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Fuzzy Choquet Integral Aggregations in Multi-Objective Emergency Service Facility Location Problem

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ABSTRACT. This paper presents the construction of a new fuzzy multi-criteria optimization model for the Emergency Facility Location Problem. A fuzzy aggregation operators approach for formation and representing of expert's knowledge on the parameters of emergency service facility location planning is developed. Based on the finite Choquet integral, objective function is constructed, which is the minimization of candidate centers' selection unreliability index. This function together with the second objective function - minimization of total cost needed to open service centers and the third objective function - minimization of number of agents needed to operate the opened service centers creates the fuzzy multi-objective facility location set covering problem. The approach is illustrated by the simulation example which looks into the problem of planning fire stations locations to serve emergency situations in specific demand points – critical infrastructure objects. *© 2018 Bull. Georg. Natl. Acad. Sci.*

Key words: facility location planning, aggregation operators, multi-objective discrete optimization

Timely servicing from emergency service centers to the affected geographical areas (demand points, such as critical infrastructure objects) is a key task of the emergency management system. Scientific research in this area focuses on distribution networks decision-making problems, which are known as a Facility Location Problem (FLP) [1,2]. FLP's models have to support the generation of optimal locations of service centers in complex and uncertain situations. There are several publications about application of fuzzy methods in the FLP. However, all of them have a common approach. They represent parameters as fuzzy values (triangular fuzzy numbers and others) [3,4] and develop methods for facility location problems called in this case Fuzzy Facility Location Problem (FFLP) [5,6]. In this work we consider a new model of FFLP based on the Choquet integral type fuzzy aggregation operators approach [7,8] for the optimal selection of facility location centers.

Definition 1. [3]. $\tilde{c}(t): R^1 \to [0;1]$ is called the Fuzzy Number (FN):

$$\tilde{c}(t)=\begin{cases}1 & \text{if } t\in[c_2';c_2''] \\ \dfrac{t-c_1}{c_2'-c_1} & \text{if } t\in[c_1,c_2'] \\ \dfrac{c_3-t}{c_3-c_2''} & \text{if } t\in[c_2'',c_3] \\ 0 & \text{otherwise}\end{cases},$$

where $c_1 \le c_2' \le c_2'' \le c_3 \in R^1$ ($\tilde{c}\equiv(c_1,c_2',c_2'',c_3)$). Fuzzy number can be considered as a generalization of the interval number.

Let us review arithmetic operations on the triangular FN (TFN) ($c_2'=c_2''$). Let $\tilde{c}$ and $\tilde{b}$ be two TFNs, where $\tilde{c}=(c_1,c_2,c_3)$ and $\tilde{b}=(b_1,b_2,b_3)$. Then 1: $\tilde{c}+\tilde{b}=(c_1+b_1,c_2+b_2,c_3+b_3)$; 2: $\tilde{c}-\tilde{b}=(c_1-b_3,c_2-b_2,c_3-b_1)$; 3: $\tilde{c}\times k=(kc_1,kc_2,kc_3),\ k>0$; 4: $\tilde{c}^k=(c_1^k,c_2^k,c_3^k),\ k>0, c_i>0$ 5: $\tilde{c}\cdot\tilde{b}=(c_1b_1,c_2b_2,c_3b_3), c_i>0, b_i>0$ 6: $1/\tilde{b}=\{1/b_3,1/b_2,1/b_1\},\ b_i>0$; 7: $\tilde{c}>\tilde{b}$ *if* $c_2>b_2$ and *if* $c_2=b_2$ *then* $\tilde{c}>\tilde{b}$ *if* $c_1+c_3>b_1+b_3$, *otherwise* $\tilde{c}=\tilde{b}$. 8. If $\tilde{c}=(c_1,c_2,c_3)$ is TFN, then the expected value of $\tilde{c}$ is defined by the formula $E(\tilde{c})=c_2+(c_3-2c_2+c_1)/4$. We say that

$$\tilde{a}>\tilde{b} \text{ if } a_2>b_2 \text{ and if } a_2=b_2 \text{ then } \tilde{a}>\tilde{b} \text{ if } \frac{a_1+a_3}{2}>\frac{b_1+b_3}{2} \text{ otherwise } \tilde{a}=\tilde{b}\ .$$

