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Cybernetics

An Open Priority Queuing System for Two Maintenance Operations

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ABSTRACT. In the paper a multi-component redundant system with unreliable, repairable units is considered. Two types of maintenance operations are performed in the system:1) the replacement of the failed main unit by the redundant one; 2) the repair of the failed unit. The open priority queuing model for the system's dependability and performability analysis is constructed in the form of infinite system of ordinary linear differential equations. In steady state it is reduced to the infinite system of linear algebraic equations. *© 2017 Bull. Georg. Natl. Acad. Sci.*

Key words: replacement, repair, open queuing model

In classical reliability theory and practice the equipment (device) reliability provisioning was the main direction. That is why in its framework the replacement problem for single-unit systems has been studied so thoroughly. Also the repairman problem has been studied thoroughly mainly for such complex systems which are reducible to simple two state failure criterion: serviceable (operative) and non-serviceable (failed) (the same is "all or nothing" with Ushakov, "on" or "off" with Barlow, "active" or "inactive", "good" or "bad" with Epstein, "up" or "down" with Gertsbach [1-7]).

In the models, related to the replacement problem of single-unit systems, the time required to make a replacement (replacement time) mainly has been considered to be zero. Even in the cases where replacement time was non-negligible, the replacement problem of a single-unit system was described by alternating renewal process, which has not been causing any difficulties [3-7]. As for complex systems, while analyzing them the replacement time has not been taken into consideration [5].

As a matter of fact, in traditional cases of redundancy, active and redundant units (as a rule) were territorially concentrated at the same place and the replacement of the failed active unit with a redundant one meant the latter's switching over, which was often automatically performed and its duration was negligibly small.

In modern networks of the above type, however, redundant units are not directly attached (linked) to active ones. They are placed at specific storages and may be located at the distance of tens, hundreds and sometimes thousands of kilometers away from the active units. Therefore, the delivery time of the redundant unit to the place of the failed active one is quite essential.

In modern reliability theory the performance (effectiveness) analysis of complex systems with unreliable, repairable components is one of the most topical directions in the field [1, 2]. This is exactly the system level of investigation, unlike the component (equipment) level of classical reliability theory.

Performance analysis is related to systems for which one is not able to formulate the "all or nothing" (serviceable or non-serviceable) type of failure criterion. Effectiveness characterizes a system ability to perform its main functions even with partial capacity. Failures of some (or even majority of system components) lead only to a gradual degradation of the system ability to perform its functions (operations). Actually one deals with such indices like "partial availability", "partial system down time". These type of notions are used to describe multi-component systems (e.g., global terrestrial systems, computer, telecommunication and transportation networks, gas and oil distribution systems, power systems, defense systems, etc.) or the so called systems with embedded "functional redundancy", where there are optional ways to perform system tasks [1, 2].

At the same time, while studying the mentioned systems, traditional mathematical models of classical renewal theory, reliability theory and queuing theory in many cases proved to be unsuitable, and there has arisen an urgent necessity for the construction and investigation of completely new types of models for them [1,2,8-14].

Subject of Study and its Initial Mathematical Description

The investigation subject of this paper is a multi-component redundant system with unreliable, repairable units (Fig.1).

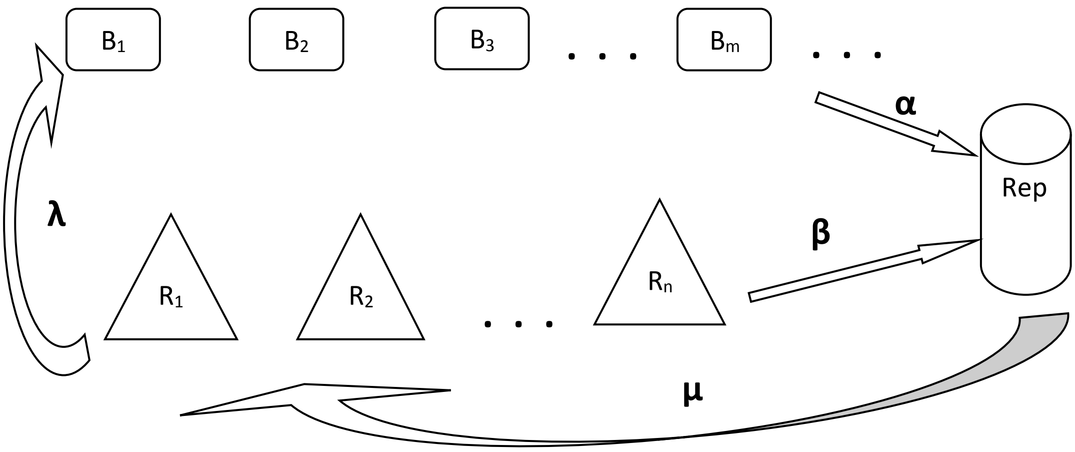

Fig. Operation scheme of the redundant system (B_i – active units; R_i - redundant units; Rep - repair facility).

