Economics

Dependence of Aggregate Supply on the Main Factors Prices of Production

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ABSTRACT. The article deals with the Keynesian aggregate supply model for the case of incomplete use of production factors. The amount of labour and capital involved in this model is determined at the labour and capital markets, respectively. The paper discusses the assumption by which the demand for labour and capital depends on the effective demand (coming from the goods market), and the labour supply is an increasing function of real wage, and the supply of capital – the increasing function of the real rental price of capital. It is shown that the labour supply in such a situation is an increasing function of nominal wage and a decreasing function of the nominal rental price of capital; capital supply is, on the contrary, a decreasing function of nominal wage and an increasing function of the nominal rental price of capital. Because of the above noted, generally, it is not known a priori in what direction the value of the Keynesian aggregate supply will change in case of the increase of prices level, when this growth is only associated with the increase of either nominal wage or the nominal rental price of capital. In the paper we prove that under certain conditions, for example, in case when indirect taxes included in the prices of products are other than zero, there are such combinations of simultaneous growth of nominal wage and nominal rental price of capital that it will lead to simultaneous growth of labour and capital supply and at the same time to the increase of aggregate supply. One of these combinations implies the equivalence of relative increments of nominal wage and the nominal rental price of capital.

Key words: aggregate supply, labour supply, capital supply, real wage, real rental price of capital, nominal wage, nominal rental price of capital

Aggregate supply plays an important role in macroeconomic analysis. Its several models are known in economic theory. In general, difference is made between classical and Keynesian models. The classical model describes producers' behaviors for the full use of production factors and is displayed as a vertical line on the coordinate plane of the price level and output volume. The Keynesian model is applicable to the case of the production factors' incomplete use. It has two options, depending on how resources are attracted to production. When the inclusion of these resources does not increase expenditures on the product unit, then the Keynesian supply model corresponds to the horizontal line; this is a special case of the Keynesian model. More frequently, companies can make the purchase of additional amount of resources only under the conditions of increased prices. Due to this circumstance, the second option of the Keynesian model of aggregate supply is the increasing function of the price level, which is displayed as an upward line on the corresponding coordinate plane. It is noteworthy that mostly this latter version of the model is implied under aggregate supply and that is why it is the subject of our further consideration.

Analysis of the classical as well as the Keynesian model of aggregate supply is usually based on the consideration of the labour market model and aggregate production function, in which the main determinant of the total output is two factors of production - labour and capital [1]. We also use this approach with the only difference that we focus on the capital market together with the labour market, and consider both markets from the Keynesian position.

Basically the Keynesian model of the labour market differs from the classical one in terms of how the demand of labour is determined. The demand for labour in the classical model is a decreasing function of real wage and its curve is derived from the assumption that firms strive to maximize their profits. According to Keynes, this kind of curve of demand for labour has a sense when it is full employment within an economy and the output is at a potential level [2]. In cases of incomplete employment, companies, despite their attempts, cannot get the maximum profit because they do not sell as much goods as they want to sell due to the different output from the potential one [2]. Therefore, in the Keynesian model, in case of the existence of balanced prices, the “effective demand” or the demand coming from goods and service market is considered as the determinant of the demand for labour. It is absolutely logical to extend this feature of the Keynesian labour market also to the capital market and consider that in case of incomplete employment, the demand for capital, like labour, is mainly determined by effective demand. It follows from the above noted that in case of the existence of appropriate aggregate demand (effective demand), under the conditions of the given production function (which is increasing with regard to both factors in standard situation), the identification of the peculiarities of the behavior of aggregate demand is related to the consideration of labour and capital supply functions [3].

At present there are many theoretical and empirical researches in which the peculiarities of labour and capital supply functions are studied [4-8]. It can be said that there is no general idea about the types and determining factors of these functions. One of the versions of the Keynesian approach which is of interest to us, like classical one, considers that labour supply is an increasing function of real wage [2]. In the present paper we accept this proposition without alternative. Furthermore, the capital supply is considered as an increasing function of the real rental price of capital and it is analyzed how the change of nominal wage and nominal rental price of capital impacts specifically on the supply of factors and by means of them, on the value of aggregate supply.