The set of all TFNs is denoted by Ψ and nonnegative TFNs ($a_i\ge 0$) by Ψ_0^+. Note that on the lattice Ψ_0^+ $1_{\Psi_0^+}=(1,1,1)$ and $0_{\Psi_0^+}=(0,0,0)$. The latest notion of inequality induces the total ordering t on the lattice Ψ_0^+ and we shall say that $\tilde{a}\ge_t\tilde{b}$ iff $\tilde{a}>\tilde{b}$ or $\tilde{a}=\tilde{b}$. We define the operations of max and min based on the total ordering $\ge_t$. We say that $\max_t\{\tilde{a};\tilde{b}\}=\tilde{a}$ and $\min_t\{\tilde{a};\tilde{b}\}=\tilde{b}$ iff $\tilde{a}\ge_t\tilde{b}$.

Fuzzy Choquet Integral Operator's Approach for the Selection of Facility Location Centers

Location planning for candidate centers is vital in minimizing traffic congestion arising from facility movement in extreme environment. In recent years, transport activity has grown tremendously and this has undoubtedly affected the travel and living conditions in difficult and extreme urban areas. Considering the growth in the number of freight movements and their negative impacts on residents and the environment, municipal administrations are implementing sustainable freight regulations like restricted delivery timing, dedicated delivery zones, congestion charging etc. With the implementation of these regulations, the logistics operators are facing new challenges in location planning for service centers. For example, if service centers are located close to customer locations, then they increase traffic congestion in the urban areas. If they are located far from customer locations, then the service costs for the operators result to be very high. Under these circumstances, it is clear that the location planning for service centers in extreme environment is a complex decision that involves consideration of multiple attributes like maximum customer coverage, minimum service costs, least impacts on geographical points' residents and the environment, and conformance to freight regulations of these points.

At first, we are focusing on a multi-attribute decision making approach for *location planning for selection of service centers under uncertain and extreme environment*. We develop a fuzzy multi-group multi-attribute decision making approach for the service center location selection problem for which a fuzzy aggregation operators' approach is used. The formation of expert evaluations for the attributes with respect to candidate centers is an important task of the centers' selection problem. To decide on the location of service centers, it is assumed that a set of candidate sites already exists. This set is denoted by $CC=\{cc_1,cc_2,...,cc_m\}$ where we can locate service centers and let $S=\{s_1,s_2,...,s_n\}$ be the set of all attributes which define service centers selection (see step 1). Let us assume that $DP=\{dp_1,dp_2,...,dp_l\}$ is the set of all demand points (customers). For each expert e_k from invited group of experts $E=\{e_1,e_2,...,e_t\}$; let $\tilde{a}'^k_{ij}$ be the rating of his/her evaluation for each candidate center cc_i, $(i=1,...,m)$, with respect to each attribute s_j, $(j=1,...,n)$. Let $\tilde{W}_k=\{\tilde{w}^k_1,\tilde{w}^k_2,...,\tilde{w}^k_n\}$ be the ratings of attributes' weights evaluated by the expert e_k. For the expert e_k we construct binary relation $\tilde{A}'_k=\{\tilde{a}'^k_{ij},\ i=1,...,m;\ j=1,...,n\}$. Elements of $\tilde{A}'_k$ and $\tilde{W}_k$ are represented in TFNs.

In fuzzy set theory [3,4], conversion scales are applied to transform the linguistic terms into fuzzy numbers. In our approach, we apply a scale of 1–9 for rating the attributes. Table 1 presents the linguistic variables and fuzzy ratings for the attributes (with respect to candidate centers), attributes' and experts' weights.

