. \tag{3}$$

For the nonlinear manufacturing systems with positive feedback, it was obtained (Arefiev, 1969), that for the system stability in the steady state, the realization of the following conditions is sufficient:

$$0 < c_1 < 1, \;\; 0 < \mathrm{c}_2, \;\; \bar{u} < \frac{1-c_1}{2c_2}, \tag{4}$$

where $\bar{u}$ – the steady state value of the input signal.

The Wiener model with unit feedback is used to model the feedback channel of a manufacturing system operating with positive feedback. Therefore, we assume that the nonlinear static element of the model is described by a function expressed in (2), and the transfer function of the linear part is represented by the expression (3).

Using the expression (2), we get:

$$f^{-1}(y) = -\beta + \sqrt{\beta^2 + \frac{1}{c_2} y}, \tag{5}$$

where there is a positive sign before the radical due to physical considerations and $\beta = c_1/2c_2$.

If we take (5) into account in equation (1), after several transformations, we get:

$$\frac{dv}{dt} = -\frac{1}{T}v + \frac{c_2}{T}v^2 + \frac{1}{T}u(t) + \frac{1}{T}\left(\beta - c_2\beta^2\right), \tag{6}$$

$$y = c_2 v^2 - c_2 \beta^2 . \tag{7}$$

Mathematical Expressions of Output Forced Oscillations

To solve the problem of parameter identification in the frequency domain, the following type of harmonic signal is supplied to the input of the research system:

$$u(t) = A\sin \omega t + B\,. \tag{8}$$

In addition, we assume that $B > A$, because if the input signal is zero, no signal will be received at the system output.

If we consider (8) in (6), we get the equation:

$$\frac{dv}{dt} = -\frac{1}{T}v + \frac{c_2}{T}v^2 + \frac{A}{T}\sin \omega t + \frac{B}{T} + \frac{1}{T}\left(\beta - c_2\beta^2\right). \tag{9}$$

It follows from the conditions (4) that $c_2 << c_1$ and c_2 can be considered as a small parameter, and the small parameter method can be used to solve Ricati equation (9), so the solution (9) can be found in the form of the series:

$$v(t) = \sum_{n=0}^{\infty} \mu^n v_n(t)\,, \tag{10}$$

where μ a small parameter, and we can assume that $\mu = c_2$.

If we take into account the smallness of c_2 and in (10), we neglect the small values of the second and higher order; we will have two terms in the series (10). Inserting expression (10) into equation (9) and equating the terms on its right and left sides, which have coefficients with the same degree of c_2, we get:

$$\frac{dv_0}{dt} = -\frac{1}{T}v_0 + \frac{1}{T}A\sin \omega t + \frac{1}{T}B + \frac{1}{T}\left(\beta - c_2\beta^2\right), \tag{11}$$

$$\frac{dv_1}{dt}(t) = -\frac{1}{T}v_1(t) + \frac{1}{T}v_0^2\,. \tag{12}$$

By solving equations (11) and (12) under zero initial conditions and taking into account (7), in the steady state, as a result of a series of calculations, we obtain:

$$\begin{aligned} y = {} & c_2B_1^2 - c_2\beta^2 + \frac{c_2A^2}{2\left(1+\omega^2T^2\right)} + \frac{2c_2AB_1}{1+\omega^2T^2}\sin \omega t + \frac{2c_2AB_1\omega T}{1+\omega^2T^2}\cos \omega t - \\ & - \frac{2c_2A^2\omega T}{2\left(1+\omega^2T^2\right)^2}\sin 2\omega t - \frac{c_2A^2\left(1-\omega^2T^2\right)}{2\left(1+\omega^2T^2\right)^2}\cos 2\omega t, \end{aligned} \tag{13}$$

where $B_1 = B + \beta - c_2\beta^2$.

Parameter Identification

Estimates of static parameters. Estimates of static parameters were previously obtained (Salukvadze, & Shanshiashvili, 2013) by the least squares method in the stationary mode:

$$\hat{c}_1 = 1 - \frac{\left(\sum_{i=1}^{n} x_i^4\right)\left(\sum_{i=1}^{n} u_i x_i\right) + \left(\sum_{i=1}^{n} x_i^3\right)\left(\sum_{i=1}^{n} u_i x_i^2\right)}{\left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} x_i^4\right) - \left(\sum_{i=1}^{n} x_i^3\right)^2}, \tag{14}$$

$$\hat{c}_2 = \frac{\left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} u_i x_i^2\right) + \left(\sum_{i=1}^{n} x_i^3\right)\left(\sum_{i=1}^{n} u_i x_i\right)}{\left(\sum_{i=1}^{n} x_i^2\right)\left(\sum_{i=1}^{n} x_i^4\right) - \left(\sum_{i=1}^{n} x_i^3\right)^2}, \tag{15}$$

where u_i, y_i $(i = 1, 2, \ldots, n)$ are the values input and output variables of the system at the t_i $(i = 1, 2, \ldots, n)$ moment in the steady-state regime.

