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Informatics

Method for Determining the Resonant Frequency of Low-Quality Oscillatory Systems and Structural Implementation

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Abstract. This paper proposes an unconventional method for determining the resonant frequency of oscillatory systems with low quality factor based on an n-th order extrapolating polynomial. The method improves the measurement accuracy and expands the functional capabilities of the measuring system. The results of theoretical analysis and simulation show the advantage of the proposed approach over traditional methods, especially in cases where the quality factor of the investigated RS is low ($3 < Q < 10$). The paper also presents the structural implementation of the measuring system built on the proposed method, which has multifunctional properties, such as the ability to determine additional information parameters of the investigated oscillatory system. The structural scheme of the measuring system is given and the principle of its operation is explained. Its reliability and applicability in a dynamic environment are emphasized.

Keywords: resonant frequency, low-quality (Q-factor), oscillatory system, extrapolating polynomial, amplitude-frequency characteristic

Introduction

In modern conditions, numerous studies aim to improve devices and search for perfect methods for solving the task, since the methods known today either do not meet the requirements, or their application is generally impossible to solve the tasks. Accordingly, the task of creating highly accurate and reliable devices for determining the parameters of technological processes that can operate in difficult operating conditions is relevant. A promising direction in the development of methods and means for determining the informational parameters of oscillating systems is the improvement of existing methods and means by expanding their operational capabilities. Therefore, it is relevant to create a device that provides highly accurate determination of the main informational parameters of the object of study – the oscillating system (OS) in extreme conditions (Kriksunova & Lunkin, 1991; Viktorov et al., 1978).

Theoretical Foundation

The essence of the proposed method is as follows: according to the amplitude-frequency characteristic (AFC) of the oscillating system, its amplitude U_m is determined and voltage levels $U_1<U_2<\ldots<U_n<U_m$ are formed. Frequencies are measured at the crossover points of the formed voltage levels and AFC, and the corresponding averaged frequencies $f_1, f_2, \ldots f_n$ are calculated at each level (Viktorov et al., 1978; Azmaiparashvili & Otkhozoria, 2021).

According to the extremum points $M_1(U_1,f_1)$, $M_2(U_2,f_2), \ldots M_3(U_n,f_n)$, we find the extrapolating function, for example, by means of a polynomial of degree n Pn(U). Using a microprocessor-based computing device, the value of the obtained function at the **Um** (maximum) point (Fig. 1), which is the resonant frequency of the oscillating system $f_0 = P_n(U_m)$ is calculated (Azmaiparashvili et al., 2017).

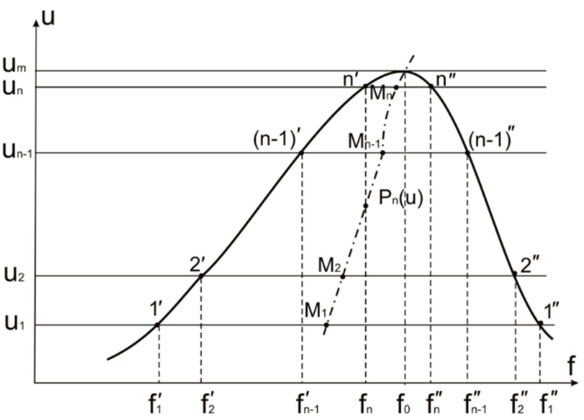

Fig. 1. Determination of the value of the extrapolating function $\boldsymbol{P_n(U)}$ at point $\boldsymbol{U_m}$.

The proposed method assumes maintaining high accuracy of determining the resonant frequency in the conditions of wide amplitude variations of the AFC of the oscillating system and large electromagnetic losses of the oscillating system (Azmaiparashvili et al., 2024).

Let us consider the methodological errors of the existing method closest to the proposed method. Our goal is to determine the main error of the resonant frequency at low quality of the oscillating system, in the range $3<Q<18$, when $Q=3,5,8,10,15$ and the frequency overlap range is $10<f<40$ MHz.

