Informatics

Analysis of Post-Quantum Cryptography Use in Practice

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ABSTRACT. Quantum computers are able to destroy most, if not absolutely all conventional cryptosystems that are widely used in practice, specifically, systems based on the problem of factoring integers (e.g., RSA). Some cryptosystems like RSA system with 4 000-bit keys are considered useful to protect classic computers from attacks, but probably absolutely useless against attacks on quantum computers. One of the alternatives are post-quantum systems, systems based on lattices. These systems are known for high security levels based on the worst-case hardness. They are based on the complexity of the problems grids, the main of which is the problem of the shortest vector (SVP). In fact, we consider the approximate option when we find a lattice vector, with the length of $\alpha(n)$ times more than the shortest nonzero vector. $\pm(n)$ is the approximation coefficient, $n$ is the lattice size. The best known algorithm for grids problems is LLL algorithm. This algorithm needs polynomial time with approximation factor $2^{O(n)}$. In 1987, Schnorr extended LLL algorithm and improved this approximation ratio, but he increased the performance of the algorithm. Schnorr replaced core of LLL algorithm by blocks of larger size. We analyze the advantages and disadvantages of the lattice based crypto systems. We consider the attacks on these systems and propose defenses against these attacks. These defenses decrease the efficiency of the systems and make the systems inefficient. Cryptosystems based on NTRU allow to implement a directional encryption as well as the digital signature, so it is possible to build a public key infrastructure, which will be fully based on the NTRU cryptosystem. It makes this cryptosystem very important for practical use. So in the article we analyze whether NTRU can be implemented in practice. From our results we can conclude that crypto-system NTRU has such advantages as faster encryption and decryption of the messages, faster key generation and cryptographic resistance compared to RSA. The main advantage of this cryptographic system is resistance to quantum computer attacks. Thus, it can be argued that the crypto system NTRU is prospective. But it is also evident from these results that the key in NTRU system is bigger than in RSA that causes loss of efficiency. It should be also noted that the size of the signature in NTRU is not constant. It is also necessary to use concrete parameters for NTRU safety. It is also worth to note that even the right formed signature does not always pass the verification. In the article we also check the safety of NTRUSign without perturbation techniques before it has lost its efficiency in different threat models and show that the system is not secure in CPA model. So, we show that despite the fact that lattice-based cryptosystems for post-quantum period are proposed, the attacks on them are still fixed, and they are not effective enough. Thus, for the creation and implementation of safe and effective lattice-based post-quantum cryptosystems, it is necessary to conduct quite a big work.

Key words: attacks, security, cryptosystems, post-quantum, lattice, encryption
Quantum computers are able to destroy most, if not absolutely all conventional cryptosystems that are widely used in practice. Specifically, systems based on the problem of factoring integers (e.g., RSA). One of the alternatives are post-quantum systems, systems based on lattices. They are based on the complexity of the problems grids, the main of which is the problem of the shortest vector (SVP). The best known algorithm for grids problems is LLL algorithm \([1]\). This algorithm needs polynomial time with approximation factor \(2^O(n)\).

In 1987, Schnorr extended LLL algorithm and improved this approximation ratio, but he increased the time of performance of the algorithm \([2]\). Collision resistant hash functions based on lattices were proposed.

Hash Functions

Ajtai suggested the family of one-way functions \([3]\) the security of which is based on the worst case of the SVP with the approximation ratio of \(n^c\), where \(n\) is constant. Later Goldreich showed that these functions are resistant to the collision. To construct a family of hash functions the following integers are used as parameters: \(n, m, q\) and \(d\). Selection of parameter \(n\) defines the safety of hash function. The key to a hash function specified by the matrix \(M\) is chosen uniformly from \(\mathbb{Z}_q^{n \times m}\). Hash function is \(f_M: \{0, \ldots, d-1\}^m \rightarrow \mathbb{Z}_q^n\). The function maps mlog\(d\) bits to nlog\(q\) bits for input compression, \(m > \log q / \log d\). This hash function is very easy to implement, because we use only addition and multiplication modulo \(q\), whose size is \(O(\log n)\).

