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Pressure Drop Distribution at High Power Perturbation over the Mountainous Territory

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ABSTRACT. Study of the spatial-temporal propagation of the air flow generated by the action of high-power phenomenon has great theoretical and practical value, especially for the mountainous territories because even the low hills slow down the velocity of flow motion and change its direction, sometimes even to the opposite direction. In the present paper the air flow generated by high-power pulse and its spatial-temporal propagation in the atmosphere above the uniform and non-uniform terrains are investigated. Some results of theoretical and numerical investigations are given. Received results can be useful in military and mining operations, especially in the process of open career works in populated places or near to them. *© 2015 Bull. Georg. Natl. Acad. Sci.*

Key words: *modelling, explosion, air flow, advective propagation, mountainous territory.*

The main characteristic features of explosion are: pressure drop, formation of explosive cloud and a blast wave, expanding with the lapse of time [1,2]. Study of real explosions is rather expensive, and theoretical methods or numerical modeling represent the cheapest and more reliable approaches, though they require experimental or some modern knowledge in the adjacent areas of science. For instance, numerical modeling of explosion, requires experimental data, very skilful numerical schemes and detailed theoretical knowledge of explosive phenomena [3].

There are many interesting numerical studies devoted to the blast wave transportation and its impact on environment [3-6]. Over the last two decades considerable attention received terrorist explosions [7-9]. Recently, on the basis of modern computer technology it became possible to perform numerical simulations and to carry out successful numerical analysis of explosions [5, 10,11]. There are numerous works devoted to investigation of the craters produced by explosions on the soil surface [3] and the blast stress waves induced from the large scale underground explosion [5].

The main goal of this work is to study "perturbed" (as a result of high power explosions) airflow advective propagation in the atmosphere for the small time Δt taking into account the influence of topogragraphy. As the process of pressure drop penetration in the geopotential field is analogous to the problem of gas nonstationary diffusion in the atmosphere [3, 5, 12, 13], we assume that at the moment $t = 0$ in the origin of coordinate system there is a source of a high power perturbed flow with power Q. The pressure drop, caused by explosion propagates into every direction and the flow velocity at the distance r can be defined by the following formula [14]:

$$v = \frac{Q}{4\pi r^2} . \tag{1}$$

To solve the problem we use a system of hydro-thermodynamic equations in the form of Gromeko-Lambda and the following integral of Lagrange [15]:

$$\frac{\partial \Psi}{\partial t} + \frac{p}{\rho} + \frac{v^2}{2} = p_\infty , \tag{2}$$

where Ψ is the flow function, p is pressure, ρ is air density, p_∞ is the pressure in infinity (practically at the remote distance r). From (2) we have:

$$\Delta p = p - \rho \cdot p_\infty = -\rho \frac{\partial \Psi}{\partial t} - \frac{\rho v^2}{2} . \tag{3}$$

Taking into account that the second term in the right side of the equation (3) is approximately 10^3 times smaller than the first one [14] we get:

$$\Delta p = -\rho \frac{\partial \Psi}{\partial t}. \tag{4}$$

Theoretical Solution of the Problem

As the studied process proceeds in a small time interval (seconds, minutes), and the pressure mainly depends on density, it is possible to assume the medium to be barotropic in which a flat divergence is zero. That is why for definition of $\frac{\partial \Psi}{\partial t}$ we can use the following vorticity equation of the barotropic medium [15]:

$$\frac{\partial \Omega_z}{\partial t} + u\frac{\partial \Omega_z}{\partial x} + v\frac{\partial \Omega_z}{\partial y} = D = 0 , \tag{5}$$

where $\Omega_z = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$ is the z component of vorticity, u and v are velocity components along the axes *ox* and *oy*. As in the considered case the velocity fields u and v are solenoidal, they can be described by the following Helmholtz relations [15]:

$u = -\frac{\partial \Psi}{\partial y} + \frac{\partial \varphi}{\partial x}$, $v = \frac{\partial \Psi}{\partial x} + \frac{\partial \varphi}{\partial y}$, where φ is flow potential. Taking into account D=0

$$u = -\frac{\partial \Psi}{\partial y}, \quad v = \frac{\partial \Psi}{\partial x}, \qquad \text{and } \Omega_z = \Delta \Psi \tag{6}$$

where Δ is a plane Laplacian operator. Using (6), from (5) we obtain:

$$\Delta \frac{\partial \Psi}{\partial t} = (\Delta \Psi, \Psi), \tag{7}$$

where $(\Delta \Psi, \Psi)$ is Jacobian. It is obvious that (7) represents Poisson equation with regards to $\frac{\partial \Psi}{\partial t}$ and solution of (7) in polar coordinate system (r,α) has the following form [16]:

$$\frac{\partial\Psi}{\partial t}=\frac{1}{2\pi}\int_0^{2\pi}\int_0^R \ln\frac{R}{r}A_\Omega r dr d\alpha+\frac{1}{2\pi R}\oint\frac{\partial\Psi}{\partial t}ds, \tag{8}$$

where $\ln\frac{R}{r}$ is an influence of the so called Green's function, $A_\Omega=(\Delta\Psi,\Psi)$ is a horizontal advection, $R-$ is maximum distance of perturbed flow penetration. Taking into account special character of the generated process [17], we can reduce Eq. (8) to

$$\frac{\partial\Psi}{\partial t}=\overline{A_\Omega}\int_0^R \ln\frac{R}{r}rdr=\frac{R^2}{4}\overline{A_\Omega}, \tag{9}$$

where $\overline{A_\Omega}$ is a mean value of A_Ω inside the whole circle with radius R.
When influence of orography is taken into account then we have [17]:

$$\Omega_z^{'}=\frac{1}{\eta}(\Delta\Psi+a\frac{\partial\Psi}{\partial x}+b\frac{\partial\Psi}{\partial y}), \tag{10}$$

where $\eta=\frac{P_z}{P_o}$, P_z is the value of atmospheric pressure at the height of Z from the earth surface, P_o is the standard value of pressure, $a=-\frac{\partial\ln\eta}{\partial x}$ and $b=-\frac{\partial\ln\eta}{\partial y}$ are distinctive parameters characterizing the influence of the relief along the parallel and meridian, respectively. Taking into account (10) for finding $\frac{\partial\Psi}{\partial t}$ instead of (9) we get [17]:

$$\Delta\frac{\partial\Psi}{\partial t}+\frac{\partial}{\partial t}(a\frac{\partial\Psi}{\partial x}+b\frac{\partial\Psi}{\partial y})=\frac{1}{\eta}(\Delta\Psi,\Psi)-(\ln\eta,\Psi)=A_\Omega'. \tag{11}$$

(11) is a Helmholtz-type equation, which has the following solution [12,16]

$$\frac{\partial\Psi}{\partial t}=\frac{1}{2\pi}\int_0^{2\pi}d\alpha\int_0^R A_\Omega' K_0(\xi)rdr+\frac{1}{2\pi R}\oint\frac{\partial\Psi}{\partial t}ds, \tag{12}$$

where $K_o(\xi)$ is the Bessel function for imaginary argument, the so-called Macdonald's function, which has logarithmic singularity at point $\xi\to 0$ and exponentially decreases when $\xi\to\infty$ [16]. In the frame of our task for the sufficient approximation we can write:

$$K_o(\xi)=-\ln\xi=\ln\frac{1}{\xi}, \tag{13}$$

where $\xi=\frac{r}{2}\sqrt{a^2+b^2}$. Taking into account all the above-mentioned and following [12,16] instead of (7) we have:

$$\frac{\partial\Psi}{\partial t}=\overline{A_\Omega'}\int_0^R \ln\frac{2}{r\sqrt{a^2+b^2}}rdr \tag{14}$$

and after calculation of the integral in (14) we get:

$$\frac{\partial\Psi}{\partial t}=(C-\ln\sqrt{R})\overline{A_\Omega'}R^2, \tag{15}$$