Table 1: Linguistic terms and ratings

Linguistic term	Ratings in TFNs
Very low (VL)	(1,1,3)
Low (L)	(1,3,5)
Medium (M)	(3,5,7)
High (H)	(5,7,9)
Very high (VH)	(7,9,9)

The proposed framework of location planning for candidate centers comprises the following steps:

Step 1: Selection of location attributes. Involves the selection of location attributes for evaluating potential locations for candidate centers. These attributes are obtained from literature review, and discussion with experts and members of the city transportation group. We use the following 10 attributes: $s_1=$"Accessibility"; $s_2=$"Security"; $s_3=$"Connectivity to multimodal transport"; $s_4=$"Cost"; $s_5=$ "Environmental impact"; $s_6=$"Proximity to customers"; $s_7=$"Proximity to suppliers"; $s_8=$"Resource availability"; $s_9=$"Conformance to sustainable freight regulations"; $s_{10}=$ "Possibility of expansion". (More detailed explanation of attributes see in [9]). It can be seen that attributes s_3 and s_4 belong to the cost category, that is, the lower the value, the more preferable the alternative for the best location. The remaining attributes are benefit type attributes which means the higher the value, the more preferable the alternative is for selection.

Step 2: Selection of candidate location centers. Involves selection of potential locations for implementing service centers. The decision makers use their knowledge, prior experience with the transportation or other conditions of the geographical area of extreme events and the presence of sustainable freight regulations to identify candidate locations for implementing service centers. For example, if certain areas are restricted for delivery by municipal administration, then these areas are barred from being considered as potential locations for implementing urban service centers. Ideally, the potential locations are those that cater to the interest of all city stakeholders, which are city residents, logistics operators, municipal administrations etc.

Step 3: Locations evaluation using fuzzy aggregation approach. The third step involves evaluation of candidate location centers against the selected attributes (step 1) using the technique of fuzzy approach chooses the alternative.

Step 3.1. Assignment of ratings to the attributes with respect to the candidate centers. Let $\tilde{A}'_k = \{a'^k_{ij},\ i=1,...,m;\ j=1,...,n\}$ be the performance ratings of each expert $e_k\left(k=1,2,..,t\right)$ for each candidate center $cc_i\left(i=1,2,..,m\right)$ with respect to attributes $s_j\left(j=1,2,..,n\right)$ and $\tilde{W}_k = \{\tilde{w}^k_1, \tilde{w}^k_2, ..., \tilde{w}^k_n\}$ be importance weights of attributes presented in TFNs.

Step 3.2. Compute aggregated fuzzy ratings for the attributes and the candidate centers. Let the fuzzy ratings of all experts be described by triangular fuzzy numbers $\tilde{q}^k = \left(q^k_1, q^k_2, q^k_3\right), k=1,2,...,t$. If the fuzzy performance ratings and importance weights of the attributes evaluated by the k-th expert are $\tilde{a}'^k_{ij} = \left(a'^k_{ij1}, a'^k_{ij2}, a'^k_{ij3}\right)$ and $\tilde{w}^k_j = \left(w^k_{j1}, w^k_{j2}, w^k_{j3}\right),\ i=1,...,m:\ j=1,2,..,n$, respectively, then the aggregated fuzzy ratings $\left(\tilde{a}'_{ij}\right)$ of candidate centers with respect to each attribute are given by $\tilde{a}'_{ij} = \left(a'_{ij1}, a'_{ij2}, a'_{ij3}\right)$ where

$$a'_{ijl} = \left(\sum_{k=1}^{t} q^k_l a'^k_{ijl}\right) / \left(\sum_{k=1}^{t} q^k_l\right),\quad l=1,2,3;\ i=1,...,m;\quad j=1,...,n.\ . \tag{1}$$

The aggregated fuzzy weights of attributes $\left(\tilde{w}_j\right),\ j=1,...,n$ are calculated as $\tilde{w}_j = \left(w_{j1},\ \omega_{j2},\ \omega_{j3}\right)$ where

$$w_{jl} = \left(\sum_{k=1}^{t} w^k_{jl} q^k_l\right) / \left(\sum_{k=1}^{t} q^k_l\right),\ i=1,2,3;\ \ j=1,...,n\ . \tag{2}$$