Identification of dynamic parameters. Applying the Fourier approximation (Hamming, 1987) to the forced oscillation output of the Wiener model in the steady-state regime allows us to determine the estimates of the Fourier coefficients $\hat{a}_0/2$, a_k, b_k $(k = 1, 2)$.

If we equate the theoretical value of the coefficient $a_0/2$ from the expression (13) with its estimate obtained through the Fourier approximation, we get:

$$c_2 B_1^2 - c_2 \beta^2 + \frac{c_2 A^2}{2\left(1 + \omega^2 T^2\right)} = \frac{\hat{a}_0}{2}. \tag{16}$$

When a harmonic signal of different $\omega = \omega_i$ $(i = 1, 2, \ldots, n)$ frequencies is supplied to the input of the research system, from the expression (16) after the transformation of (16), we get:

$$\left(2\hat{c}_2 B_1^2 - 2\hat{c}_2 \beta^2 - \hat{a}_{0i}\right)\omega^2 T_0 + \varepsilon_i = \hat{a}_{0i} - \hat{c}_2 A^2 - 2\hat{c}_2 B_1^2 + 2\hat{c}_2 \hat{\beta}^2, \tag{17}$$

where $\hat{a}_{0i}$ $(i = 1, 2, \ldots, n)$ estimates of the Fourier coefficient $\hat{a}_0$ $\hat{c}_2$, $\hat{\beta}$ – estimates of a_0, c_2, β coefficients, ε_i $(i = 1, 2, \ldots, n)$ – errors of approximations and measurements, at the ω_i frequency, and $T_0 = T^2$.

According to the least-squares method, which we use to estimate the dynamic parameter, the sum of the error squared is:

$$S = \sum_{i=1}^{n} \varepsilon_i^2 = \sum_{i=1}^{n} \left[\left(\hat{a}_{0i} - \hat{c}_2 A^2 - 2\hat{c}_2 B_1^2 + 2\hat{c}_2 \hat{\beta}^2\right) - \left(2\hat{c}_2 B_1^2 - 2\hat{c}_2 \beta^2 - \hat{a}_{0i}\right)\omega^2 T_0\right]^2. \tag{18}$$

If we differentiate (18) by T_0 and equating the received results to zero, we get the expression for estimating the value of the parameter T_0:

$$\begin{aligned}
&\sum_{i=1}^{n}\left(\hat{a}_{0i} - \hat{c}_2 A^2 - 2\hat{c}_2 B_1^2 + 2\hat{c}_2 \hat{\beta}^2\right)\sum_{i=1}^{n}\left(2\hat{c}_2 B_1^2 - 2\hat{c}_2 \beta^2 - \hat{a}_{0i}\right)\omega_i^2 = \\
&= \left[\sum_{i=1}^{n}\left(2\hat{c}_2 B_1^2 - 2\hat{c}_2 \beta^2 - \hat{a}_{0i}\right)\omega_i^2\right]^2 T_0.
\end{aligned} \tag{19}$$

Taking into account that $T_0 = T^2$, we get an estimate of the parameter T from the expression (19):

$$\hat{T} = \sqrt{\frac{\sum_{i=1}^{n}\left(\hat{a}_{0i} - \hat{c}_2 A^2 - 2\hat{c}_2 B_1^2 + 2\hat{c}_2 \hat{\beta}^2\right)}{\sum_{i=1}^{n}\left(2\hat{c}_2 B_1^2 - 2\hat{c}_2 \beta^2 - \hat{a}_{0i}\right)^2 \omega_i^4}}. \tag{20}$$

An estimate of the parameter T can also be calculated from the values of the Fourier coefficients a_k, b_k $(k = 1, 2)$ and then the obtained results can be compared with each other.

Let us calculate the estimate of the parameter T using the values of the coefficients a_1 and b_1.

Equating the estimated Fourier coefficients with their theoretical values from the expression (13), we get:

$$\hat{a}_1 = \frac{2c_2 A^2 \omega T}{2\left(1 + \omega^2 T^2\right)^2}, \quad \hat{b}_1 = \frac{2c_2 A B_1}{1 + \omega^2 T^2}. \tag{21}$$

Through the expression (21), we obtain:

$$\widehat{T}=-\frac{\sum_{i=1}^{n}\frac{\widehat{a}_{1i}}{\widehat{b}_{1i}}\omega_i}{\sum_{i=1}^{n}\omega_i^2}. \tag{22}$$

Accuracy Question Research

Computer modeling was carried out using the MATLAB package to investigate the accuracy of the developed method. At the same time, the tools of the Simulink toolbox were used for system modeling, and the Symbolic Math Toolbox was used for mathematical calculations.