However, it should be noted that the problem of effectiveness of these functions is that the size of their key grows quadratically in \(n\), so that functions are ineffective. Due to the attacks on these functions by combinatorial method \([4,5]\), for the security of 100-bit, you need to use a key of 500,000 bits size. To increase effectiveness, matrix \(M\) can be changed by block matrix, each block of which is circulant matrix: \(M = [M^{(1)} | \ldots | M^{(m/n)}]\). Changing the structure of the matrix allows us to find the collision. When multiplying each block by constant vector \(v_i \ast 1 = (v_i, \ldots, v_i)\), the output of \(f_M\) function is also constant vector \(c \ast 1\). Since \(c\) can take \(q\) different values, the collision can be found in polynomial time \(q\) or even in \(O(\sqrt{q})\). This problem was solved by Peikert and Rosen and by Lyubashevsky and Micciancio, using ideal matrices. To construct a family of hash functions as parameters are used integers: \(n, m, q, d\) and the vector \(f \in \mathbb{Z}_n^q\). As the key we take \(m/n\) number of vectors \(a_1, \ldots, a_{m/n}\) chosen uniformly from \(\mathbb{Z}_q^n\). Hashing takes place as follows: \(f_M: \{0, \ldots, d-1\}^m \rightarrow \mathbb{Z}_q^n\), where \(f_M(y) = [F^*a_1 | \ldots | F^*a_{m/n}] y \mod q\). As \(M\) is taken as a block matrix with structured blocks \(M^{(i)} = F^*a_i\). For security, based on the “worst case”, vector \(f\) must satisfy the following conditions: 1. For two unit vectors \(u_1, u_2\) vector \([F^*u_1] u_2\) must have a small norm. 2. The polynomial \(f(x) = x^n + f_n x^{n-1} + \cdots + f_1 \in \mathbb{Z}[x]\) should be irreducible over integers. In \([6]\) the values of \(f\) are proposed, satisfying the both conditions: \(f = (1, \ldots, 1) \in \mathbb{Z}_n^q\), where \(n + 1\) is prime; \(f = (1, 0, \ldots, 0) \in \mathbb{Z}_n\), where \(n\) is power of two. The family of hash functions SWIFFT is an optimized version of the hash function described above, and is rather efficient due to the use of FFT in \(\mathbb{Z}_q^n\).

Public Key Encryption Schemes

The lattice based public key encryption schemes were proposed. Goldreich, Goldwasser, and Halevi offered cryptosystem: GGH, which is analogous of the cryptosystem McEliece, based on algebraic coding theory \([7]\). In 1999, Nguyen published an attack on the GGH, able to break the system for the matrix size up to 350. The attack is successful because of two vulnerabilities: The first one is that the error vectors are always very short compared to the lattice vectors. It makes easier to solve the closest vector problem, CVP. Second vulnerability is in the choice of the error vector. Nguyen offered several options for correcting the second vulnerability, but they all increase the first one. Because of this attack it is necessary to increase the size of the matrix, which makes the system inefficient. It should be also noted that this system is not semantically secure. In 1996 Hoffstein, Pipher and Silverman offered cryptosystem NTRU. NTRU is usually described as the ring poly-
mials cryptosystem. Nevertheless, the relationship between the public and private keys determines the grid, which is called a lattice NTRU. The basis of this lattice can be obtained from the public key. In addition, the private key of cryptosystem corresponds to certain short-vectors in this lattice. Thus, the natural attack on this system is an attempt to solve the problem of the shortest vector in the lattice. However, it is not necessary to use the lattice during encryption and decryption. The private key is a short vector \( f, g \in \mathbb{Z}^n \). Public Key
\[ h = p[R_f]g \mod q, \]
where \( p \) - is a small modulus, \( R \) is a cyclic rotation, transforming the vector \((x_1, x_2, \ldots, x_n)\), \( p \)- is a big modulus. Encryption takes place in the following way: a message is encrypted as a vector \( m \in \{1,0,-1\}^n \), for randomness vector \( r \in \{1,0,-1\}^n \) is used, containing \( d_r \) records - 1, and all the rest - 0. \( d_r \) is the boundary of an integer for \( r \). Cypher is calculated:
\[ c = m + [R^r h]r \mod q. \]
To decrypt: \( \text{red} = [R_f]c \mod q \); all red coefficients should lie in the range: \([-q/2; q/2]\).

