საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, ტ. 19(193), №3, 2025
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 19(193), no. 3, 2025

Engineering

Analysis of Hierarchical Interrelations of Berthing and Building Structures Using Fuzzy Sets

David Gurgenidze*, Merab Akhobadze, Elguja Kurtskhalia****

* *Academy Member, Georgian Technical University, Tbilisi, Georgia*
** *Informatics and Control Systems, Georgian Technical University, Tbilisi, Georgia*

Abstract. The work is devoted to the creation of a methodology for analyzing hierarchical connections of berths and building structures based on the theory of fuzzy sets. The developed algorithms allow optimizing work on the restoration of berths and building structures. *© 2025 Bull. Georg. Natl. Acad. Sci.*

Keywords: pier, building structure, hierarchical analysis, fuzzy sets

Introduction

In general, any structure, building or berth can be considered as a system consisting of a set of "structural elements" and "technological processes" united by certain features. The structure of the system under consideration, on which various dynamic processes develop, is a set of mathematical relations $\wedge = \{\lambda, \mu, ...\}$, existing between these sets. Knowledge of the relations $\lambda, \mu, ...$ allows us to detect structural connections between various elements of the building and track events occurring in a long chain of connections between elements with different functional purposes, in a spatio-temporal perspective. From a systemic point of view, we can evaluate and predict all the processes that arise in the system (Akhobadze et al., 2023).

We can consider a structure as a simplicial complex, where each "structural element" is a simplex whose vertices are the elements from which the given "structural unit" is obtained. This approach allows using the method of combinatorial topological Q – analysis to study the structure of structures and the connections between its individual elements (Atkin, 1974; Akhobadze et al., 2018).

Materials and Methods

The mathematical model of the structure presented in this way allows us to detect all sorts of connections between the elements of the structure and to trace what quantitative and qualitative changes will be caused in space-time by the effects on each of the "structural elements". Usually, in order to clearly show whether a given connection exists between any elements of these sets, we use the corresponding incidence matrix of this connection. A matrix, each element of which is either 1 (when a given connection exists between the corresponding elements) or 0 (when a given connection does not exist between the corresponding elements). In other words, in order to clearly demonstrate the presence of a given connection between the elements of two sets, we use the characteristic function of a certain subset of the Cartesian product of these sets. However, in many cases, if the relationship between some elements of two sets is insignificant (small) compared to other relationships, then for the purpose of using the $Q-$ analysis method, such a relationship is considered to be zero. Obviously, when using a model based on such assumptions, a lot of information is lost, which can lead to incorrect results, possibly even a catastrophe (Casti, 1979; Akhobadze & Kurtskhalia, 2021).

The approach we propose allows us to overcome this problem, namely, to expand the Q –analysis by introducing fuzzy sets (Zadeh, 1965). In this case, the elements of the incidence matrix will not be the numbers 0 and 1, but the membership functions of fuzzy sets. To characterize the "connection" relationship between the elements of two sets, we will use the membership function of the subset of the Cartesian product of two fuzzy sets. For example, to characterize the degree of readiness of a specific object for operation, we will use the membership functions of fuzzy subsets (e.g: "completely ready", "requires minor repairs", "requires major repairs", "this object is not subject to operation", etc.).

A mathematical model built on the basis of this approach allows for a systematic assessment of the building structure and its stability in a wide range of disturbances (Akhobadze et al., 2023; Akhobadze, 2021; Akhobadze & Kurtskhalia, 2021).