In order to determine the relationship between the aggregate supply and the nominal prices of the production factors, it is expedient to divide the total output of economy into three aggregate components – intermediate, investment and final products.The first one (Z) is used for the current need of production in the economy, the second type of product (K) involves the aggregate of the goods for investment purposes, and the third type of product (Y) is used for household and state consumption. Each product has its price level. The price level of intermediate products is marked by P_Z, the price level of investment products – by P_K and the price level of the final products – by P_Y. Before writing the determining equations of P_Z and P_Y, let us make some important assumptions.

First, let us assume that any aggregate product prices level is determined, along with demand and supply, by the cost of the used intermediate consumption products, remuneration for labour and capital and other elements that are conventionally united under the concept of indirect taxes.

Secondly, we imply that the amount of labour remuneration in the production of any product unit is determined by the labour input of this product and the wage rate whose nominal value is marked by ω. Let us say, that the labour input coefficients of the production of intermediate, investment and the final consumption products are l_z, l_K and l_Y respectively. Then, according to the above, the values of labour remuneration will be ωl_z, ωl_K and ωl_Y.

The third assumption is related to the identification of the role of capital in the price structure. Capital participates in the formation of production price in two ways. On the one hand, capital costs are reflected in the price in the form of amortization charges. On the other hand, capital, as a factor of production, receives "compensation" for participating in the creation of product, which, like labour remuneration, is involved in the price in the form of property income, rental income or profit. It should be noted that reflection of the value of amortization charges in the price is not a problem; it is difficult to determine the owned share of capital. This can be explained by the fact that the latter is influenced by many circumstances. Afterwards we will simplify the situation to some extent and assume that the participation of capital in the formation of the price of product is determined by two characteristics – capital output ratio of product and rental price of capital – ν. The rental price of capital is the balancing value of the total amount of capital supply in the economy and the total demand on this capital. Let f_z, f_K and f_Y denote the coefficients of capital output ratio of intermediate, investment and final consumption products, respectively. Then, based on our assumption, the values that reflect the reimbursement of capital according to aggregated products will be νf_z, νf_K and νf_Y.

Based on the given assumptions, the determining system of the balanced price level will be written as follows:

$$P_z = A_z P_z + \omega l_z + \nu f_z + \eta_z \,, \qquad (1)$$

$$P_K = A_K P_z + \omega l_K + \nu f_K + \eta_K \,, \qquad (2)$$

$$P_Y = A_Y P_z + \omega l_Y + \nu f_Y + \eta_Y \,, \qquad (3)$$

where η_Z, η_K, η_Y – are the aggregate standards of indirect taxes for intermediate, investment and final consumption products respectively; A_z, A_k, A_y – are the aggregate coefficients of intermediate products' technological expenditures required for producing intermediate, investment and final consumption products. In general, these coefficients are different from each other. It is meant that $0 < A_Z < 1$. This is a natural requirement that we use only for the coefficient A_z. As for the rest of the two technological coefficients A_K and A_Y such demand does not have any principal importance for them. The only natural requirement to be met by them is that $A_K > 0$ and $A_Y > 0$. However, it should be noted that the higher the values of the coefficients A_K and A_Y, in other equal conditions, the higher the relevant price level of the products produced by these technologies. As a rule, getting a value close to one or above it by A_K and A_Y negatively influences many consequential characteristics of an economy.