The message we calculate:
\[ m = [R_f]^{-1} \text{red} \mod q. \]
Coppersmith and Shamir carried out an attack on the private key of the system. The aim of the attack is to find the vectors \( f \) and \( g \) or vectors that are close to them, which can be used for cipher decryption. Vector \((f, g)\) and its pairwise rotations are the vectors in NTRU Lattice. The Euclidean norm of this vector is \((2d_q - 1 + 2d_q)^{1/2}\). NTRU lattice volume is defined as \( \det(L) = q^n \) and lattice dimension is \( 2n \). It is expected that the shortest vector has a length approximately \( \det(L)^{1/2n} = q^{1/2} \). Because \((2d_q - 1 + 2d_q)^{1/2} < q^{1/2}\), with a high probability vector \((f, g)\) is the shortest vector of the lattice, so its pairwise rotations are also candidates for the shortest vector. These vectors can also be used to decrypt the cipher. Coppersmith and Shamir have shown that if the resultant vector is greater than \((f, g)\) at some constant value, it is possible to combine several of these vectors and to decrypt the cipher. When using modern lattice reduction algorithms the approximation coefficient of the shortest vector problem is still exponential in \( n \).

Howgrave-Graham has shown that it is possible to improve the attack, if it is combined with combinatorial attack. To resist this attack, it is necessary to increase the parameters of the system. A public key of the system has the size of \( n \log_q(q) \). Algorithm execution time depends on \( n \) and on the boundaries of integers. To increase the efficiency of the system as the private key we can take \( f = u + pf \), where \( u \) is the first unit vector and records \( f \) are randomly selected from \((1, 0, -1)\) depending on the \( d_f \). In this way we increase efficiency as we get rid of the multiplication step.

NTRU cryptosystem is much more effective than the GGH and AD. AD cryptosystem was proposed by Ajtai-Dwork in 1997. This system is defined in the Euclidean setting of vector space, using the standard Euclidean norm. Later Nguyen and Stern carried out an attack on the private key of the system. Regev proposed cryptosystem LWE [8], this cryptosystem is the most effective system based on lattices. The private key of the system is matrix \( S \in \mathbb{Z}_{q \times n} \), chosen randomly. \( q, n \) and \( l \) are integers. The public key is \( (A, P = AS + E) \in Z_{q \times n} \times Z_{q \times l} \), where \( A \in Z_{q \times n} \), chosen randomly. \( E \in Z_{q \times l} \), where data is selected according to the distribution \( \Psi_{\alpha} \), over \( Z_{q} \), obtained by choosing a standard variable with a mean value 0 and with deviation \( \pm q/(2A)^{1/2} \), approximating the result to the nearest integer and decreasing it modulo \( q \). Encryption takes place as follows: given an element from the message space \( e \in Z_{q}^l \) messages and the public key \((A, P)\), vector \( v \in \{-d, -d+1, \ldots, d\}^n \) is selected randomly, \( -d \) - is integer, and the cipher is issued: \((u = A^v, c = P^v + f(e)) \in Z_{q}^l \times Z_{q}^l \) is issued. Decryption takes place as follows: given cypher \((u, c) \in Z_{q}^l \times Z_{q}^l \) and private key \( S \in Z_{q \times n} \) and \( f^l(C-S^lu) \) is issued. The above given is implemented rather easily using only addition and multiplication modulo \( q \). To optimize the execution time, it is possible to set \( t \) as the power of two and delay the operation reduction operations. This system has the following properties: Public key size: \( nl \log q \); Private key size: \( m(n+1) \log q \); Message size: \( 1 \log t \); Cypher size: \((n+1) \log q \); Encryption blowup factor: \((1+n/l)\log(\log t)\); Operations needed for one bit encryption: \( O(m(1+n/l)) \); Operations needed for one bit decryption: \( O(n) \).
Digital Signature Schemes