Step 3.3. Compute the fuzzy decision matrix. The fuzzy decision matrix $\tilde{A}'$ for the candidate centers CC and the attributes S is constructed as follows:

$$\begin{array}{c} \\ cc_1 \\ cc_2 \\ \\ cc_m \end{array}\begin{array}{c} \begin{array}{cccc} s_1 & s_2 & & s_n \end{array} \\ \begin{bmatrix} \tilde{a}'_{11} & \tilde{a}'_{12} & ... & \tilde{a}'_{1n} \\ \tilde{a}'_{21} & \tilde{a}'_{22} & ... & \tilde{a}'_{2n} \\ ... & ... & ... & ... \\ \tilde{a}'_{m1} & \tilde{a}'_{m2} & ... & \tilde{a}'_{mn} \end{bmatrix} \end{array} \tag{3}$$

Step 3.4. Normalize the fuzzy decision matrix. The raw data are normalized using a linear scale transformation to bring the various attributes scales onto a comparable scale. The normalized fuzzy decision matrix $\tilde{A} = \{\tilde{a}_{ij}\},\ i=1,2,...,m; j=1,2,...,n$ is given by fuzzy decision matrix $\tilde{A}'$, where

$$\tilde{a}_{ij} = \left(\frac{a'_{ij1}}{a'_j}, \frac{a'_{ij2}}{a'_j}, \frac{a'_{ij3}}{a'_j}\right) \text{ or } \tilde{a}_{ij} = \left(\frac{a^-_j}{a'_{ij3}}, \frac{a^-_j}{a'_{ij2}}, \frac{a^-_j}{a'_{ij1}}\right) \tag{4}$$

where $a'_j = \max_i a'_{ij3}$ (for benefit attributes) and $a^-_j = \min_i a'_{ij1}$ (for cost attributes).

Step 4: Identify constructive fuzzy measure take into account attributes importance and attributes interactions. We introduce the definition of a fuzzy measure (monotone measure) [10] adapted to the case of a finite referential:

Definition 2 [10]. Let $S = \{s_1, s_2, ..., s_n\}$ be a finite set of attributes and g be a set function $g: 2^S \to [0,1]$. We say g is a fuzzy measure on S if it satisfies

$$\begin{aligned} &(i)\quad g(\varnothing)=0;\ g(S)=1; \\ &(ii)\quad \forall A, B \subseteq S,\quad if\ \ A \subseteq B,\ then\ g(A) \le g(B). \end{aligned} \tag{5}$$

In our applications as a fuzzy measure we used the 2-order additive fuzzy measure [11] on an attributes set. The fuzzy measure can represent flexibly a certain kind of an interaction among the decision attributes and can vary from redundancy (negative interaction) to synergy (positive interaction) [11]. In [8] connections between values of the 2-order additive fuzzy measure's associated probabilities and *interaction indexes* I_{ij} among the decision attributes and importance of attribute I_i are received:

$$P_\sigma\left(s_{\sigma(i)}\right) \equiv p_{\sigma(i)} = g(\{s_{\sigma(1)},...,s_{\sigma(i)}\}) - g(\{s_{\sigma(1)},...,s_{\sigma(i-1)}\}) = I_i + (1/2)\cdot\sum_{l=1}^{i-1} I_{\sigma(i)\sigma(l)} - (1/2)\cdot\sum_{l=i+1}^{n} I_{\sigma(i)\sigma(l)}\ , \quad (6)$$

where if in (6) $i=1$ then the second addend is zero, and if $i=n$ then the third addend is zero. Representation of the associated probability distribution (6) has an interesting interpretation in terms of the representation of interaction between attributes [11]. We assume that attributes' importance values $I_j,\ j=1,...,n$ may be defined by the normalization of attributes' importance fuzzy weights as

$$I_j = E(\tilde{w}_j / (\textstyle\sum_{l=1}^{n} \tilde{w}_l)) \quad (7)$$

and an interaction index $I_{ij}, i<j$ may be defined as a normalized value

$$I_{ij} = E(\tilde{I}_{ij} / (\max_t (\tilde{I}_{lk},\ l<k)) \quad (8)$$

where $\tilde{I}_{ij}$ is a fuzzy interaction index evaluated by experts in TFNs (see Table 1).