Let us solve the equation with respect to P_z and insert the obtained value into the remaining two equations. The system presented here (1)-(3) through the simplest transformations can be rewritten in the following way:

$$P_z = \alpha_z \omega + \beta_z \nu + \gamma_z \,, \qquad (4)$$

$$P_K = \alpha_K \omega + \beta_K \nu + \gamma_K \,, \qquad (5)$$

$$P_Y = \alpha_Y \omega + \beta_Y \nu + \gamma_Y \,, \qquad (6)$$

where α_z, α_K and α_Y are positive values which characterize the full cost of labour in the production of units of intermediate, investment and final consumption products, respectively. In short, α_z, α_K and α_Y can be called the full expenditure coefficients of labour;

$\beta_z, \beta_K, \beta_Y$ – reflect the full expenditure of capital for production of intermediate, investment and final consumption products; according to the explanation, all three parameters are positive.

$\gamma_Z, \gamma_K, \gamma_Y$ – are the coefficients of the full indirect taxes which reflect both indirect taxes to be collected directly from the given intermediate, investment and final products units and the sum of indirect taxes to be collected from all those products whose production is necessary for producing the given products units. These coefficients like the rest of the coefficients provided in (4)-(6) are positive values.

(1)–(3) equations can be called ***structural*** equations for determining the level of balanced prices, and (4)–(6) figures – ***reduced*** equations for determining the level of balanced prices. As it follows from the (4)–(6) system, the price level of any aggregate product is determined essentially by the wage level ω, and the rental price of capital ν [9]. In addition, the positiveness of α and β coefficients indicates that the prices are increasing functions of ω and ν characteristics. It follows from the heterogeneousity of the (4)–(6) functions that, in other equal conditions, if the value of nominal wage ω, or nominal rental price of capital ν, increases (decreases) then the level of all three types of price will also increase (decrease), but the rate of change in the price level compared with ω and ν change rate will be less. This is a very important circumstance and we will get back to it later.

The above provided equations of the price level enable us to determine the functions of labour and capital supply. First, let us consider the labour supply function which is denoted by $L(\bar{\omega})$, where $\bar{\omega}$ is a real wage. The latter is determined by the level of nominal wage and aggregate final product price, so we can write:

$$\bar{\omega} = \frac{\omega}{P_Y} = \frac{\omega}{\alpha_Y \omega + \beta_Y \nu + \gamma_Y}. \qquad (7)$$

Therefore, the function of the labour supply in our case can be presented as follows:

$$L(\bar{\omega}) = L(P_Y, \omega) = L\left(\frac{\omega}{\alpha_Y \omega + \beta_Y \nu + \gamma_Y}\right), \ \frac{\partial L}{\partial \bar{\omega}} > 0. \ (8)$$

It is easy to check that

$$\frac{\partial L}{\partial \omega} = \frac{\partial L}{\partial \bar{\omega}} \cdot \frac{\partial \bar{\omega}}{\partial \omega} > 0 \quad \text{and} \quad \frac{\partial L}{\partial \nu} = \frac{\partial L}{\partial \bar{\omega}} \cdot \frac{\partial \bar{\omega}}{\partial \nu} < 0.$$

It follows from here that labour supply $L(\bar{\omega})$ is an increasing function of nominal wage ω and a decreasing function of the nominal rental price ν of capital.

The function of capital supply is determined the same way as the function of labour supply which is denoted by $\Phi(\bar{\nu})$. In this record $\bar{\nu}$ is a real rental wage of capital. To get the latter, it should be expressed by the ratio of the nominal rental price to the price of investment goods. Therefore,

$$\bar{\nu} = \frac{\nu}{P_K} = \frac{\nu}{\alpha_K \omega + \beta_K \nu + \gamma_K}. \qquad (9)$$

Considering this, $\Phi(\bar{\nu})$ can be written in the following way:

$$\Phi(\bar{\nu}) = \Phi(P_K, \nu) = \Phi\left(\frac{\nu}{\alpha_K \omega + \beta_K \nu + \gamma_K}\right), \frac{\partial \Phi}{\partial \bar{\nu}} > 0. \ (10)$$

If we use the partial derivatives of $\Phi(\bar{\nu})$, then we can easily make sure that

$$\frac{\partial \Phi}{\partial \omega} = \frac{\partial \Phi}{\partial \bar{\nu}} \cdot \frac{\partial \bar{\nu}}{\partial \omega} < 0 \quad \text{and} \quad \frac{\partial \Phi}{\partial \nu} = \frac{\partial \Phi}{\partial \bar{\nu}} \cdot \frac{\partial \bar{\nu}}{\partial \nu} > 0.$$

Thus, capital supply $\Phi(\bar{\nu})$ is a decreasing function of nominal wage ω and an increasing function of ν, the nominal rental price of capital.