Digital signature systems based on lattices were also offered. The private key is the secret matrix \( S \), whose columns form the basis of lattice \( L \), the base consists of short, almost orthogonal vectors. The public key is an open matrix \( B \), which forms a bad basis of the same matrix. Best of all is to use HNF (Hermite Normal Form) of matrix \( S \). To encrypt the message we map it to the point \( m \in \mathbb{R}^n \), using the secret basis, then we approximate \( m \) to the nearby point \( n \in L(S) \), using the secret basis. It takes place using Babai’s round-off procedure. \( n = S[S^{-1}m] \) To verify the signature and message pair \( (m, n) \) we must verify that \( n \in L(B) = L(S) \), using the public key \( B \), and that the distance from \( n \) to \( m \) is small. Gentry and Szydlo noticed that each signature leaks information about the secret key. In some years Nguyen and Regev showed that this leak leads to the attack on secret key. The main idea of the attack is that \( m-n \) difference is distributed uniformly. Therefore, considering a sufficient number of such pairs, we finally get to the algorithmic problem – so-called hidden parallelepiped problem. The solution of this problem leads to this attack. To protect against these attacks the most effective measures are the perturbation techniques. Hidden parallelepiped is replaced by more complex figures that helps to prevent the given attacks. These protection techniques slow down the generation of the signature, and increase the size of the private key. Gentry, Peikert and Vaikuntanathan identified a scheme called «preimage sampleable trapdoor functions» and showed how to construct it on the basis of the “worst case” lattice problems. This construction can be used as an option of GGH with the security proof. This scheme does not leak information about the secret base. This is achieved by replacing Babai’s round-off procedure with Gaussian sampling procedure. System has quadratic complexity, as in the size of the key, also in the case of verification time. Lyubashevsky and Micciancio proposed a scheme of an electronic signature with security, based on the “worst case” lattice problems, the scheme is asymptotically efficient as in the case of the key size, also in the case of the verification time it has a linear complexity [10]. This system uses a new one-time signature system based on hashing. This type of schemes can be converted into full signature schemes using standard tree construction with only logarithmic loss of efficiency. One-time signature scheme is based on a hash function, collision resistant, based on ideal lattices. The hash function \( h \), may be selected during key generation, or be a fixed parameter. The input data of function are vectors \( v_1, \ldots v_{mn} \in \mathbb{Z}_q^n \). The private key of the function is randomly chosen pair \( x_1, \ldots x_{mn} \in \mathbb{Z}_q^n \). Messages are presented in the form of short vectors: \( m \in \mathbb{Z}_q^n \). Signature is calculated as follows: \( \text{sig} = (\text{sig}_1, \ldots \text{sig}_{mn}) = ([F^*m]x_1, v_1, \ldots [F^*m]x_{mn} + v_{mn}) \mod q \). To verify the signature, it is checked whether \( \text{sig} \) is the sequence of short vectors which is hashed to \( [F^*m]X + V \mod q \). Security of the scheme is based on the fact that even after the attacker sees the signature, private key value is hidden from him.

NTRU Use in Practice.

Cryptosystems based on NTRU allow to implement a directional encryption as well as the digital signature, so it is possible to build a public key infrastructure, which will be fully based on the NTRU cryptosystem. It makes this cryptosystem very important for practical use.

From our results we can conclude that crypto-system NTRU has such advantages as faster encryption and decryption of the messages, faster key generation and cryptographic resistance compared to RSA. The main advantage of this cryptographic system is resistance to quantum computer attacks. Thus, it can be argued that the crypto system NTRU is perspective.
To check the possibility of NTRU use in practice, we compared it with RSA system which is widely used in practice. Table 1 provides a comparison of time needed for key generation in the cases of RSA and NTRU.