Step 5: Compute candidate center's selection unreliability index by the fuzzy Choquet integral operator. For our reasoning based on the Def. 1 we present an extension of the Choquet Integral [7] operator on the lattice Ψ_0^+.

Definition 3. Let we have a fuzzy measure g on S and a fuzzy variable of expert evaluations $\tilde{a}: S \Rightarrow \Psi_0^+$ such that $\tilde{a}(s_i) \equiv \tilde{a}_i \in \Psi_0^+, i=1,2,...,n$. Then the aggregation

$$FCA_g(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \sum_{j=1}^{n} p_{\sigma(j)} \tilde{a}_{\sigma(j)}\ , \quad (9)$$

where

$$p_{\sigma(j)} = g\left(\left\{ s_{\sigma(1)},.....,s_{\sigma(j)} \right\}\right) - g\left(\left\{ s_{\sigma(1)},.....,s_{\sigma(j-1)} \right\}\right),$$
$$g\left(\left\{ s_{i(0)} \right\}\right) \equiv 0,$$

is called a Finite Fuzzy Choquet Averaging (FCA) operator. In the proceeding σ is index permutation such that $\tilde{a}_{\sigma(j)}$ is the j-th largest of the $\{\tilde{a}_i\}_{i=1}^{n}$ in the sense of the total ordering $\geq_t$.

Our task is to build aggregation operators' approach, which for each candidate center $cc_i,\ (i=1,...,m)$ aggregates presented objective and subjective data into scalar values – center's selection unreliability index. This aggregation we define as a Fuzzy Choquet Averaging reverse value:

$$\tilde{\delta}_{\mathrm{i}} \equiv \delta(\mathrm{cc_i}) = -FCA_g(\tilde{a}_{i1}, \tilde{a}_{i2}, ..., \tilde{a}_{in})\ , \quad i=1,...,m. \quad (10)$$

where g is – the 2-order additive fuzzy measure which take into account interactions between attributes and important indexes (weights) of attributes in its construction.

Multi - Objective Optimization Model of Fuzzy Facility Location Set Covering Problem

We are focusing on the multi-objective optimization problems [12,13]. The location set covering problem (LSCP) was proposed by C. Toregas and C. Revell in 1972, which seeks a solution for locating the least number of facilities to cover all demand points within the service distance. Fuzzy extension of LSCP for facility location was given in [9]. Using the fuzzy TOPSIS approach, in this work we constructed new fuzzy LSCP model for emergency service facility location planning. In this section we construct fuzzy aggregation operators' approach for the LSCP model.

The center's selection unreliability index reflects expert evaluations with respect to the center, considering all actual attributes. If $x=\{x_1,x_2,...,x_m\}$ is Boolean decision vector, which defines some selection from candidate centers $CC=\{cc_1,cc_2,...,cc_m\}$ for facility location, we can build centers' selection unreliability index as linear sum of triangular fuzzy values - $\tilde{\delta}_j x_j$: As a result, new fuzzy objective function – centers' selection unreliability index $\sum_{j=1}^{m}\tilde{\delta}_j x_j$ is constructed. Minimization will select group of centers with the minimal unreliability index from admissible covering selections. Classical facility location set covering problem tries to minimize the total cost needed to open of service centers - $\sum_{j=1}^{m}C_j x_j$, where C_j is a cost necessary for opening of the candidate center - cc_j . We assume that we know in advance the approach number of people needed to make a candidate center operate as a service center. This number is denoted $M_i,\ i=1,...,n$. Our goal is to locate service facilities centers with the minimal number of agents needed to operate the opened service centers - $\sum_{j=1}^{m}M_j x_j$. The problem aims to locate service facilities in minimal travel time from candidate centers. Let experts evaluate movement fuzzy times between demand points and candidate centers $\tilde{t}_{ij},\ i=1,...,l;\ j=1,...,m$. In extreme environment for emergency planning radius of service center is not defined based on distance but it is defined based on maximum allowed time T for movement, since the rapid help and servicing is crucial for demand points in such situations. Respectively, a set of candidate centers N_i , covering customer $dp_i\in DP=\{dp_1,dp_2,...,dp_l\}$, is defined as $N_i=\{cc_j,\ cc_j\in CC/E(\tilde{t}_{ij})\le T\}$. Then we can state Multi-objective facility location set covering problem:

