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Cybernetics

On Network Maintenance Problem. Open Markovian Queuing System with Bifurcation of Arrivals

Archil Prangishvili*, Hamlet Meladze, Revaz Kakubava§, Nino Svanidze#**

**Academy Member, Georgian Technical University (GTU) Tbilisi, Georgia*
***Georgian Technical University (GTU), Tbilisi, Georgia*
§Georgian Technical University (GTU), Tbilisi, Georgia
#Faculty of Physics-Mathematics and Computer Sciences, Batumi Shota Rustaveli State University, Batumi, Georgia

ABSTRACT. In the present paper a multi-component redundant system with unreliable, repairable units is considered. Two types of maintenance operations are performed in the system: 1) the replacement of the failed main unit by the redundant one; 2) the repair of the failed unit. The numbers of redundant units, replacement and repair facilities in the system are arbitrary. The open exponential queuing model for the system dependability and performability analysis is constructed in the form of infinite system of ordinary linear differential equations. In steady state, it is reduced to the infinite system of linear algebraic equations. At present the system is still being investigated.

Key words: replacement, repair, queuing model

During the last 70 years, the number of works in the reliability field has become huge (textbooks, handbooks, monographs and many other materials).

Among them, in classical Mathematical Theory of Reliability (MTR) one book is the most prominent. It is Richard E. Barlow and Frank Proschan "Mathematical Theory of Reliability' first published by John Wiley & Sons, New York, 1965, and republished in 1996 by SIAM (thereafter referred to as BP). It should be noted that this book became the basis of the modern MTR.

Since BP is the most prominent in the English speaking countries, in our paper we shall mainly use the terms and language of BP with some necessary changes for our objectives.

The most important aspects in BP concerning complex systems are considered within the framework of repairman (maintenance) problem. According to BP, we assign repair and replacement to the maintenance operations. To clearly understand our further statement, we quote an excerpt from BP: "Suppose we are given m identical units stochastically independent of one another and supported by n spare units. Suppose that each fails according to some lifetime distribution. Furthermore, suppose that we have a repair facility capable of repairing k units simultaneously. Obviously, we could consider the facility as consisting of k repairmen.

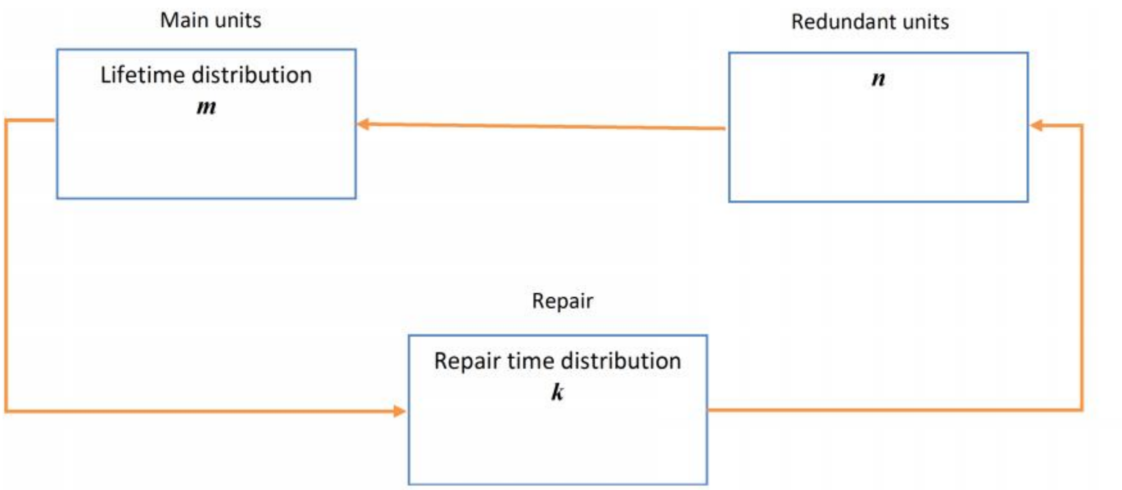

Fig. 1. Diagram illustrating repairman (maintenance) problems. From Richard E. Barlow, Frank Proschan, Mathematical Theory of Reliability. SIAM, 1996.

