საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, ტ. 3, №1, 2009
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 3, no. 1, 2009

Informatics

On the Stability of Locally Optimal Solution in Boolean Optimization Problems

Bezhan Ghvaberidze

I. Javakhishvili Tbilisi State University

(Presented by Academy Member V. Chavchanidze)

ABSTRACT. The notion of the set of stability of a locally optimal solution is introduced for Boolean optimization problems; its properties are studied. A formula is obtained for calculation of the stability radius. *© 2009 Bull. Georg. Natl. Acad. Sci.*

Key words: *Boolean optimization problem, locally optimal solution, set of stability, stability radius.*

In the paper, the question of stability of the locally optimal solution for Boolean optimization problems is researched. The same question for ε-approximate solution is considered in [1], using the approach proposed in [2, 3]. As is known [4, 5], the most part of discrete optimization problems belongs to the class of NP-complete problems, therefore local search methods are often used to solve such problems.

I. Let B^n be a set of all Boolean vectors $x = (x_1, \cdots, x_n)$; $R_+^n = \{c = (c_1, \cdots, c_n) : c_i \geq 0, c_i \in R, i = 1, \cdots, n\}$

Let us consider the problem:

$$\min \langle c, x \rangle, \tag{1}$$

$$x \in X. \tag{2}$$

where $c \in R^n$ is a given vector, $X \subset B^n$ is a nonempty set of admissible solutions, $\langle \cdot ; \cdot \rangle$ is a scalar product of vectors. Problem (1)-(2) always has the solution:

$$x^* \in X : \langle c, x^* \rangle = \min_{x \in X} \langle c, x \rangle .$$

Let us explain the function of neighbourhood $N : X \to 2^X$ for each $x \in X$. The function compares each admissible solution to the set of solutions neighboured in certain sense [4]. We call the set $N(x)$ a neighbourhood of admissible solutions x in the set X.

II. Let $x^N \in X$ be an admissible solution of problem (1)-(2).

Definition 1. We call an admissible solution $x^N \in X$ locally optimal solution of problem (1)-(2) with respect to the function *N* if $\langle c, x^N \rangle \leq \langle c, y \rangle$ for each $y \in N(x)$.

Definition 2. We call a set of vectors, $c \in R^n$ for which x^N is locally optimal solution of the problem (1)-(2) with respect to the function *N*, a stability domain of the solution x^N and denote it $F(x^N)$.

Proposition 1. $F(x^N)$ *is a closed convex cone*.

Definition 3. Let $\delta \geq 0$. The problem

$$\min\langle c+b, x\rangle,$$
$$x \in X,$$

where $\| b \| < \delta, \quad \| b \| = \max_{i=1,\dots,n} | b_i |$.

We call the problem (1)-(2) δ-perturbed.

Definition 4. A locally optimal solution is called δ-stable if it is a locally optimal solution of any δ-perturbed problem.

Definition 5. A closed kernel $S_\rho(c)$ with centre $c \in F(x^N)$ and radius ρ is called a kernel of stability of a locally optimal solution x^N if

$$S_\rho(c) \bigcap R_+^n \subset F(x^n),$$

and maximal value of radius ρ of the stability kernel is called stability radius and is denoted by $\rho(x^N, c)$.

Theorem. *The following formula*

$$\rho(x^N, c) = \min_{z \in N(x) \setminus x^N} \frac{|\langle c, x^N - z\rangle|}{\sum_{i=1}^{n} | x_i^N - z_i |} \tag{3}$$

is valid.

III. Finally, let us make some remarks.

1. Formula (3) is valid when x^N is the only locally optimal solution in the set N(x);
2. When $N(x) = X$, formula (3) allows us to calculate a stability radius of the optimal x^* solution;
3. The principle of construction of $N(x)$ can be various and often depends on the type of the concrete problem. Therefore, completeness of calculation of the stability radius by formula (3) depends on the power of the set $N(x)$;
4. The equality $\rho(x^N, c) = 0$ means that there exist certain directions in the area R^n along which changes of values of the vector **C** causes a loss of solution property, in particular, x^N will not be locally optimal.

ინფორმატიკა

ლოკალურად ოპტიმალური ამონახსნის მდგრადობის შესახებ ბულის ოპტიმიზაციის ამოცანებში

ბ. ღვაბერიძე

ი.ჯავახიშვილის სახ. თბილისის სახელმწიფო უნივერსიტეტი

(წარმოდგენილია აკადემიკოს ვ.ჭავჭანიძის მიერ)

ბულის ოპტიმიზაციის ამოცანებისათვის შემოღებულია ლოკალურად ოპტიმალური ამონახსნის მდგრადობის სიმრავლის ცნება, შესწავლილია მისი თვისებები. მიღებულია მდგრადობის რადიუსის გამოსათვლელი ფორმულა.

REFERENCES

1. *B. Ghvaberidze* (2003), Bull.Georg.Acad.Sci., **167,** 2: 219-222.
2. *V.K. Leont'ev* (1979), In: Problemy kibernetiki, **35**, M. (in Russian).
3. *V.K. Leont'ev* (1976), DAN SSSR, **228**, 1 (in Russian).
4. *Kh. Papadimitriu, K. Staiglits* (1985), Kombinatornaya optimizatsiya, M. (in Russian).
5. *M.R. Geri, D.S. Jonson* (1982),Vychislitel'nye mashiny i trudnoreshaemye zadachi, M. (in Russian).

Received December, 2008