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Mathematics

On General Solutions of Particular Classes of Ordinary Nonlinear Differential Equations

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ABSTRACT. This article contains the formulae of general and subgeneral solutions for particular classes of ordinary nonlinear second and high order differential equations. *© 2009 Bull. Georg. Natl. Acad. Sci.*

Key words: *ordinary nonlinear differential equations, general and subgeneral solutions.*

We introduce the following notations: $I =]t_1,\ t_2[\subset]-\infty,+\infty[,\ \dot{x} = dx/dt, x^{(k)} = d^k x/dt^k, k = \overline{2,n}.$

Lemma 1. *Let us consider the equation*

$$xx^{(n)} - \dot{x}x^{(n-1)} - a(t)x^2 = 0,\ t \in I, \tag{1}$$

where $a = a(t),\ t \in I$ *is an arbitrary continuous function. If the following condition holds*

$$\varphi^{(n-1)} - (c_0 + \int a dt)\varphi = 0,\ \ t \in I, \tag{2}$$

where $c_0 - \forall\text{const.}, \varphi = \varphi(t), t \in I$ *is an arbitrary admissible function, i.e. a function for which the presented operations hold, then the function* $\varphi = \varphi(t), t \in I$ *is a solution of equation* (1).

Proof. From formula (2) it follows

$$\varphi^{(n)} - (c_0 + \int a dt)\dot{\varphi} - a(t)\varphi = 0,\ t \in I \Rightarrow \varphi\varphi^{(n)} - \varphi(c_0 + \int a dt)\dot{\varphi} - a(t)\varphi^2 = 0, t \in I \Rightarrow$$
$$\varphi\varphi^{(n)} - \varphi^{(n-1)}\dot{\varphi} - a(t)\varphi^2 = 0, t \in I.$$

Lemma 1 is proved. □

Lemma 1 implies the following assertion.

Theorem 1. *The general solution of the equation*

$$xx^{(2)} - \dot{x}^2 - a(t)x^2 = 0,\ t \in I, \tag{3}$$

where $a = a(t),\ t \in I$ *is an arbitrary continuous function, has the form*

$$x = c_1 e^{\int (c_0 + \int a dt) dt}, c_0, c_1 - \forall\text{const.}, t \in I. \tag{4}$$

(See [1]).

Proof. Let n=2. In this case, equation (1) has the form (3) and condition (2) has the form:

$$\dot{\varphi} - (c_0 + \int a dt)\varphi = 0,\ \exists\varphi^{(2)},\ t \in I.$$

Consequently, $\varphi = c_1 e^{\int (c_0 + \int a dt) dt}, \quad t \in I.$

where $c_0, c_1 - \forall$const., is a solution of equation (3).

Let t_0 be an arbitrary fixed point of the interval I and x_0, x_1 be arbitrary fixed constants. Let the following conditions hold: $\varphi(t_0) = x_0, \dot{\varphi}(t_0) = x_1$.

Then

$$x_0 = c_1 e^{\int (c_0 + \int adt)dt}\,|_{t=t_0}, x_1 = (c_0 + \int adt)\,|_{t=t_0}\, x_0 \Rightarrow c_0 = x_1 / x_0 - \int adt\,|_{t=t_0}, c_1 = x_0 / c,$$

where $c = e^{\int (c_0 + \int adt)dt}\,|_{t=t_0}$.

Theorem 1 is proved. □

Theorem 2. *The equation*

$$xx^{(3)} - \dot{x}x^{(2)} - \left[b^2(t) + \dot{b}(t)\right]^{\bullet} x^2 = 0,\ t \in I, \tag{5}$$

where $b = b(t), t \in I$ *is an arbitrary admissible function, has the following subgeneral solution*

$$x = e^{\int bdt}(c_1 + c_2 \int e^{-2\int bdt}dt),\ t \in I, \tag{6}$$

where $c_1, c_2 - \forall$const.

Proof. Let n=3. In this case equation (1) has the form (5), where $\left(b^2 + \dot{b}\right)^{\bullet} = a, t \in I$ and condition (2) has the form:`

$$\varphi^{(2)} - (c_0 + c + b^2 + \dot{b})\varphi = 0,\ \ \exists \varphi^{(3)},\quad t \in I \tag{7}$$

where $c - \forall$const. Let $c = -c_0$. Then condition (7) has the form

$$\varphi^{(2)} - (b^2 + \dot{b})\varphi = 0,\ \ \exists \varphi^{(3)},\ t \in I$$

Consequently (see [2], Theorem 6),

$$\varphi = e^{\int bdt}\left(c_1 + c_2 \int e^{-2\int bdt} dt\right),\ t \in I \tag{8}$$

where $c_1, c_2 - \forall$const.

From Lemma 1 it follows that formula (8) gives a solution of equation (5).

Let t_0 be an arbitrary fixed point of the interval I and x_0, x_1 be arbitrary fixed constants. Let the following conditions hold: $\varphi(t_0) = x_0, \dot{\varphi}(t_0) = x_1$.

Then

$$x_0 = e^{\int bdt}\,|_{t=t_0}\,(c_1 + c_2 \int e^{-2\int bdt}dt\,|_{t=t_0}),$$

$$x_1 = b\,|_{t=t_0}\, x_0 + c_2 e^{-\int bdt}\,|_{t=t_0} \Rightarrow c_2 = (x_1 - b(t_0)x_0)e^{\int bdt}\,|_{t=t_0},$$

$$c_1 = x_0 e^{-\int bdt}\,|_{t=t_0} - c_2 \int e^{-2\int bdt} dt\,|_{t=t_0}.$$

Consequently, formula (6) gives the subgeneral solution of equation (5).