The following queue discipline is observed. If all repairmen are busy, each new failure joins a waiting line and waits until a repairman is freed. We assume that the repair times are also independent, identically distributed random variables with some repair time distribution".

The diagram in Fig.1 will be helpful in explaining various models of repairman problems.

Unfortunately, classical mathematical theory of reliability does not allow to solve the problems of dependability and performance analysis of large scale territorially distributed networks, because this theory is mainly equipment (machine) oriented [1- 8]. Fortunately, in this classical theory, the basics was set for creating new methods and models corresponding to the above problems. For these purposes, further extending, development and deepening of classical repairman (maintenance) models are necessary. In particular, we need to consider in the diagram illustrating repairman problem which takes into account the following factors [9-16]:

1) The case $m = \infty$. We give an explanation of a specific example: the number of Radio Base Station (RBS) in modern mobile communication networks may be hundreds, thousands and more. That means that in mathematical models we can consider the set of RBS as infinite ($m = \infty$) source of failures. Due to the same factor, we can consider the total failure rate to be constant. Consequently, we will have a Poisson stream of requests to maintenance facilities. As it is known, this is very important for the construction of suitable mathematical models and also for their investigation;

2) We need to include additional rectangle in Fig.1 pointing replacement operation, as particular independent maintenance operation, which is not negligible (it requires some finite time, which is necessary to be taken into consideration).

The fact is that in traditional cases of redundancy the main and redundant units, as a rule, are territorially concentrated at the same place and the replacement of the failed main unit with a redundant one means the latter's switching over, which is often automatically performed, and its duration is negligibly small. In other cases, for example, in two unit systems, main and redundant units are connected in parallel and both are operating, although only one is in actual service. In such cases, after failure of the main unit, redundant one continues servicing without any replacement.

In modern networks of the above type, however, redundant units are not directly linked to the main ones. They are placed at specific storages and may be located at the distance of tens, hundreds and sometimes

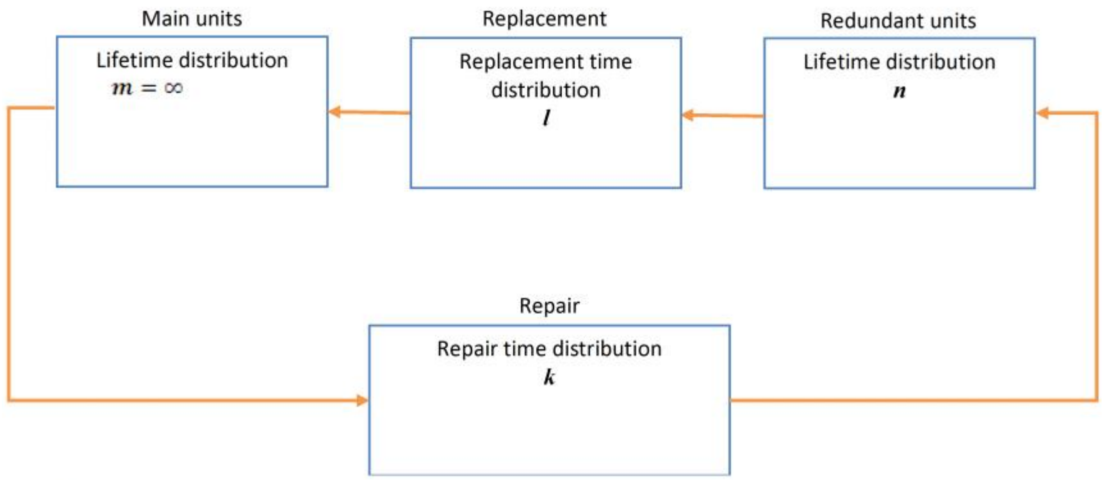

Fig. 2. Diagram illustrating the extended repairman (maintenance) problem.

thousands of kilometers away from the active units. Therefore, the delivery time of the redundant unit to the place of the failed main one is quite essential.