Table 1. Pressure drop changeability according to distance and angle of inclination above the uniform terrain

r(m)	Δp (pa)		
	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
5	1×10^6	1.2×10^6	1×10^6
10	1.6×10^4	1.9×10^4	1.6×10^4
20	257	297	257
30	22.6	26	22.6
40	4	4.6	4
50	1.05	1.2	1.05
60	0.4	0.5	0.4
70	0.1	0.2	0.1
100	0.02	0.03	0.02

where $C = \frac{1}{2}\ln\frac{2}{\sqrt{a^2+b^2}} - 0.25$ and the value of C for certain regions are calculated individually. For this reason we represent a mountain massif as a body with regular geometric form, for example, as a triangular pyramid, the length, width and height of which are real magnitudes.

Results and Discussion

Now we are going to represent horizontal advections $\overline{A_\Omega}$ and $\overline{A'_\Omega}$ in different approximations. Let us consider the case when there is a pinpoint blasting on a plane terrain. Taking into account (1) we have $\overline{A_\Omega} = -\frac{3\cdot Q^2}{32\cdot\pi^2}\cdot\frac{\sin 2\alpha}{r^6}$, $\frac{\partial\Psi}{\partial t} = -\frac{3\cdot R^2\cdot Q^2}{128\cdot\pi^2}\cdot\frac{\sin 2\alpha}{r^6}$ and for pressure drop we get:

$$\Delta p = -\rho\frac{\partial\Psi}{\partial t} = \rho\frac{3\cdot R^2\cdot Q^2}{128\cdot\pi^2}\cdot\frac{\sin 2\alpha}{r^6} \quad . \tag{16}$$

From (16) it follows that above the plane terrain the pressure drop decreases proportionally according to r^6. Pressure drop was calculated from (16) for each value of distances r=5, 10, 20 ... 100m, for three different values of angle α - ($\frac{\pi}{6},\frac{\pi}{4},\frac{\pi}{3}$) and for common experimental parameters of the task R=500 m, r = 1,3 kg/m ^{3} and Q=5000 m^3/s . The results of calculations presented in Table 1 show that the values of the pressure drop decrease with the increase of distances, and at the distance of r=100m the disturbed atmospheric flow vanishes (r=100m, Δp = 0,02 (pa)). Also, Table 1 shows that when $\alpha = \frac{\pi}{4}$ for any value of distance the values of pressure drop are slightly greater than for the angles $\alpha = \frac{\pi}{6}, \alpha = \frac{\pi}{3}$. This indicates that in case we are not taking into account the influence of orography, the disturbed atmospheric flow mainly spreads towards 45^0 angle to the ground level. Also, calculations show that the value of pressure nearby the explosion is rather great (at r=10m, Δp = $1.9\cdot10^4$ pa), and significantly decrease according to distance (at r=50m the value of pressure changes Δp = 1.2 pa). Indeed, this result is natural for the open atmosphere space (without any obstacles) where the pressure drop is rapidly decreasing according to the distance. In order to

Table 2. Pressure difference changeability depending on the distance and inclination angle taking into account three deferent configurations of relief

r (m)	Δp (pa)								
	a>b (a=10^{-3}, b=10^{-4})			a<b (a=10^{-4}, b=10^{-3})			a=b (a=10^{-3}, b=10^{-3})		
	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$
10	$2\cdot10^4$	$3\cdot10^4$	$2\cdot10^4$	$2\cdot10^4$	$3\cdot10^4$	$2\cdot10^4$	$1\cdot10^4$	$2\cdot10^4$	$1\cdot10^4$
20	346	391	328	473	504	420	279	295	240
30	28	28	20	85	79	61	48	42	30
50	-0.6	-0.7	-2.3	21	17.3	12	11	8.4	5
100	-0.2	-0.6	-0.9	4.9	3.9	2.7	2.6	1.9	1
150	-0.09	-0.3	-0.4	2.1	1.7	1.2	1.1	0.8	0.4
200	-0.05	-0.2	-0.2	1.2	0.9	0.7	0.6	0.5	0.2
300	-0.02	-0.07	-0.1	0.5	0.4	0.3	0.3	0.2	0.1