The system consists of identical active and redundant unit. The numbers of active units are infinite and the numbers of redundant ones are $n \geq 0$. The redundant units are designated for permanent replacement of main components in case of their failure. It is supposed that for the normal operation of the system, the serviceability of all active units is desired. However, if their number is less, then the system continues to function, but with lower economic effectiveness.

The total failure rate of all active units is α, the redundant ones are β. A failed active unit is replaced by a serviceable redundant one if there is an available unit in the system. In the opposite case the replacement will be performed after the availability of the redundant unit. The failed units are repaired. They become identical with the new ones and pass to the group of redundant units. The system has one replacement and repair facility. The replacement and repair times have exponential distribution functions with parameters λ and μ, respectively. When maintenance facility is busy, requests for replacement or repairs are queued. Note that we consider the system with absolute priority of replacement. As we see, in a natural way we have a open queuing system with two types of maintenance operations " replacement and repair.

To this day, neither in the reliability theory nor in the queuing theory the above problems have been investigated in general. At the same time modern research methods of Markov and semi-Markov processes make it possible to construct and to analyze such models in the framework of the mathematical theory of reliability and queuing theory [15,16].

For the last 10-12 years the experts of Georgian Technical University (GTU) have achieved notable success in this direction [8-14]. Namely, the queuing systems of above type were first introduced by GTU experts and have not been yet considered by other authors.

The Mathematical Model

To describe the considered system we introduce the random processes, which determine the states of the system at the time t;

$i(t)$ – the number of units missed in the group of active units;

$j(t)$ – the number of non-serviceable (failed) units in the system.

Denote, $P(i,j,t)=P\{i(t)=i;\ j(t)=j\},\quad i>0,\ j=i,\dots,n+i.$

Proceeding in the usual way, we can set up the basic difference equations which relate the probability of being in a certain state at time $t+\Delta t$ to the probabilities of being in various states at time ***t***. From these difference equations we obtain the infinite systems of ordinary linear differential equations.

For $n>0$ we have:

$$\frac{d}{dt}p(0,0,t)=-p(0,0,t)(\alpha+\beta n)+p(0,1,t)\mu$$

$$\frac{d}{dt}p(0,j,t)=-p(0,j,t)(\alpha+\beta(n-j)+\mu)+p(0,j-1,t)\beta(n-(j-1))+p(0,j+1,t)\mu+p(1,j,t)\lambda$$
$$j=1,\dots,n-1;$$

$$\frac{d}{dt}p(0,n,t)=-p(0,n,t)(\alpha+\mu)+p(0,n-1,t)\beta+p(1,n,t)\lambda$$

$$\frac{d}{dt}p(i,i,t)=-p(i,i,t)(\alpha+\beta n+\lambda)+p(i-1,i-1,t)\alpha \tag{1}$$

$$\frac{d}{dt}p(i,n+i,t)=-p(i,n+i,t)(\alpha+\mu)+p(i,n+i-1,t)\beta+p(i-1,n+i-1,t)\alpha$$
$$+p(i+1,n+i,t)\lambda$$

$$\frac{d}{dt}p(i,n+i-1,t)=-p(i,n+i-1,t)(\alpha+\beta+\lambda)+p(i,n+i-2,t)2\beta$$
$$+p(i-1,n+i-2,t)\alpha+p(i+1,n+i-1,t)\lambda+p(i,n+i,t)\mu$$

$$\frac{d}{dt}p(i,j,t) = -p(i,j,t)(\alpha+\beta(n+i-j)+\lambda)+p(i-1,j-1,t)\alpha$$
$$+\,p(i,j-1,t)\beta(n+i+1-j)+p(i+1,j,t)\lambda, \qquad j=i+1,...,n+i-2.$$

For n=0 we have:

$$\frac{d}{dt}p(0,0,t) = -p(0,0,t)\alpha + p(1,0,t)\lambda$$

$$\frac{d}{dt}p(i,i-1,t) = -p(i,i-1,t)\lambda + p(i,i,t)\mu \qquad (2)$$

$$\frac{d}{dt}p(i,i,t) = -p(i,i,t)(\lambda+\mu)+p(i-1,i-1,t)\alpha + p(i+1,i,t)\mu, \quad i=1,2,...$$

It can be proved that for these systems the limit of $p(i,j,t)$, as $t\to\infty$ exists for all i,j, if $\alpha<\lambda$ and $\alpha<\mu$.