At the same time, in practical cases, due to various reasons, before the start of the delivery operation of the redundant unit, there passes quite some time, which is often many times greater than the delivery time itself. In addition, the replacement operation, apart from the delivery of the redundant unit to the main unit's place, includes other sub-operations, which are necessary for the redundant unit to continue the main unit functions. Under such circumstances the mean replacement time is not insignificant, and it often reaches 20-40 % of the mean repair time. Moreover, the replacement operation, as a rule, is performed not by a repair facility, but by a special replacement channel. Therefore, the replacement of the failed main unit by the redundant one quite naturally becomes an independent maintenance operation;

3) We need to construct repairman models for cases when redundant units are not identical with main ones, in the sense that they replace main units temporarily until their repair. As a rule, this factor creates additional difficulties in modelling;

4) We also need to consider the cases where redundant units themselves may be subject to failure and, consequently, be failed when requested.

In other similar cases, if the main unit fails, it may be temporarily replaced by a mobile redundant unit until its repaired if such is available at that time.

Taking into consideration all the above four factors (or some of them in different variants) in mathematical models of large scale territorially distributed systems is of paramount importance.

The diagram in Fig. 2 will be helpful in explaining various models of the extended repairman problem (network maintenance problem).

As a matter of fact, we are talking about network maintenance problem instead of classical machine maintenance problem.

As we see, the network maintenance problem is a peculiar combination of classical replacement problem and classical repairman one. We believe this combination to be productive.

Subject of the Study and its Initial Mathematical Description

The investigation subject of this paper is a multi-component redundant system with unreliable repairable units.

The system consists of identical active and redundant units, their numbers are *m* and *n* respectively. The redundant units are designated for permanent replacement of main components in case of their failure. It is supposed that for the normal operation of the system, the serviceability of all active units is desirable. However, if their number is less, then the system continues to function, but with lower economic effectiveness.

The total failure rate of all active units is α. The redundant ones are not the subject to failures. A failed active unit is replaced by a serviceable redundant one if there is an available unit in the system. In the opposite case the replacement will be performed after the availability of the redundant unit. The failed units are repaired, become identical with the new ones and pass to the group of redundant units. The system has one replacement facility and one repair facility. The replacement time and repair time are random variables with distribution functions *F* and *G*, respectively. When maintenance facilities are busy, requests for replacement or repairs are queued. Service discipline is FCFS (first come, first served). As we see, in a natural way we have queuing system with two types of maintenance operations – replacement and repair. We consider here the case, where *m* is a large number (in practice it might be tens, hundreds, thousands and more), and we will suppose that we have an infinite source of requests and will get an open queuing system for two parallel maintenance operations - replacements and repairs.

The request for the replacement arises due to failure of the active unit. The same event generates a request for repair. In this way, the necessity of two parallel service operations arose.

To this day, neither in the reliability theory nor in the queuing theory the above problems have been investigated in general. At the same time, modern research methods of Markov and semi-Markov processes make it possible to construct and to analyze such models in the framework of the mathematical theory of reliability and queuing theory [10, 11].

During the last 10-12 years the experts of Georgian Technical University (GTU) achieved notable success in this direction [9-16].

Namely, the queuing systems of the above type (where in the arriving stream of homogenous events there arise demands for two parallel service operations) were first introduced by GTU experts and have not been yet considered by the other authors.

The present paper is further generalization of the model in [9].

Mathematical Model

In this section, we construct and investigate the mathematical model for the case where $m=\infty$, *n* is arbitrary. The replacement and repair times have exponential distribution functions with parameters ª and «, respectively.

To describe the considered system we introduce the random processes, which determine the states of the system at the time *t*;

$i(t)$ - the number of units missing in the group of active units;

$j(t)$ - the number of non-serviceable (failed) units in the system.