$$\min_t \quad z_1=\sum_{j=1}^{m}\tilde{\delta}_j x_j \quad (11),\qquad \min \quad z_2=\sum_{j=1}^{m}C_j x_j \quad (12),\qquad \min \quad z_3=\sum_{j=1}^{m}M_j x_j \qquad (13)$$

$$\sum_{s_j\in N_i}x_j\ge 1\ \ (i=1,2,...,l)\ ;\ x_j\in\{0,1\}\quad j=1,2,...,n\ . \qquad (14)$$

Simulation of Emergency Service Facility Location Model

We illustrate the effectiveness of the constructed optimization model by the numerical example. Let us consider an emergency management administration of a city that wishes to locate some fire stations with respect to timely servicing of critical infrastructure objects. Assume that there are 30 demand points (critical infrastructure objects) and 8 candidate facility centers (fire stations) in the urban area. Let us have 4 experts from Emergency Management Agency (EMA) of a Country for the evaluation of the travel times and the ranking indexes of candidate facility centers. The travel times between demand points and candidate centers are evaluated in triangular fuzzy numbers (omitted here because it has a large dimension). According to the standards of the EMA, let the principle of location fire stations be that the fire station can reach the area edge within 5 minutes after receiving the dispatched instruction. Therefore, we set covering radius $T=5$ minutes.

Each expert $e_k\ (k=1,2,3,4)$ with fuzzy rating $\tilde{q}^k$ presented the ratings $\tilde{a}'^k_{ij}$ for each candidate center cc_i , $(i=1,...,8)$, with respect to each attribute s_j , $(j=1,...,10)$ and weights $\tilde{w}^k_j$ for each attribute. Let also experts evaluated interactive indexes between attributes $\tilde{I}^k_{ij},\ i,j=1,...,10;\ i<j$ (experts' evaluations are omitted) Using formulas (2), (3) normalized decision matrix $\tilde{A}$ and attributes weights $\tilde{W}$ were obtained. Using formula (6) associated probabilities were calculated. By formulas (9) (10) candidate sites unreliability indexes were calculated (calculations are omitted). Experts also evaluated cc-costs C_j and cc-

agents M_j. Movement fuzzy times between demand points and candidate centers $\tilde{t}_{ij},\ i=1,...,30;\ j=1,...,8$ are defined by experts. Therefore, the subsets of service demand points $N_i, i=1,...,8$ are received. At the ending, the Multi-objective set covering optimization problem (11)-(14) was constructed:

$$\begin{cases} z_1 = \tilde{\beta}_1 x_1 + \tilde{\beta}_2 x_2 + \tilde{\beta}_3 x_3 + \tilde{\beta}_4 x_4 + \tilde{\beta}_5 x_5 + \tilde{\beta}_6 x_5 + \tilde{\beta}_7 x_7 + \tilde{\beta}_8 x_8 \Rightarrow \min_t, \\ z_2 = 35x_1 + 47x_2 + 55x_3 + 39x_4 + 70x_5 + 62x_6 + 46x_7 + 57x_8 \Rightarrow \min \\ z_3 = 27x_1 + 19x_2 + 31x_3 + 18x_4 + 23x_5 + 29x_6 + 25x_7 + 20x_8 \Rightarrow \min \\ Ax^T \geq (1,1,1,1,1,1,1,1)^T \\ x \equiv \{x_1,...,x_8\},\ \ x_i \in \{0,1\},\ i=1,...,8 \end{cases}$$

where triangular fuzzy coefficients are presented in Table 2, matrix of covering constraints A is constructed but omitted here. Matrix A is a concatenation of vectors N_i in which the covering of demand point is presented by "1" and no covering by "0". Coefficients of objective function z_2 are presented in thousand unit.