make the question clear concerning the pressure drop dependence upon the explosion intensity we also made some calculations. Dependence of the pressure drop on the distance calculated for as three different values of explosions intensity (Q=500, 5000, 50000 m^3/s) are presented in Fig.1. As it was expected and as Fig.1 shows, when Q=50000 m^3/s the disturbed atmospheric flow spreads to the maximum distance from the center of explosion r=150m. Behavior of the other curves are almost identical.

Fig.1 illustrates rapid decrease of Δp according to distance which was stipulated by free distribution of the blast wave in the open atmosphere, deprived of any barriers.

When the perturbed air flow penetrated over the mountainous territory the calculations were performed by formula (15). In this case $\overline{A'_\Omega} = -\frac{Q}{4\pi r^2}\left(\frac{a\cdot\sin(\alpha)}{4} - b\cdot\cos(\alpha)\right) - \frac{1}{\eta^2}\frac{18\cdot Q^2\sin 2\alpha}{512\cdot\pi^2 r^6}$ and for Δp we get

$$\Delta p = -\rho\left(\frac{1}{2}\ln\left(\frac{2}{\sqrt{a^2+b^2}}\right) - 0{,}25 - \ln\sqrt{R}\right)\overline{A'_\Omega}R^2 . \quad (17)$$

Formula (17) shows if we take into considiration the influence of topography and assumption that the air streams behavior is difference along the terrestrial meridian and parallel then the pressure drop is inversely proportional to a squared distance. Once again for the following values of parameters R=500 m, r=1.3 kg/m^3 and Q=5000 m^3/s, we calculated the values of Δp at the distances r=5, 10, 20 ... 300 m, for the angles $\alpha = (\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3})$, and for different configuration of the relief (*a>b; a<b; a=b*). The results of such calculations are presented in Table 2. Analysis of Table 2 shows that when *a>b* (mountainousness in the direction of a terrestrial parallel is less than in the direction of a meridian) the values of Δp are decreasing quickly until r=30m, and further its values are expressed by negative numbers. This fact clearly indicates that when *a>b* the main atmosphere masses are not able to overcome the barrier. They are rebounded from the obstacle and are spreading backword for any values of angles. When *a<b* (when hilliness in the direction of a terrestrial parallel is more than in the direction of a meridian) then the values of Δp are considerably decreassing until r=30-50m, further their values are decreassing quite smoothly and the maximum distance of the disturbed flow penetration is about 200m (when r=200m then $\Delta p = 1.2(pa)$, and further the disturbed atmospheric flows are

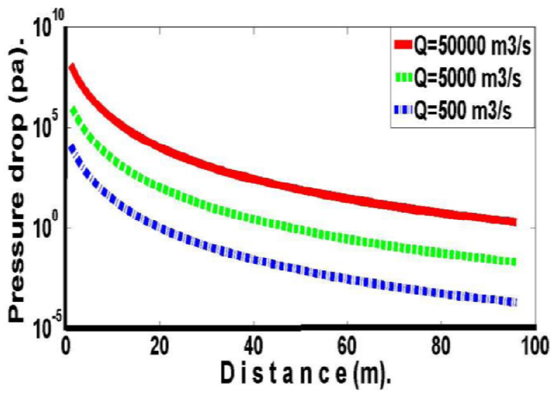

Fig. 1. Pressure drop changeability according to the distance for the three different values of explosion intensity above the uniform terrain.