Denote, $P(i,j,t)=P\{i(t)=i;\quad j(t)=j\},\quad i=\overline{1,m}\qquad j=\overline{0,n+i}.$

Proceeding in the usual way, we can set up the basic difference equations which relate the probability of being in a certain state at time $t+\Delta t$ to the probabilities of being in various states at time *t*. From these difference equations we obtain the infinite systems of the ordinary linear differential equations.

Theorem 1. *For n>0 probabilities P(i,j,t) satisfy the following infinite system of the ordinary linear differential equations (Kolmogorov equations).*

$$\frac{dP(0,0,t)}{dt}=-(\alpha+n\beta)P(0,0,t)+\lambda P(1,0,t)+\mu P(0,1,t),$$

$$\frac{dP(i,0,t)}{dt}=-(\alpha+n\beta+i\lambda)P(i,0,t)+(i+1)\lambda P(i+1,0,t)+\mu P(i,1,t),\quad 0<i<k,$$

$$\frac{dP(i,0,t)}{dt}=-(\alpha+(n+i-k)\beta+k\lambda)P(i,0,t)+k\lambda P(i+1,0,t)+\mu P(i,1,t),\quad i\geq k,$$

$$\frac{dP(0,j,t)}{dt}=-(\alpha+(n-j)\beta+j\mu)P(0,j,t)+(n-j+1)\beta P(0,j-1,t)+\lambda P(1,j,t)+$$
$$+(j+1)\mu P(0,j+1,t),\quad 1\leq j<l,$$

$$\frac{dP(0,j,t)}{dt}=-(\alpha+(n-j)\beta+l\mu)P(0,j,t)+(n-j+1)\beta P(0,j-1,t)+\lambda P(1,j,t)+$$
$$+l\mu P(0,j+1,t),\quad l\leq j<n,$$

$$\frac{dP(0,n,t)}{dt}=-(\alpha+l\mu)P(0,n,t)+\beta P(0,n-1,t)+\lambda P(1,n,t),\tag{1}$$

$$\frac{dP(i,j,t)}{dt}=-(\alpha+(n-j)\beta+i\lambda+j\mu)P(i,j,t)+\alpha P(i-1,j-1,t)+(n-j+1)\beta P(i,j-1,t)+$$
$$+(i+1)\lambda P(i+1,j,t)+(j+1)\mu P(i,j+1,t),\quad 1\leq i<k,\quad 1\leq j<l,$$

$$\frac{dP(i,j,t)}{dt}=-(\alpha+(n-j)\beta+i\lambda+l\mu)P(i,j,t)+\alpha P(i-1,j-1,t)+(n-j+1)\beta P(i,j-1,t)+$$
$$+(i+1)\lambda P(i+1,j,t)+l\mu P(i,j+1,t),\quad 1\leq i<k,\quad l\leq j\leq n,$$

$$\frac{dP(i,j,t)}{dt}=-(\alpha+(n+i-j)\lambda+l\mu)P(i,j,t)+\alpha P(i-1,j-1,t)+(n+i-j+1)\lambda P(i+1,j,t)+$$
$$+l\mu P(i,j+1,t),\quad 1\leq i<k,\quad n<j<n+i,$$

$$\frac{dP(i,n+i,t)}{dt}=-(\alpha+l\mu)P(i,n+i,t)+\alpha P(i-1,n+i-1,t)+\lambda P(i+1,n+i,t),\quad i\geq 1,$$

$$\frac{dP(i,j,t)}{dt}=-(\alpha+(n+i-j-k)\beta+j\mu+k\lambda)P(i,j,t)+(n+i-j+1-k)\beta P(i,j-1,t)+$$
$$+\lambda kP(i+1,j,t)+(j+1)\mu P(i,j+1,t),\quad i\geq k,\quad 1\leq j<l,$$

$$\frac{dP(i,j,t)}{dt}=-(\alpha+(n+i-j-k)\beta+l\mu+k\lambda)P(i,j,t)+\alpha P(i-1,j-1,t)+$$
$$+(n+i-j+1)\beta P(i,j-1,t)+k\lambda P(i+1,j,t)+l\mu P(i,j+1,t),\quad i\geq k,\quad l\leq j\leq n+i-k,$$

$$\frac{dP(i,j,t)}{dt}=-(\alpha+(n+i-j)\lambda+l\mu)P(i,j,t)+\alpha P(i-1,j-1,t)+(n+i+1-j)\lambda P(i+1,j,t)+$$
$$+l\mu P(i,j+1,t),\quad i\geq k,\quad n+i-k<j<n+i.$$