Denote $p(i,j)=\lim_{t\to\infty}p(i,j,t)$. Let $t\to\infty$ in (1) and (2) we obtain an infinite system of linear algebraic equations with respect to $p(i,j)$.

For n>0 we have (together with normalizing condition):

$$p(0,0)(\alpha+\beta n) = p(0,1)\mu$$
$$p(0,j)(\alpha+\beta(n-j)+\mu) = p(0,j-1)\beta(n-(j-1))+p(0,j+1)\mu+p(1,j)\lambda, \quad j=1,...,n-1$$
$$p(0,n)(\alpha+\mu) = p(0,n-1)\beta + p(1,n,t)\lambda$$
$$p(i,i)(\alpha+\beta n+\lambda) = p(i-1,i-1)\alpha \qquad (3)$$
$$p(i,n+i)(\alpha+\mu) = p(i,n+i-1)\beta + p(i-1,n+i-1)\alpha + p(i+1,n+i)\lambda$$
$$p(i,n+i-1)(\alpha+\beta+\lambda) = p(i,n+i-2)2\beta + p(i-1,n+i-2)\alpha + p(i+1,n+i-1)\lambda + p(i,n+i)\mu$$
$$p(i,j)(\alpha+\beta(n+i-j)+\lambda) = p(i-1,j-1)\alpha + p(i,j-1)\beta(n+i+1-j)+p(i+1,j)\lambda,$$
$$j=i+1,...,n+i-2$$

$$\sum_{i=0}^{\infty}\sum_{j=i}^{n+i}p(i,j)=1.$$

For n=0 we have (together with normalizing condition):

$$p(0,0)\alpha = p(1,0)\lambda$$

$$p(i,i-1)\lambda + p(i,i)\mu$$
$$p(i,i)(\lambda+\mu) = p(i-1,i-1)\alpha + p(i+1,i)\mu, \qquad i=1,2,... \qquad (4)$$

$$p(0,0)+\sum_{i=1}^{\infty}\sum_{j=i-1}^{i}p(i,j)=1.$$

After finding the probabilities $p(i,j)$, it is easy to calculate all steady-state dependability and performance measures for the considered system.

The results of the investigation of systems (3) and (4) will be published in the nearest future.

Conclusion

In the given paper an open priority queuing system for two parallel service operations is discussed. The problem, which arises here is how to investigate the infinite system of linear algebraic equations.

In general the study and solution of infinite system of equations, as a rule, is a very complex problem, often insurmountable. But the matrices of our systems (3) and (4) are highly sparse and this gives us a chance to advance in their investigation. Namely, the problem of existence and uniqueness of the solution has been investigated. Also, the numerical algorithms have been developed, making it possible to find the approximate solution by means of finite arithmetical operations. Finally, the error of the approximate solution has been estimated.

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კიბერნეტიკა

ღია პრიორიტეტული რიგების სისტემა ორი პარალელური მომსახურების ოპერაციებისთვის

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წარმოდგენილ ნაშრომში განხილულია მრავალკომპონენტიანი დარეზერვებული სისტემა, რომელიც შედგება არასაიმედო, აღდგენადი ელემენტებისგან. ამ სისტემაში სრულდება მომსახურების ორი პარალელური ოპერაცია: 1) მტყუნებული ელემენტის ჩანაცვლება სარეზერვოთი; 2) მტყუნებული ელემენტის აღდგენა. აგებულია რიგების ღია პრიორიტეტული მოდელი საკვლევი სისტემის საიმედოობისა და ეფექტიანობის ანალიზისათვის. ის წარმოადგენს ჩვეულებრივ წრფივ დიფერენციალურ განტოლებათა უსასრულო სისტემას. მისგან სტაციონარულ მდგომარეობაში მიღებულია წრფივ ალგებრულ განტოლებათა უსასრულო სისტემა. ამჟამად მიმდინარეობს ამ სისტემის გამოკვლევა.

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