Based on the above consideration, ω and ν characteristics have an impact on L and Φ functions in different directions. In other equal conditions, for example, if ω increases, then it will simultaneously incur the increase of labour supply and reduce the supply of capital. The rise of ν gives an opposite result – in this case labour supply decreases and capital supply increases. Since the aggregate supply is an increasing one with respect to used labour and capital, the noted indicates that in general, it is not known a priori in what direction its value will change in case of the changes of ω or ν separately. In spite of this, we can prove that when:

- *Balanced prices of aggregate products are determined by (1)–(3) equations where* *aggregate coefficients of indirect taxes are positive* ($\eta_z > 0$, $\eta_k > 0$ *and* $\eta_y > 0$)*;*

- *The supply of labour and capital is carried out in accordance with (8) and (10) functions,*

then there are such positive $d\omega$ *and* $d\nu$ *increments of nominal wage and nominal rental price of capital, for which the simultaneous growth of labour and capital supply is possible.*

To prove this result let us compose the following equations determining the total increments of $L(\bar{\omega})$ and $\Phi(\bar{\nu})$ functions:

$$dL = \frac{\partial L}{\partial \bar{\omega}} \times \frac{\partial \bar{\omega}}{\partial \omega} d\omega + \frac{\partial L}{\partial \bar{\omega}} \times \frac{\partial \bar{\omega}}{\partial \nu} d\nu ; \qquad (11)$$

$$d\Phi = \frac{\partial \Phi}{\partial \bar{\nu}} \times \frac{\partial \bar{\nu}}{\partial \omega} d\omega + \frac{\partial \Phi}{\partial \bar{\nu}} \times \frac{\partial \bar{\nu}}{\partial \nu} d\nu . \qquad (12)$$

Based on the (7) and (9) formulas we have:
$\partial\bar{\omega}/\partial\omega = (\beta_Y \nu + \gamma_Y)/P_Y^2$, $\partial\bar{\omega}/\partial\nu = -\beta_Y \omega/P_Y^2$,
$\partial\bar{\nu}/\partial\omega = -\alpha_K \omega/P_K^2$, $\partial\bar{\nu}/\partial\nu = (\alpha_K \nu + \gamma_K)/P_K^2$.

If we take into consideration these values of partial derivatives in the expressions (11) and (12) of the total increments and make simple transformations, we will get:

$$dL = \frac{\partial L}{\partial \bar{\omega}} \times \frac{1}{P_Y^2}\left[(\beta_Y \nu + \gamma_Y) d\omega - \beta_Y \omega d\nu\right], \qquad (13)$$

$$d\Phi = \frac{\partial \Phi}{\partial \bar{\nu}} \times \frac{1}{P_K^2}\left[-\alpha_K \nu d\omega + (\alpha_K \omega + \gamma_K) d\nu\right]. \qquad (14)$$

Let us consider (13)–(14) in the role of equations system, in which dL and $d\Phi$ are the given positive values and, $d\omega$ and $d\nu$ are endogenous variables. Because, according to the explanation

$$\frac{\partial L}{\partial \bar{\omega}} > 0, \ \frac{\partial \Phi}{\partial \bar{\nu}} > 0, \ P_Y^2 > 0, \ P_K^2 > 0,$$

therefore, the matrix Ω determines the existence and character of the system's solutions:

$$\Omega = \begin{pmatrix} (\beta_Y \nu + \gamma_Y) & -\beta_Y \omega \\ -\alpha_K \nu & (\alpha_K \omega + \gamma_K) \end{pmatrix}.$$

According to the definition, α, β and γ parameters are positive. In these conditions we can easily check that the Ω matrix determinant is positive and there is non-negative inverse Ω^{-1} matrix. This indicates that the (13)–(14) system has the only positive solution $(d\omega,\ d\nu)$ for each $(dL > 0,\ d\Phi > 0)$ couple.