Theorem 2. *For n=0 probabilities P(i,j,t) satisfy the following infinite system of ordinary linear differential equations (Kolmogorov equations):*

$$\frac{dP(0,0,t)}{dt} = -\alpha P(0,0,t) + \lambda P(1,0,t),$$

$$\frac{dP(i,0,t)}{dt} = -(\alpha + i\lambda)P(i,0,t) + (i+1)\lambda P(i+1,0,t) + \mu P(i,1,t), \quad 0 < i < k,$$

$$\frac{dP(i,0,t)}{dt} = -(\alpha + k\lambda)P(i,0,t) + k\lambda P(i+1,0,t) + \mu P(i,1,t), \quad i \ge k,$$

$$\frac{dP(i,j,t)}{dt} = -(\alpha + (i-j)\lambda + j\mu)P(i,j,t) + \alpha P(i-1,j-1,t) + (i+1)\lambda P(i+1,j,t) +$$
$$+ (j+1)\mu P(i,j+1,t), \quad 1 \le i-j < k, \quad 0 < j < l, \qquad (2)$$

$$\frac{dP(i,j,t)}{dt} = -(\alpha + k\lambda + j\mu)P(i,j,t) + \alpha P(i-1,j-1,t) + k\lambda P(i+1,j,t) + (j+1)\mu P(i,j+1,t),$$
$$i-j \ge k, \quad 0 < j < l,$$

$$\frac{dP(i,j,t)}{dt} = -(\alpha + (i-j)\lambda + l\mu)P(i,j,t) + \alpha P(i-1,j-1,t) + (i-j+1)\lambda P(i+1,j,t) + l\mu P(i,j+1,t),$$
$$1 \le i-j < k, \quad j \ge l,$$

$$\frac{dP(i,j,t)}{dt} = -(\alpha + k\lambda + l\mu)P(i,j,t) + \alpha P(i-1,j-1,t) + k\lambda P(i+1,j,t) + + l\mu P(i,j+1,t),$$
$$i-j \ge k, \quad j \ge l,$$

$$\frac{dP(i,i,t)}{dt} = -(\alpha + i\mu)P(i,i,t) + \alpha P(i-1,i-1,t) + \lambda P(i+1,i,t), \quad 1 \le i < l,$$

$$\frac{dP(i,i,t)}{dt} = -(\alpha + l\mu)P(i,i,t) + \alpha P(i-1,i-1,t) + \lambda P(i+1,i,t), \quad i \ge l.$$

It can be proved that for these systems the limit of $P(i,j,t)$, with $t \Rightarrow \infty$ exists for all i,j, if ~<kª and ~<l«.

Denote $P(i,j) = \lim_{t\to\infty} P(i,j,t)$ Letting $t \Rightarrow \infty$ in (1) and (2), we obtain an infinite system of linear algebraic equations with respect to $P(i,j)$.

For n>0 we have (together with normalizing condition):

$$(\alpha + n\beta)P(0,0,t) = \lambda P(1,0,t) + \mu P(0,1,t),$$

$$(\alpha + n\beta + i\lambda)P(i,0,t) = (i+1)\lambda P(i+1,0,t) + \mu P(i,1,t), \ 0 < i < k,$$

$$(\alpha + (n+i-k)\beta + k\lambda)P(i,0,t) = k\lambda P(i+1,0,t) + \mu P(i,1,t), \quad i \ge k,$$

$$(\alpha + (n-j)\beta + j\mu)P(0,j,t) = (n-j+1)\beta P(0,j-1,t) + \lambda P(1,j,t) + (j+1)\mu P(0,j+1,t),$$
$$1 \le j < l, \qquad (3)$$