Discussion

Let $A=\{a_1;a_2;...;a_s\}$ and $B=\{b_1;b_2;...;b_R\}$ be some sets, and we characterize the connection between these sets of some linguistic variable $X^1;X^2;...;X^p$. We denote by symbols X^p the fuzzy sets corresponding to the linguistic variable under consideration, and by $\mu_p : A\times B \to [0,1]$ (for each $p\in\overline{[1;P]}$) the membership function of the fuzzy set X^p. We consider the case when it is possible to order the sets X^p by levels. In other words, if $p_1 > p_2$ then X^{p_1} denotes a higher level of connectivity than X^{p_2}. We construct the matrix Λ corresponding to this relation as follows: for each $s\in\overline{[1;S]}$, $r\in\overline{[1;R]}$, the element at the intersection of the corresponding row and column of the matrix Λ will be the vector $\overrightarrow{(a_s;b_r)}=\left(\mu_1(a_s;b_r)\right);\mu_2(a_s;b_r);...;\mu_p(a_s;b_r)\big)$:

$$\Lambda=\begin{pmatrix} \overrightarrow{(a_1;b_1)} & \overrightarrow{(a_1;b_2)} & ... & \overrightarrow{(a_1;b_R)} \\ \overrightarrow{(a_2;b_1)} & \overrightarrow{(a_2;b_2)} & ... & \overrightarrow{(a_2;b_R)} \\ .... & .... & ... & .... \\ \overrightarrow{(a_s;b_1)} & \overrightarrow{(a_s;b_2)} & & \overrightarrow{(a_s;b_R)} \end{pmatrix}.$$

To construct a vector of the corresponding structure for a given relation, we need to determine the "Zadeh product" of two matrices.

Suppose we are given two matrices: $M=\overrightarrow{(m_{ij})}_{qxs}$ and $N=\overrightarrow{(n_{kr})}_{sxt}$, whose elements are P-dimensional vectors. Using matrices M and N, for each $p\in\overline{[1;P]}$, we construct numerical matrices $M_p=\left(m_{ik}^p\right)_{qxs}$ and $N_p=\left(n_{ik}^p\right)_{qxs}$, whose elements will be $M=\overrightarrow{(m_{ij})}_{qxs}$ and $N=\overrightarrow{(n_{jk})}_{sxt}$ and $P-$coordinates of the corresponding matrix elements – m_{ij}^p and n_{jk}^p. We calculate the product of matrices M_p and N_p according to the rules proposed by Zadeh. In particular, we call the product of matrices M_p and N_p matrix E_p, whose elements $f_{ik}^p=\max\limits_j\left\{\min\left\{m_{ij}^p;n_{jk}^p\right\}\right\}$.

The matrix F, each element of which is a $P-$dimensional vector whose P-coordinate is the corresponding element of the matrix E_p, is called the "Zadeh product" of the matrices M and N, and is written as: $F=M\otimes N$.

Once we have determined the scalar product of two matrices, we can construct a vector of the corresponding structure for a given relation.

Let us construct the transposed matrix of the given matrix:

$$\Lambda^T = \begin{pmatrix} \overrightarrow{(a_1;b_1)} & \overrightarrow{(a_2;b_1)} & \dots & \overrightarrow{(a_s;b_1)} \\ \overrightarrow{(a_1;b_2)} & \overrightarrow{(a_2;b_2)} & \dots & \overrightarrow{(a_s;b_2)} \\ \dots\dots\dots & \dots\dots\dots & \dots & \dots\dots\dots \\ \overrightarrow{(a_1;b_R)} & \overrightarrow{(a_2;b_r)} & \cdots & \overrightarrow{(a_s;b_R)} \end{pmatrix}.$$

Let $C = \Lambda \otimes \Lambda^T$. Each element of the matrix C is a finite-dimensional (P – dimensional) vector $\overrightarrow{c_{ij}}$, so for each ij there will be one or more $p \in \overline{1;P}$ such that the P -th coordinate of the vector $\overrightarrow{c_{ij}}$ is not less than the other coordinates of the vector $\overrightarrow{c_{ij}}$. Let $p_0(i,j)$ denote the largest among such P . Let us construct a matrix X whose element at the intersection of the *i*-th row and the *j*-th column is the fuzzy set $X^{p_0(i,j)}$. Since the subsets X^p are ordered by levels, then the element x_{ss} of the matrix X is a fuzzy set X^p for which there is at least one element b_r from the set B such that $\mu_p(\mathrm{a}_s;b_r) = \max\limits_{1\le k\le P}\{\mu_k(\mathrm{a}_s;b_r)\}$, where P is the largest among such numbers. In other words, the element x_{ss} is a fuzzy subset X^p to which the element $(\mathrm{a}_s;b_r)$ with the largest "weight" for some b_r belongs, where P is the largest among such numbers.