A fairly accurate approximation of the normalized amplitude-frequency characteristic of the oscillating system is known, as an asymmetric function:

$$\frac{U(f)}{U(f_0)} = \frac{1}{\sqrt{1+Q^2(1-f_0^2/f^2)^2}}\,, \tag{1}$$

where $U(f)$is the voltage applied to the oscillating system, f – the input frequency, f_0 – the resonant frequency, and Q – the quality: $U(f_0) = S \cdot Q$, and S is the sinusoidal voltage applied to the oscillating system under investigation.

The second-order derivative of the amplitude-frequency characteristic of equation (1) with respect to frequency is expressed by the following relationship:

$$\frac{\dfrac{\partial^2 U(f)}{\partial f^2}}{\dfrac{\partial^2 U(f_0)}{\partial f^2}} = \frac{\left(\dfrac{f_0}{f}\right)^4 \left[3-5\left(\dfrac{f_0}{f}\right)^2 + Q^2\left(1-\dfrac{f_0^2}{f^2}\right)^2 \cdot \left(3+\dfrac{f_0^2}{f^2}\right)\right]}{2\cdot\sqrt{[1+Q^2(1-f_0^2/f^2)^2]^\delta}}, \tag{2}$$

where $\partial^2 U(f_0)/\partial f^2$ is the second-order derivative of the frequency and is equal to:

$$\frac{\partial^2 U(f_0)}{\partial f^2} = \frac{4\cdot 5\cdot Q^3}{f_0^2}\,. \tag{3}$$

To determine the parameters of the OS, it is necessary to identify the extremum points of the inflection points of the amplitude-frequency characteristic curve, for which we set the second-order derivative of the above image equal to *zero*.

$$3-5\cdot(f_0/f)^2+Q^2\cdot(1-f_0^2/f^2)^2\cdot(3+f_0^2/f^2)=0. \quad (4)$$

The resulting equation (9) is of the 8th degree, the solution of which in analytical form is associated with difficulties, therefore the resulting equation can be approximated with a certain accuracy by a quadratic parabola:

$$4\cdot Q^2\cdot(1-f_0^2/f)^2-2=0. \quad (5)$$

Solving this equation allows us to determine the of extremum points of the inflection point:

$$f_0/f_{1,2}=\pm\sqrt{1-2/Q\sqrt{2}}\,. \quad (6)$$

Expression (5) and its solutions (6) allow us to determine the quality factor (Q) of the oscillating system with the following expression:

$$Q=\frac{1}{\sqrt{2}(|1-f_0^2/f_{1,2}|)}=\frac{f_{1,2}^2}{\sqrt{2}(|f_{1,2}^2-f_0^2|)}\,. \quad (7)$$

With some error, we can represent the image (7) as follows:

$$Q=\frac{f_{1,2}^2}{\sqrt{2}(|f_{1,2}+f_0|)(f_{1,2}-f_0)}\approx\frac{f_0}{2\sqrt{2}(f_{1,2}-f_0)}\approx\frac{f_1+f_2}{2\sqrt{2}(f_1-f_2)}\,. \quad (8)$$

At this time the resonant frequency is

$$f_0=(f_1+f_2)/2\,. \quad (9)$$

The bandwidth is at the 0.707 level

$$\triangle F_{0,707}=\sqrt{2}(f_1-f_2). \quad (10)$$

Based on the obtained results, we can conclude that the methodological error of the considered method mainly depends on the quality factor (*Q*) of the oscillating system, with its decrease the methodological error increases, and the bandwidth of the oscillating system expands.

When operating in dynamic mode, where the frequency is rearranged according to a linear law in time, the dynamic resonance curve of the oscillating system is shifted relative to the static resonance curve. If we denote the rate of linear frequency change by β, the shift error can be estimated by the following expression:

$$\gamma=\frac{80\cdot\beta^2\cdot Q^4}{\sqrt{2}\cdot w_0^4}. \quad (11)$$

It is evident from the Figure that the shift of the dynamic curve is proportional to the frequency tuning speed and quality factor *(Q*) (Kriksunova & Lunkin, 1991; Viktorov et al., 1978).