$$(\alpha + (n-j)\beta + l\mu)P(0,j,t) = (n-j+1)\beta P(0,j-1,t) + \lambda P(1,j,t) + l\mu P(0,j+1,t), \ \ l \le j < n,$$

$$(\alpha + l\mu)P(0,n,t) = \beta P(0,n-1,t) + \lambda P(1,n,t),$$

$$(\alpha + (n-j)\beta + i\lambda + j\mu)P(i,j,t) = \alpha P(i-1,j-1,t) + (n-j+1)\beta P(i,j-1,t) + (i+1)\lambda P(i+1,j,t) +$$
$$+ (j+1)\mu P(i,j+1,t), \ \ 1 \le i < k, \ \ 1 \le j < l,$$

$$(\alpha + (n-j)\beta + i\lambda + l\mu)P(i,j,t) = \alpha P(i-1,j-1,t) + (n-j+1)\beta P(i,j-1,t) + (i+1)\lambda P(i+1,j,t) +$$
$$+ l\mu P(i,j+1,t), \ 1 \le i < k, \ l \le j \le n,$$

$$(\alpha+(n+i-j)\lambda+l\mu)P(i,j,t)=\alpha P(i-1,j-1,t)+(n+i-j+1)\lambda P(i+1,j,t)+l\mu P(i,j+1,t),$$

$$(\alpha+(n+i-j)\lambda+l\mu)P(i,j,t)=\alpha P(i-1,j-1,t)+(n+i-j+1)\lambda P(i+1,j,t)+l\mu P(i,j+1,t),\ \ n<j<n+i,$$

$$(\alpha+l\mu)P(i,n+i,t)=\alpha P(i-1,n+i-1,t)+\lambda P(i+1,n+i,t),\ \ i\geq 1,$$

$$(\alpha+(n+i-j-k)\beta+j\mu+k\lambda)P(i,j,t)=(n+i-j+1-k)\beta P(i,j-1,t)+\lambda kP(i+1,j,t)+ \\ +(j+1)\mu P(i,j+1,t),\ \ i\geq k,\ 1\leq j<l,$$

$$(\alpha+(n+i-j-k)\beta+l\mu+k\lambda)P(i,j,t)=\alpha P(i-1,j-1,t)+(n+i-j+1)\beta P(i,j-1,t)+ \\ +k\lambda P(i+1,j,t)+l\mu P(i,j+1,t),\quad i\geq k,\ l\leq j\leq n+i-k,$$

$$(\alpha+(n+i-j)\lambda+l\mu)P(i,j,t)=\alpha P(i-1,j-1,t)+(n+i+1-j)\lambda P(i+1,j,t)++l\mu P(i,j+1,t), \\ i\geq k,\ n+i-k<j<n+i,$$

$$\sum_{i=0}^{\infty}\sum_{j=0}^{n+i}P(i,j)=1.$$

For n>0 we have (together with the normalizing condition):

$$\alpha P(0,0,t)=\lambda P(1,0,t),$$

$$(\alpha+i\lambda)P(i,0,t)=(i+1)\lambda P(i+1,0,t)+\mu P(i,1,t),\quad 0<i<k,$$

$$(\alpha+k\lambda)P(i,0,t)=k\lambda P(i+1,0,t)+\mu P(i,1,t),\quad i\geq k,$$

$$(\alpha+(i-j)\lambda+j\mu)P(i,j,t)=\alpha P(i-1,j-1,t)+(i+1)\lambda P(i+1,j,t)+(j+1)\mu P(i,j+1,t), \\ 1\leq i-j<k,\ 0<j<l,$$

$$(\alpha+k\lambda+j\mu)P(i,j,t)=\alpha P(i-1,j-1,t)+k\lambda P(i+1,j,t)+(j+1)\mu P(i,j+1,t), \\ i-j\geq k,\ 0<j<l,$$