If $s \neq j$, then x_{sj} is a fuzzy subset X^p for which there is at least one element b_r from the set B such that:

$$\min\left\{\mu_p\left(a_s;b_r\right);\mu_p\left(a_j;b_r\right)\right\} = \max_{1\le k\le P}\left\{\max_{1\le i\le R}\left(\min\left\{\mu_k\left(a_s;b_i\right);\mu_p\left(a_j;b_i\right)\right\}\right)\right\}$$

and, moreover, P is the largest of such numbers.

We say that elements a_s and a_j of the set A are P - linked if $x_{sj} = X^m$ and $m \ge p$.

For each natural number P , $1 \le p \le P$, we construct a subset A^p of the set A as follows: $a_s \in A^p$ if $x_{ss} = X^m$ and $m \ge p$. We introduce the following relationship between the elements of the set A^p : we say that the element a_s of the set A^p is linked to the element a_j of the set A^p .

With an element if a_s and a_j are P relations. It is easy to show that the above relation is an equivalence relation. So this relation divides the set A^p into equivalence classes. Let Q_p denote the number of equivalence classes of the set A^p .

Vector $Q = \left(Q_0;Q_1;Q_2;\dots;Q_p\right)$ is called the structural vector corresponding to a given relation between sets A and B (Akhobadze et al., 2018), (Akhobadze, 2021). For each $1 \le p \le P$, the coordinate of the structure vector Q_p shows the geometric constraints for the exchange of information of level P (quality P) between the elements of set A^p . The longer the number Q_p , the greater the resistance to the implementation of the connection of level P (degree P) between the elements of set A^p .

Often, there is no direct connection between two sets A and C, but they are connected to each other through some set or sets (intermediate links). This means that any change made to set A reaches C after some time. Therefore, it is important to study the composition of relations, in particular to answer the question: how, using matrices corresponding to these relations, to construct a matrix corresponding to the composition of these relations?

Let $A \xrightarrow{\lambda} B, \mathrm{B} \xrightarrow{\delta} \mathrm{C}$, and we characterize the relations λ and δ of the same linguistic variable. Let $X^1;X^2;\dots;X^p$ denote the fuzzy sets corresponding to the linguistic variable under consideration, and let $\mu_p : (A\times B)\bigcup(B\times C) \to [0;1]$ (for each $p \in \overline{[1;\mathrm{P}]}$ denote the assignment function of the fuzzy set X^p).

Let us define the relation ν between the sets A and C (the composition of the relations λ and μ). To do this, we need to know how to calculate the value of the assignment function of the fuzzy set X^p for each element of the set $A \times C$ for each $p \in \overline{[1;\mathrm{P}]}$. We naturally assume that for each $a_i \in A$ and $c_k \in C$:

$$\mu_p\left\{a_i;c_k\right\} = \max_r\left\{\min\left\{\mu_p\left(a_i;b_r\right);\mu_p\left(b_r;c_k\right)\right\}\right\}. \quad (1)$$

Let $\Lambda^{(1;2)}$ and $\Lambda^{(2;3)}$ denote the matrices of the relations λ and μ, respectively. If we recall the rule for calculating the "Zadeh product" of two matrices, then based on (1) we obtain that the matrix $\Lambda^{(1;3)}$, corresponding to the relation ν between the sets A and C, is obtained by multiplying the matrices $\Lambda^{(1;2)}$ and $\Lambda^{(2;3)}$ according to the "Zadeh rule": $\Lambda^{(1;3)} = \Lambda^{(1;2)} \otimes \Lambda^{(2;3)}$.