Table 2. Candidate centers selection unreliability indexes

$\tilde{\beta}_1$	$\tilde{\beta}_2$	$\tilde{\beta}_3$	$\tilde{\beta}_4$	$\tilde{\beta}_5$	$\tilde{\beta}_6$	$\tilde{\beta}_7$	$\tilde{\beta}_8$
(0.8,4.1,5.5)	(0.5,2.9,5.0)	(-0.5.2,4.8)	(-0.7,2.7,4.5)	(-1.1,2.5,4.7)	(-1,2.1,4.7)	(0,3.3,5.6)	(0.3,3,5.1)

For the constructed problem Pareto solutions are founded. There are
(four Pareto solutions):
1) $x_2, x_4, x_7 \rightarrow z_1 = (-0.1, 8.9, 15.4),\ z_2 = 132,\ z_3 = 62;$ 2) $x_3, x_5, x_8 \rightarrow z_1 = (-0.3, 7.6, 14.8),\ z_2 = 182, z_3 = 74;$
3) $x_3, x_6, x_7 \rightarrow z_1 = (-0.6, 7.4, 15.1),\ z_2 = 163,\ z_3 = 85;$ 4) $x_3, x_6, x_8 \rightarrow z_1 = (-1.2, 7.2, 14.7),\ z_2 = 147, z_3 = 80;$
5) $x_4, x_6, x_7 \rightarrow z_1 = (-1.7, 8.1, 15.0),\ z_2 = 147,\ z_3 = 72;$

It is clear that, decreasing of the unreliability index of covering in Pareto solutions gives us more worse level of the second objective function - *the total cost needed to open of service centers* or of the third objective function - *number of agents needed to operate the opened service centers*. But the decision on the choice of the fire stations as service centers is depend on the decision making person's preferences with respect to risks of administrative actions.

Conclusions. The paper presented new approach for fuzzy facility location problem for selection of the locations of service centers in extreme and uncertain situations. The approach utilizes experts knowledge represented by triangular fuzzy numbers and considers the suitability of central location (i.e. affordability, security, etc.) using Choquet integral based fuzzy aggregation approach. On the other hand, the model also considers the necessity to reach all critical infrastructure points and time that is required to reach them, also presented by triangular fuzzy numbers. Experts also evaluated attributes interaction indexes and important weights in TFNs. Therefore, the fuzzy measure's associated probabilities is calculated and candidate sites unreliability indexes is obtained. As a result, Fuzzy Multi-Objective Set Covering Problem is constructed. The constructed methodology is illustrated by a numerical example for locating fire stations servicing critical infrastructure points in a city. For the constructed problem Pareto solutions are obtained. In our future studies (large dimension cases of the problem) the epsilon-constraint approach for the Pareto front obtaining will be constructed.

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აგებულია ობიექტების განთავსების ამოცანის ახალი ფაზი-მოდელი. გამოყენებულია შოკეს ინტეგრალზე დაფუძნებული ფაზი-აგრეგირების ოპერატორი. განვითარებულია საგანგებო სიტუაციების ობიექტების განთავსების დაგეგმვის პარამეტრების შეფასების ექსპერტული ცოდნის წარმოდგენისა და ფორმირების ფაზი - მიდგომა. შექმნილია ახალი მიზნობრივი ფუნქცია, კერძოდ, ცენტრების შერჩევის არასაიმედოობის ინდექსის მინიმიზაცია. ეს კი მეორე მიზნობრივ ფუნქციასთან - შერჩეული ცენტრების გახსნის ჯამური ფასის მინიმიზაციასა და მესამე მიზნობრივ ფუნქციასთან - შერჩეულ ცენტრებში მომუშავე პერსონალის მინიმიზაციასთან ერთად ქმნის ობიექტების განთავსების მრავალკრიტე-რიუმიან ამოცანას. აგებული მოდელი ილუსტრირებულია საგანგებო სიტუაციაში დახმარების ობიექტების განთავსების დაგეგმვის სიმულაციურ მაგალითზე. კონკრეტულად კი, საგანგებო სიტუაციის შემთხვევაში თუ როგორ დაიგეგმოს სახანძრო სადგურების განთავსება კრიტიკული ინფრასტრუქტურის ობიექტების მოთხოვნების გათვალისწინებით.

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