Based on the results of the study of the mathematical model compiled according to the literature data, the Table was obtained that shows the results of the known method. The calculations indicate that for $3<Q<10$, the relative error of the eigenfrequency is $0.347\leq\gamma\leq 4.573$. All results are acceptable when $Q=10$, while in the case of $Q\leq 5$ the error exceeds 1.5%. This is almost 13 times higher than its value in the case of $Q=10$. Therefore, the use of the existing method is not advisable for oscillating systems with a low quality factor.

Table. Dependence of error on nonlinearity coefficients, results obtained based on traditionally known methods

№	K1(%)	K2	K3(%)	K4	γ(%)			
					Q=3	Q=5	Q=8	Q=10
1	0	0	0	0	4.541	1.546	0.593	0.378
2	0.1	0	0	0	4.542	1.547	0.594	0.379
3	0	1	0	0	4.513	1.517	0.564	0.348
4	0	0	0.1	0	4.543	1.547	0.593	0.380
5	0	0	0	1	4.573	1.554	0.600	0.380
6	0.1	0	0.1	0	4.544	1.547	0.593	0.378
7	0.1	1	0.1	0	4.513	1.516	0.564	0.349
8	0.1	1	0.1	1	4.543	1.524	0.570	0.350
9	1	0	1	0	4.542	1.547	0.594	0.378
10	0	1	0.02	0	4.513	1.517	0.564	0.349
11	1	1	1	1	4.516	1.527	0.577	0.347

The above calculations demonstrate the advantages of the methods developed by us over the methods known to date, which are also caused by the asymmetry and low quality factor (Q) of the amplitude-frequency characteristic of the oscillating system.

Experimental Implementation

The structural scheme of the device implementing the proposed method is given in Fig. 2. Its operating principle is as follows: at the output of the synchronous block 1 of the variable frequency generator, a linearly changing frequency signal is generated in time, which is supplied to the studied oscillating system 2 and the frequency measurement block 8. At the output of the block 2 of the studied oscillating system, a radio signal modulated according to the amplitude-frequency characteristic of the oscillating system is received, which is detected by the amplitude detector 3, at its output a low-frequency component is generated - a resonant video pulse. The received video pulse is supplied to the amplitude capture and level generation block 4. The latter block fixes the amplitude value $U_m$of the resonant video pulse and forms the voltage levels $U_1 < U_2 < \cdots < U_n < U_m$ (for example, by means of a voltage divider). The logic signal formed at each moment of equality of the voltage levels $U_1, U_2, \dots, U_n$ and the instantaneous value of the resonant video pulse by the comparator block 5 connected to the microprocessor block causes the microprocessor block 6 to generate a frequency measurement control signal, through which the frequency is fixed at the output of the synchronous block 1 of the variable frequency generator for a certain time, and during this time interval the frequency measurement block – 8 measures the fixed frequencies $f_1'; f_1''; f_2'; f_2'' \dots f_{n-1}' f_{n-2}''$. The acquired data are supplied to the microprocessor block – 6 and the average frequencies are calculated in sequence:

$$f_1 = \frac{f_1'+f_1''}{2}, f_2 = \frac{f_2'+f_2''}{2}, \dots, f_n = \frac{f_n'+f_n''}{2}.$$

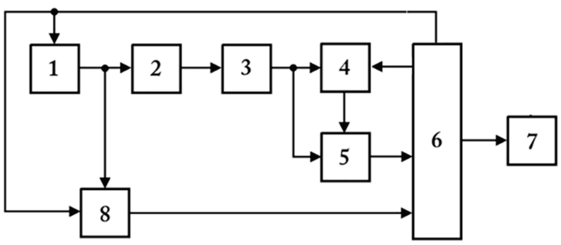

Fig. 2. Structural diagram of the measuring system.
1 – synchronous block of the oscillatory frequency generator, 2 – investigated oscillatory system, 3 – amplitude detector, 4 – amplitude capture and level generation block, 5 – comparator block, 6 – microprocessor block, 7 – data visualization unit, 8 – frequency measurement block.