**Table 1. Relation between key length and generation time**

<table>
<thead>
<tr>
<th>Cryptosystem</th>
<th>Key length</th>
<th>Generation time (ms)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSA</td>
<td>512</td>
<td>360</td>
</tr>
<tr>
<td></td>
<td>1024</td>
<td>1280</td>
</tr>
<tr>
<td></td>
<td>2048</td>
<td>4195</td>
</tr>
<tr>
<td>NTRU</td>
<td>1169</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>1841</td>
<td>7.5</td>
</tr>
<tr>
<td></td>
<td>4024</td>
<td>17.5</td>
</tr>
</tbody>
</table>

![Fig. 1. Illustrates a comparison of time needed for key generation in the cases of RSA and NTRU.](image)

**Table 2. Encryption and decryption time**

<table>
<thead>
<tr>
<th>Cryptosystem</th>
<th>Key length</th>
<th>Message encryption (block/s)</th>
<th>Message Decryption (block/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RSA</td>
<td>512</td>
<td>2440</td>
<td>120</td>
</tr>
<tr>
<td></td>
<td>1024</td>
<td>930</td>
<td>20</td>
</tr>
<tr>
<td></td>
<td>2048</td>
<td>310</td>
<td>3</td>
</tr>
<tr>
<td>NTRU</td>
<td>1169</td>
<td>5940</td>
<td>2820</td>
</tr>
<tr>
<td></td>
<td>1841</td>
<td>3680</td>
<td>1620</td>
</tr>
<tr>
<td></td>
<td>4024</td>
<td>1470</td>
<td>610</td>
</tr>
</tbody>
</table>

Table 2 provides a comparison of time needed for encryption and decryption of message block in RSA and NTRU cases.

Table 3 provides comparison of security in RSA and NTRU cases.

Table 4 provides comparison of efficiency between RSA and NTRU.

From our results we can conclude that crypto-system NTRU has such advantages as faster encryption and decryption of the messages, faster key generation and cryptographic resistance compared to RSA. The main advantage of this cryptographic system is resistance to quantum computer attacks. Thus, it can be argued that crypto system NTRU is perspective.
But it is also evident from these results that the key in NTRU system is bigger than in RSA that causes loss of efficiency. It should be also noted that the size of the signature in NTRU is not constant. It is also necessary to use concrete parameters for NTRU safety. It is also worth to note that even the right-formed signature does not always pass the verification.

Analysis of NTRUsign in Different Threat Models.

Let us analyze NTRUsign without perturbation techniques before it has lost its efficiency in different threat models.

In the treat model “ciphertext-only attack” (coa), attacker sees only the plain text and he cannot say anything about \( m \) and \( n \), so the difference between \( n \) and \( m \) does not give him any information. So in this treat model the scheme is secure.

To get computational indistinguishability in the treat model “Known-plaintext attack” (KPA), we fix alg - algorithm, a – attacker and define a randomized experiment: 1. \( a(\{1\}) \) outputs \( m_0, m_1 \in \{0,1\} \) of equal length; 2. \( \text{ans}' \leftarrow A(c) \); at succeeds if \( \text{ans} = \text{ans}' \), and experiment evaluates to 1 in this case. The scheme is secure if: \( \Pr[\text{Exp}_{a,\text{alg}}(n) = 1] \leq 1/2 + \varepsilon(n) \). In this case attacker signs two messages, and the oracle encrypts one of them, the experiment succeeds, if the attacker provides a correct guess with probability more than \( 1/2 + \varepsilon(n) \). \( \varepsilon(n) \) is negligible, \( n \) is security parameter. In the treat model “Chosen-plaintext attack”, CPA, the attacker can encrypt as many messages as he wants and only afterwards can apply the experiment. In this case according [11], 400 tries will be absolutely enough to break the system, so the system is not secure in CPA threat model.

Conclusion

As we can see, despite the fact that on the proposed lattice-based cryptosystems for post-quantum period the attacks are fixed and they are not effective. Thus, for the creation and implementation of safe and effective lattice-based post-quantum cryptosystems, it is necessary to conduct quite a big work.