In general, by a simplicial complex, a system, we mean a quadratic system $X;Y;p;\pi$, where X and Y are sets, $\rho \subset X \times Y$, and π is a mapping $\pi : K_p \to R$, i.e. a correspondence in which each simplex corresponds to a real number. π is called a model of K_p on the simplicial complex.

Suppose we characterize the relationship between two sets by some linguistic variable and let $X^1; X^2;...;X^p$ be the corresponding fuzzy sets. For each $p \in \overline{[1;\mathrm{P}]}$, a model π_p. π_p can be defined, for example, by the reflection $f_p : \rho \to R$

$$\pi_p\left[x_0; x_1;...;x_k\right] = \sum_{y;\forall_j (x_j;y)\in\rho} \sum_{j=0}^{k} f_p(x_j; y)$$

f_p and π_p have the following physical meaning: $f_p(x;y)$ is the cost (time, money) incurred in implementing a p – level connection between elements x and y in the system and $\pi_p(\sigma)$ is the cost of establishing all connections with σ. The growth of the π_p model reflects the dynamics of the system when an action is performed on it. The π_p model allows optimizing rehabilitation work according to various criteria (Akhobadze et al., 2023; Akhobadze, 2021).

Conclusion

We modified the classical Q-analysis method using fuzzy set theory, which allowed us to more accurately assess not only the stability of berths and building structures, but also to fairly accurately predict the dynamics of damage development with small disturbances and defects in structural elements. The developed algorithms, based on fuzzy sets and Fuzzy technology, allow optimizing the work on the restoration of berths and building structures.

ინჟინერია

ნავმისადგომის, შენობა-ნაგებობის კონსტრუქციების იერარქიული ბმულობის ანალიზი არამკაფიო სიმრავლეთა გამოყენებით

დ. გურგენიძე*, მ. ახობაძე, ე. კურცხალია****

** აკადემიის წევრი, საქართველოს ტექნიკური უნივერსიტეტი, თბილისი, საქართველო*
*** საქართველოს ტექნიკური უნივერსიტეტი, ინფორმატიკისა და მართვის სისტემების ფაკულტეტი, თბილისი, საქართველო*

ნაშრომი ეხება ნავმისადგომების, სამშენებლო ნაგებობების კონსტრუქციების იერარქიული ბმულობის ანალიზის მეთოდოლოგიის შექმნას, არამკაფიო სიმრავლეთა თეორიის ბაზაზე. შემუშავებული ალგორითმები საშუალებას გვაძლევს მოვახდინოთ ნავმისადგომების, სამშენებლო ნაგებობათა სარეაბილიტაციო სამუშაოების ოპტიმიზაცია.

REFERENCES

Akhobadze, M. (2021). *Issues of mathematical modeling of macrosystems (based on the principle of entropy maximization).* Tbilisi.

Akhobadze, M., Gurgenidze, D., Tsikarishvili, M., Bulia T. (2023). New optimal planning of restoration and technical diagnostics of Sea berths on the basis Q analysis and fuzzy set theory. *Bull. Georg. Natl. Acad. Sci., 17*(1), 38-43.

Akhobadze, M., Kurtskhalia, E. (2021). Fuzzy relationships in an urban system*. Bull. Georg. Natl. Acad. Sci., 15*(1), 7-11.

Akhobadze, M., Mesablishvili, B., Kurtskhalia, E. (2018). *Structural analysis and management of complex macrosystems.*

Atkin, R. H. (1974). *Mathematical structure in Human affairs, Heinemann Educational Books.* LTD. London.

Casti, J. (1979). *Connectivity, complexity, and catastrophe in large-scale systems.* New York.

Zadeh, L.A. (1965). *Fuzzy sets. Information and control,* p. 338-353.

Received April, 2025