The same block determines the point of extremum points of the $P_n(U)$ extrapolation function $M_1(U_1, f_1),\ M_2(U_2, f_2), \dots, M_n(U_n, f_n)$ according to which the extrapolation function f(U)=$P_n(U)$ is found and its value is calculated at the point U_m. The calculated value of the polynomial at the point U_m represents the resonant frequency of the oscillating system $f_0 = P_n(U_m)$. The last information is supplied to the data visualization unit – 7, where the measurement results are displayed.

Conclusion

A new method is proposed for determining the resonance frequency of an investigated oscillatory system with distributed parameters and low quality factor. This method utilizes an *n*-th order extrapolating polynomial, providing increased accuracy in measuring the primary informative parameter of the oscillatory system and expanding overall functional capabilities compared to traditional methods.

The resonance frequency is determined by finding the abscissa (coordinate) of the intersection point between the extrapolation function and the amplitude response curve (frequency response curve) of the investigated oscillatory system. When determining the amplitude of the resonance curve, *n* constant amplitude levels are formed. For each level, the frequencies at two points where these levels intersect the frequency response curve are recorded and measured. The average values of the two frequencies (coordinates of these points) are then calculated for each level. Based on the set of these coordinates, an *n*-th order extrapolating polynomial is determined.

An analysis of the main errors of both the existing and proposed methods is performed, demonstrating the advantage of the proposed method under conditions of high electromagnetic losses in the investigated oscillatory system with distributed parameters. A block diagram of the device implementing the method is presented and its operating principle is explained.

ინფორმატიკა

დაბალი ვარგისიანობის რხევითი სისტემების რეზონანსული სიხშირის განსაზღვრის მეთოდი და სტრუქტურული რეალიზაცია

ა. ფრანგიშვილი*, ზ. აზმაიფარაშვილი, ნ. ოთხოზორია**, ა. ჯავახიშვილი****

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მოცემულ ნაშრომში, შემოთავაზებულია დაბალი ვარგისიანობის მქონე რხევითი სისტემების რეზონანსული სიხშირის განსაზღვრის არატრადიციული მეთოდი, რომელიც აგებულია n-

რიგის ექსტრაპოლაციური პოლინომის საფუძველზე. წარმოდგენილი მეთოდი აუმჯობესებს გაზომვის სიზუსტეს და აფართოებს საზომი სისტემის ფუნქციურ შესაძლებლობებს. თეორიული ანალიზისა და სიმულაციის შედეგები აჩვენებს შემოთავაზებული მიდგომის უპირატესობას ტრადიციულ მეთოდებთან შედარებით, განსაკუთრებით იმ შემთხვევებში, როდესაც გამოსაკვლევი რს-ის ვარგისიანობა (Q-ფაქტორი) დაბალია (3<Q<10). სტატიაში ასევე წარმოდგენილია შემოთავაზებულ მეთოდზე აგებული საზომი სისტემის სტრუქტურული რეალიზაცია, რომელსაც გააჩნია მულტიფუნქციური თვისებები – გამოსაკვლევი რხევითი სისტემის დამატებითი ინფორმაციული პარამეტრების განსაზღვრის შესაძლებლობა. კერძოდ, ასმ-ის ამპლიუდური მნიშვნელობის, გატარების ზოლის და ვარგისიანობის (Q-ფაქტორის) დადგენის შესაძლებლობა. მოცემულია საზომი სისტემის სტრუქტურული სქემა და ახსნილია მისი მოქმედების პრინციპი. ხაზგასმულია მისი საიმედოობა და გამოყენებადობა დინამიურ გარემოში.

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