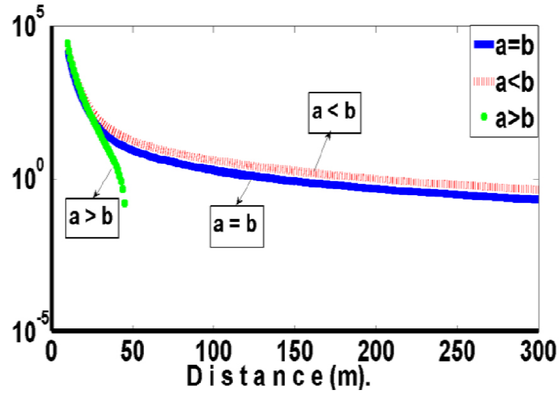

Fig. 2. Pressure difference changeability depending on the distance and inclination angle for three deferent configurations of relief and advection penetration.

almost eliminated. Also, Table 2 clearly shows that the values of pressure drop till 20m are slightly more for $\alpha = \frac{\pi}{4}$ and further air masses are spreading towards 30^0 angle. The latter indicates that after intial moment of time the main atmosphere masses are accumulated there near to the 45° angle Δp and reaches its maximum, further the values of pressure increase with the reduction of the angle value. This fact is natural as the obstacle (orography) block the way to the flow and the pressure changes slower than at the angles of 60° or 45°. At last when $a=b$ the values of Δp are considerably decreassing until r=20m, and the stream extends at the angle of 45°. Further after r=30m values of Δp are decreassing quite smoothly, the stream extends at the angle of 30° and the maximum distance of the disturbed flow penetration is about 150m (when r=150m then Δp=1,1(pa). The curves presented in Fig. 2 illustrate the behavior of pressure drops according to distance for three different configurations of orography ($a>b$; $a<b$; $a=b$) and calculated for the parameters of the task R=500m, ρ=1,3 kg/m^3, $\alpha = \frac{\pi}{4}$. Fig. 2 shows that when $a>b$ reduction of pressure drop proceeds very fast in comparison with other configurations ($a<b$; $a=b$). When $a \le b$ behaviour of the curves are almost identical but it must be noted that reduction of pressure happens very quickly till r=30m, further its changability has slower character. Behaviour of the curves in Fig. 2 confirms justice of the above given arguments (Table 2).

Conclusion

Taking modern global warming into consideration it is topical to study the processes associated with the blasts. As usual, these phenomena propagate in a little time on the relatively small territory, but their results are important. Especially interesting is the advective propagation of harmful air masses above the mountainous territories. Even lower hills slow down the velocity of flow motion and often change its direction, sometimes even to the opposite direction. Namely such a kind of natural phenomenon is characteristic for some regions of Georgia (Tskhinvali and Sachkhere territories), where military actions took place in 2008. Theoretical justification of such processes, are given in this article. Received results can be useful in military operations, mining operations, especially during open career works in the populated areas or near them.

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"შეშფოთებული" ჰაერის ნაკადის სივრცესა და დროში გავრცელების შესწავლას უდიდესი თეორიული და პრაქტიკული მნიშვნელობა აქვს განსაკუთრებით მთაგორიანი ტერიტორიებისთვის, ვინაიდან მცირე სიმაღლის ბურცობებიც კი სწრაფად ანელებს ნაკადის მოძრაობის სიჩქარეს, უცვლის მას მიმართულებას და ხშირად აბრუნებს საწინააღმდეგო მიმართულებითაც კი. მოცემულ ნაშრომში შესწავლილია მძლავრი "შეშფოთებით" წარმოშობილი ჰაერის ნაკადის სივრცესა და დროში გავრცელება, როგორც ერთგვაროვან ასევე არაერთგვაროვან ტერიტორიებზე ოროგრაფიის გავლენის გათვალისწინებით. მოცემულია თეორიული და რიცხვითი კვლევების ზოგიერთი შედეგი. მიღებული შედეგები სასურველია მხვედველობაში იქნეს მიღებული საომარი მოქმედებებისა თუ სამთო და ღია კარიერებზე მუშაობისას, მითუმეტეს თუ თუ ასეთი სამუშაოები მიმდინარეობს დასახლებულ პუნქტებში ან მათ სიახლოვეს.

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