Therefore, we have proven that when in the (1)-(3) equations of prices levels $\eta_Z > 0$, $\eta_K > 0$ and $\eta_Y > 0$, then there are always such combinations of the increase in nominal wage and rental price of capital that will generate the simultaneous growth of labour and capital supply and at the same time – the increase of aggregate supply. One of such combinations is, for example, equivalence of ω and ν relative increments:

$$\frac{d\omega}{\omega} = \frac{d\nu}{\nu}. \qquad (15)$$

It should be noted that when in the (1)–(3) equations of prices levels the coefficients of indirect taxes η_Z, η_K and η_Y are zero, then there are not such $(d\omega > 0, d\nu > 0)$, *which will contribute to the simultaneous growth of labour and capital*. At the same time, when the relative increment of nominal wage exceeds the relative increment of the nominal rental price of capital, then $dL > 0$ and $d\Phi < 0$. On the contrary, when $d\omega/\omega < d\nu/\nu$, then the supply of labour is reduced and the capital supply is increased, i.e. $dL < 0$, $d\Phi > 0$. It is obvious that in such conditions the impact of growth of factors prices on aggregate supply is not explicitly determined. At the same time, if (15) holds, i.e. if $\nu d\omega = \omega d\nu$, then the change of aggregate supply is zero, because in this case $dL = 0$ and $d\Phi = 0$. Naturally, the question arises: how to explain the fact that in case of zero indirect taxes, the proportionate growth of ω and ν does not affect the supply of labour and capital but in case of the existence of taxes other than zero contributes to the increase of the both factors supply?

The answer is simple enough. The point is that in case of zero indirect taxes the reduced equations of price levels (4)-(6) are homogeneous, because of this the simultaneous increase or decrease of ω and ν in a certain proportion, leads to a corresponding proportional change of price levels. In such a situation, based on the (7) and (9) formulas the real wage $\bar{\omega}$ and the real rental price of capital $\bar{v}$ remain unchanged, therefore, the existing values of the L and Φ functions (and of the function of aggregate supply depending on them) are also unchanged. It is noteworthy that the formation of vertical curve of aggregate supply which is well-known in classical theory actually is based on the consideration of this particular case [1].

In the second case, when indirect taxes are not zero and therefore the (4)–(6) equations of prices are non-homogeneous, the increase of ω and v in any given rate leads to the rise of price levels, but the change rate of the latter is less as compared with the change rate of ω and v. In the given situation this means that the corresponding real $\bar{\omega}$ and $\bar{v}$ characteristics and L and Φ functions depending on them, as well as the function of aggregate supply, change in the same direction as ω and v.

Since η_Z, η_K and η_Y characteristics of indirect taxes in the price structure are always positive, the following conclusion follows from the analysis carried out above: in the conditions of incomplete employment the increase of ω alone or the increase of v alone, despite the fact that this will incur the increase of prices level may not be enough to achieve the increase of aggregate supply. At the same time, the simultaneous growth with a certain degree of correspondence of the values of ω and v, will lead to the increase of price levels as well as the increase of aggregate supply.