The diagram of the output signal of Wiener's electronic model, when experimenting with the following parameters: $T=1,\ c_1=0,5,\ c_2=0,1$, is shown in Fig. 2.

Using the algorithm and program compiled according to the developed method, the following parameter values were calculated: T = 1.0634, c_1 = 0.4819, c_2 = 0.0949.

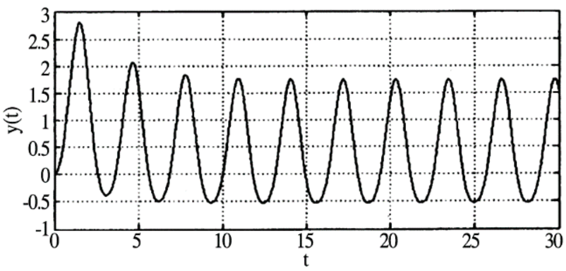

Fig. 2. Diagram of the output's oscillation of the Wiener model.

When using the developed identification method under industrial conditions, it is necessary to consider that systems operating with positive feedback usually work in noisy environments and have measurement errors. In such a case, the parameter estimation accuracy depends on the systems' input and output signal measurements during the experiment as well as on the mathematical processing of the obtained experimental data. In addition, the least squares method used for the parameter estimation gives reliable results under certain restrictions on the probabilistic characteristics of the noise.

Conclusions

In this work, a method for parameter identification of nonlinear dynamic systems operating with positive feedback has been developed in the frequency domain under of active experiment conditions. It is implied that the model structure is known as a Wiener model with unit feedback. The differential equation describing the Wiener model is obtained considering the conditions of the functioning of the real manufacturing systems with positive feedback and their characteristics. When experimenting on a system with input harmonic signals, it is necessary to take into account that stable motion at the system output is obtained under certain restrictions on the parameters and input signals.

The developed method allows us to estimate the dynamic parameters in the frequency domain using the least squares method. It considers the Fourier approximation of the periodic oscillations obtained at the output of the research system and the known estimates of the static parameters obtained in the stationary mode of the system operation.

The issues of the accuracy of the developed method and its application to practical manufacturing processes are investigated through theoretical analysis and computer modeling.

ინფორმატიკა

ერთი კლასის დადებითი უკუკავშირიანი არაწრფივი სისტემების პარამეტრული იდენტიფიკაცია

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მათემატიკური მოდელის აგება სისტემის იდენტიფიკაციის მეთოდების გამოყენებით მოითხოვს სხვადასხვა ამოცანის გადაწყვეტას სისტემის შესახებ აპრიორულ ინფორმაციაზე დამოკიდებულებით. დადგენილი მოდელის სტრუქტურის შემთხვევაში პარამეტრული იდენტიფიკაციის ამოცანის წარმატებით გადაწყვეტა მნიშვნელოვნად განაპირობებს სისტემის ადეკვატური მოდელის აგებას. წინამდებარე ნაშრომში დადებითი უკუკავშირით მოქმედი არაწრფივი დინამიკური სისტემების პარამეტრის იდენტიფიკაციის ამოცანა განხილულია სიხშირულ არეში, როდესაც ჰარმონიული სიგნალები მოქმედებენ კვლევის სისტემის შესასვლელზე. აღნიშნული იდენტიფიკაციის ამოცანა განიხილება იმ პირობებში, როდესაც საკვლევი სისტემა წარმოდგენილია ვინერის მოდელით ერთეული უკუკავშირით. მოდელის აღმწერი არაწრფივი ჩვეულებრივი დიფერენციალური განტოლება მიღებულია წარმოების ასეთ პირობებში მოქმედი სისტემების ფუნქციონირების თავისებურებების გათვალისწინებით. ამ მახასიათებლების გათვალისწინება მცირე პარამეტრის მეთოდის გამოყენების საშუალებას იძლევა დიფერენციალური განტოლების ამოსახსნელად. პარამეტრული იდენტიფიკაციის ამოცანა დაიყვანება ალგებრული განტოლებების ამოხსნაზე ფურიეს აპროქსიმაციის გამოყენებისას პერიოდული სიგნალებისთვის, რომლებიც მიიღება საკვლევი სისტემის გამოსასვლელზე დამყარებულ რეჟიმში. პარამეტრების შეფასება ხორციელდება უმცირესი კვადრატების მეთოდით. შემუშავებული პარამეტრული იდენტიფიკაციის მეთოდი გამოკვლეულია მიღებული შედეგების სიზუსტის თვალსაზრისით.

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Received April, 2025