$$(\alpha+(i-j)\lambda+l\mu)P(i,j,t)=\alpha P(i-1,j-1,t)+(i-j+1)\lambda P(i+1,j,t)+l\mu P(i,j+1,t), \\ 1\leq i-j<k,\quad j\geq l, \tag{4}$$

$$(\alpha+k\lambda+l\mu)P(i,j,t)=\alpha P(i-1,j-1,t)+k\lambda P(i+1,j,t)++l\mu P(i,j+1,t),\ \ i-j\geq k,\ \ j\geq l,$$

$$(\alpha+i\mu)P(i,i,t)=\alpha P(i-1,i-1,t)+\lambda P(i+1,i,t),\ \ 1\leq i<l,$$

$$(\alpha+l\mu)P(i,i,t)=\alpha P(i-1,i-1,t)+\lambda P(i+1,i,t),\ \ i\geq l.$$

$$\sum_{i=0}^{\infty}\sum_{j=0}^{i}P(i,j)=1.$$

After finding the probabilities $P(i,j)$, it is easy to calculate all steady-state dependability and performability measures for the considered system.

Conclusion

Up to now, there was no general approach of classic repairman problem. There exists a relatively complete investigation for the case, where *F* and *G* functions are exponential. This case generates classical birth-and-death processes. The solutions often are given in analytical form and their practical application is rather easy.

As for semi-Markov repairman models, only special cases have been solved up to now. The moreover, it must be obvious that it is impossible to create a general model for network maintenance problem. We need to construct and investigate a set of special models (tens and more) for particular types of *H* and *G* functions as well as for particular values of parameters *n, k* and *l*.

At the same time, in our case the bifurcation of failures is particularly important.

As a result, unlike the classical queueing theory, we need multi-dimensional Markov processes with two discrete components. It is the bifurcation factor, that is the reason of the above situation.

The given paper is a further generalization of the model in [9]. Namely, the number **k** of replacement facilities, and the number l of repair facilities in the system are arbitrary.

In this case we need to use two-dimensional Marcov processes unlike the classical exponential queueing models, where the birth-and-death process is very productive tool.

It is natural that the investigation of new, two-dimensional models is much more complicated. In most cases solution of such models can be only by numerical methods. It is clear that this work requires huge efforts of highly competent mathematicians. On the other hand, it is worth being carried out due to first-rate importance of the network maintenance problem.

Note that the study and solution of infinite system of equations, as a rule, is a very complex problem, often unsurmountable. However the matrices of our systems (3) and (4) are highly sparse and this allows us to advance in their investigation.

Namely, the problem of existence and uniqueness of the solution has been investigated. Also, the numerical algorithms have been developed, making it possible to find the approximate solution by means of finite arithmetical operations. Finally, the error of the approximate solution was estimated.

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კიბერნეტიკა

ქსელის მომსახურების პრობლემის შესახებ. ღია მარკოვული რიგების სისტემა შემოსვლების ბიფურკაციით

ა. ფრანგიშვილი*, ჰ. მელაძე, რ. კაკუბავა§, ნ. სვანიძე#**

** აკადემიის წევრი, საქართველოს ტექნიკური უნივერსიტეტი, თბილისი საქართველო*
*** საქართველოს ტექნიკური უნივერსიტეტი, თბილისი, საქართველო*
§ საქართველოს ტექნიკური უნივერსიტეტი, თბილისი, საქართველო
 # ბათუმის შოთა რუსთაველის უნივერსიტეტი, ფიზიკა-მათემატიკისა და კომპიუტერულ მეცნიერებათა ფაკულტეტი, ბათუმი, საქართველო

წარმოდგენილ სტატიაში განხილულია მრავალკომპონენტიანი დარეზერვებული სისტემა, რომელიც შედგება არასაიმედო, აღდგენადი ელემენტებისაგან. ამ სისტემაში სრულდება მომსახურების ორი პარალელური ოპერაცია: 1) მტყუნებული ელემენტის ჩანაცვლება სარეზერვოთი; 2) მტყუნებული ელემენტის აღდგენა. ჩანაცვლებისა და რემონტის ორგანოთა რაოდენობები ნებისმიერია.

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