ეკონომიკა

ერთობლივი მიწოდების დამოკიდებულება წარმოების ძირითადი ფაქტორების ფასებზე

ი. ანანიაშვილი

ივანე ჯავახიშვილის სახელობის თბილისის სახელმწიფო უნივერსიტეტი, თბილისი, საქართველო

(წარმოდგენილია აკადემიის წევრის ვ. პაპავას მიერ)

სტატიაში განხილულია კეინზიანური ერთობლივი მიწოდების მოდელი წარმოების ფაქტორების არასრული გამოყენების შემთხვევისთვის. მოდელში ჩართული შრომისა და კაპიტალის რაოდენობები, შესაბამისად, შრომისა და კაპიტალის ბაზრებზე განისაზღვრება. ნაშრომში გამოყენებულია დებულება, რომლის თანახმადაც შრომასა და კაპიტალზე მოთხოვნა ეფექტიან (საქონელთა ბაზრის მხრიდან წამოსულ) მოთხოვნაზეა დამოკიდებული, ხოლო შრომის მიწოდება არის რეალური ხელფასის, კაპიტალის მიწოდება კი – კაპიტალის გაქირავების რეალური ფასის ზრდადი ფუნქცია. ნაჩვენებია, რომ ასეთ სიტუაციაში შრომის მიწოდება წარმოადგენს ნომინალური ხელფასის ზრდად და კაპიტალის გაქირავების ნომინალური ფასის კლებად ფუნქციას; კაპიტალის მიწოდება პირიქით, ნომინალური ხელფასის კლებადი და კაპიტალის გაქირავების ნომინალური ფასის ზრდადი ფუნქციაა. აღნიშნულის გამო, ზოგადად, აპრიორულად ცნობილი არ არის, რა მიმართულებით შეიცვლება კეინზიანური ერთობლივი მიწოდების მნიშვნელობა ფასების დონის ზრდის

შემთხვევაში, როცა ეს ზრდა მხოლოდ ნომინალური ხელფასის ან მხოლოდ კაპიტალის გაქირავების ნომინალური ფასის ზრდას უკავშირდება. ამავე დროს, სტატიაში დასაბუთებულია, რომ გარკვეული პირობების შესრულებისას, მაგალითად იმ შემთხვევაში, როცა პროდუქტების ფასებში შემავალი არაპირდაპირი გადასახადები ნულისგან განსხვავებულია, არსებობს ნომინალური ხელფასისა და კაპიტალის გაქირავების ნომინალური ფასის ერთდროული ზრდის ისეთი კომბინაციები, რომლებიც გამოიწვევს შრომისა და კაპიტალის მიწოდების ერთდროულ ზრდას, ამის პარალელურად კი – ერთობლივი მიწოდების ზრდას. ერთ-ერთი ასეთი კომბინაცია ნომინალური ხელფასისა და კაპიტალის გაქირავების ნომინალური ფასის შეფარდებითი ნაზრდების ერთმანეთთან ტოლობას გულისხმობს.

REFERENCES

1. Sachs J. D. and Larrain F. (1996) Macroeconomics in the Global Economy, 67-103 (Transl. from English). M., Delo, (in Russian).
2. Harris L. (1990) Monetary Theory 268-315; 441-472 (Transl. from English). M., Progress (in Russian).
3. Ananiashvili I. and Papava V. (2014) Laffer-Keynesian Synthesis and Macroeconomic Equilibrium. 106, Nova Science Publishers, Inc. New York.
4. Blanchard O. J. and Fischer S. (2014) Lectures on Macroeconomics, 430-508 (Transl. from English). M., Delo (in Russian).
4. Barro R.and Sala-i-Martin X. (2010) Economic Growth, 490-551 (Transl. from English). M., BINOM (in Russian).
6. G. J. Borjas (2013) Labour Economics, 7th edition. New York: McGraw-Hill.
7. Killingworth M. R. (1983) Labour Supply. Cambridge: Cambridge University Press.
8. Wikens M. (2012) Macroeconomic Theory: A Dynamic General Equilibrium Approach. Second Edition. Princeton, Princeton University Press.
9. Balatsky E. and Ekimova N. (2016) Distribution Models of Market Economy. *Terra Economicus,* **14**, 2: 48-69 (in Russian).

Received October, 2017