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�ანტიკორორი კომპიუტერების ანგარიში კომპიუტერული გამოყენების პრაქტიკაში

გასაღები კომპიუტერების ანგარიში კომპიუტერული გამოყენების პრაქტიკაში. ჩოგრძელავდები, სახელწოდები, როდესაც გადაიქცა იმპლური პრობლემის ფენადოს რუკაზე (შედეგით კავშირით RSA). სხვა კომპიუტერებს ჩამოვალ არა RSA იმპლური ხანის გადამდინარეთ თეორემა გარკვიეთ კატასტროფული კომპიუტერული პრობლემების ათაშში წარდგენს. გახადო შებიძა ამოღვაწეთ გამოყენების საფუძველმა ჰქონოდან. რეალური ალტერნატიული სისტემები არის პორტალური ქალაქები რომლებიც დაფუძნებული სისტემები, როდესაც გამოყენების სისტემათმსოფლის მაღალი დონით, და ლოკალური აღრიცხვა გარემო ჰქონდა (worst-case hardness). სისტემის ფუნქციას მეტნიდან პრობლემების ხორციდან, პასუხის დროს პრობლემები მდგომარეობდნენ სქემოთური ფუნქცია (SVP) ნებისმიერები. აქვთილი, ვენას პატივით გავრცელებით, ხოლო გარკვეთ წარმოშობით გამოთქმის, რომ სიხშირე α(n) ჯერ უნდა მიიღო გამოყენების სწრაფი გამოცდის სიხშირე. Α(�) ნებისმიერები არსებობს კომპლექსურ ქვეყანაში, რომლიც სხვა გარკვეთით გამოყენების ქვეყანაში. მეორე, პრობლემები წარმოადგენდნენ გამოყენებით LLL ალგორითმზე, რომლის სიდიდეებში გამოყენების რაქმის სზრება გაქონდა. მოქმედ LLI ალგორითმის სიდიდე დიდი ქვეყანის სახით გამოყოფა გამოყენების სიდიდე ძალიან მაღალი მდგომარეობდა. სიხშირე პრობლემის პასუხები გამომდინარეობდა წარმოადგენდა გარკვეთით ქვეყანის ქვეყანაში. 1987 წლის მონაცემი გამოყენებით LLI ალგორითმზე, რომ ბაზარი გაუწვო არსებობა პრობლემების ქვეყანაში. მაქს ამოღვაწეთ არაფრითი ჰქონდა რამდენიმე შეცდომა გამოცდის, ტრადიციული ქვეყანის გარკვეთით NTRU-ზე დაფუძნებული ქონაქმუნი გამოყენების თავის ქონაქმუნი მოქმედირების ფერდობების, მეორე წარმოადგენდა პრობლემებში წარმოადგენდა. პრობლემებს სეიზონში გამოყენებით, ფოლადი თუ არა NTRU-ზე გამოყენება პრობლემებზე. მეორე შეფასებას გამოყენება დაფხვევად იყო, რომ სინამდვილი NTRU-ზე გამოყენება გამოყენების სრული წარმოადგენდა და გამოყენება გამოყენებით გამოყენების ჰქონდა. წარმოადგენდა RSA-ზე გამოყენება ჯილდო გამოყენება სისტემათმსოფლის არა დაფხვილი სახელწოდებით კომპლექსური არსებობის წარმოადგენდა. ამდენ გამოყენება ხარ, რომ NTRU არის გამოყენებული კომპიუტერზე. გამოცდა, რომ გამოყენება NTRU-შემთხვევაში არის უფრო დიდი გამოყენებით, რაც თუ არა განსაზღვრა სარგებლობები გამოყენება. იშვიათი ხარ, რომ NTRU სწორი გამოყენება შთამოვანი არ არის ბუქვა. აქვთილი,
Avtandil Gagnidze, Maksim Iavich, Giorgi Iashvili

mniSvnelovania konkretuli parametrebis gamoyeneba NTRU sistemis usafrTxoebis Tvis.

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