Theorem 2 is proved. □

Theorem 3. *Let the following conditions hold*:

$$\dot{a}_1 + a_1^2 + a_1 a_2 + 2\dot{a}_2 + a_2^2 = 0,\ t \in I, \tag{9}$$

where $a_1 = a_1(t),\ a_2 = a_2(t),\ t \in I$ *are arbitrary admissible functions. Then the subgeneral solution of the equation*

$$xx^{(4)} - \dot{x}x^{(3)} + \left\{(a_1 a_2 - \dot{a}_2)^{(2)} + \left[(a_1 + a_2)(a_1 a_2 - \dot{a}_2)\right]^{\bullet}\right\}x^2 = 0, t \in I \tag{10}$$

has the form

$$x = e_2 s(c_0 + \int e_2^{-1} s^{-1} Y dt),\ t \in I,$$

where

$$Y = e_1 s^{-1}(c_1 + c_2 \int e_1^{-2} e_2^{-1} s dt), \quad e_i = e^{\int a_i dt}, \; i = 1,2, \; s = \int e_1 e_2^{-1} dt, \; c_0, c_1, c_2 - \forall \text{const}; \; t \in I.$$

Proof. Let n=4. In this case, equation (1) has the form (10), where

$$-a(t) = (a_1 a_2 - \dot{a}_2)^{(2)} + [(a_1 + a_2)(a_1 a_2 - \dot{a}_2)]^{\bullet}, \; t \in I$$

and condition (2) has the form

$$\varphi^{(3)} + \left[c_0 + c + (a_1 a_2 - \dot{a}_2)^{\bullet} + (a_1 + a_2)(a_1 a_2 - \dot{a}_2)\right]\varphi = 0, \exists \varphi^{(4)}, t \in I, \tag{11}$$

where $c - \forall$const.

Let $c = -c_0$. Then condition (11) has the form

$$\varphi^{(3)} + [(a_1 a_2 - \dot{a}_2)^{\bullet} + (a_1 + a_2)(a_1 a_2 - \dot{a}_2)]\varphi = 0, \; \exists \varphi^{(4)}, \; t \in I.$$

Consequently (see [2], Theorem 10),

$$\begin{cases} \varphi = e_2 s(c_0 + \int e_2^{-1} s^{-1} Y dt), \\ \text{where} \\ Y = e_1 s^{-1}(c_1 + c_2 \int e_1^{-2} e_2^{-1} s dt), \\ e_i = e^{\int a_i dt}, \; i = 1,2, \; s = \int e_1 e_2^{-1} dt, \; c_0, c_1, c_2 - \forall \text{const}; \; t \in I. \end{cases} \tag{12}$$

From Lemma 1 it follows that formula (12) gives a solution of equation (10).

Let t_0 be an arbitrary fixed point of the interval I and x_0, x_1, x_2 be arbitrary fixed constants. Let the following conditions hold:

$$\varphi(t_0) = x_0, \dot{\varphi}(t_0) = x_1, \varphi^{(2)}(t_0) = x_2.$$

We have

$$\dot{\varphi} = a_2 \varphi + e_1 e_2^{-1} s^{-1} \varphi + c_2 e_1^{-1} e_2^{-1} + Y, \; t \in I,$$

$$\varphi^{(2)} = \left(a_2 + e_1 e_2^{-1} s^{-1}\right)^{\bullet} \varphi + \left(a_2 + e_1 e_2^{-1} s^{-1}\right)\dot{\varphi} + c_2 \left(e_1^{-1} e_2^{-1}\right)^{\bullet} + \dot{Y} \Rightarrow$$

$$\varphi^{(2)} = \left(\dot{A} + A^2\right)\varphi + c_2\left(AB + \dot{B}\right) + a_1 Y - \frac{\dot{s}}{s} Y + c_2 B, \; t \in I,$$

where $A = a_2 + e_1 e_2^{-1} s^{-1}, B = e_1^{-1} e_2^{-1}, \; t \in I$.

Hence,

$$Y = \dot{\varphi} - A\varphi - c_2 B, \; t \in I, \tag{13}$$

$$\varphi^{(2)} = (\dot{A} + A^2)\varphi + c_2(AB + \dot{B} + B) + (a_1 - e_1 e_2^{-1} s^{-1})Y, t \in I \Rightarrow$$

$$\varphi^{(2)} = (\dot{A} + A^2)\varphi + c_2(AB - a_1 B - a_2 B + B) + (a_1 + a_2 - A)(\dot{\varphi} - A\varphi - c_2 B), t \in I \Rightarrow$$

$$c_2(2AB - 2(a_1 + a_2)B + B) = \varphi^{(2)} - (\dot{A} + A^2)\varphi - (a_1 + a_2 - A)(\dot{\varphi} - A\varphi), t \in I \Rightarrow$$

$$c_2(2A - 2(a_1 + a_2) + 1)B|_{t=t_0} = x_2 - (\dot{A} + A^2)|_{t=t_0} x_0 - (a_1 + a_2 - A)(x_1 - Ax_0)|_{t=t_0} \Rightarrow c_2 K = M,$$

where $K = (2A - 2(a_1 + a_2) + 1)e_1^{-1} e_2^{-1}|_{t=t_0}$,

$$M = x_2 - (\dot{A} + A^2)|_{t=t_0} x_0 - (a_1 + a_2 - A)(x_1 - Ax_0)|_{t=t_0}.$$

Now let us prove that $K \neq 0$.

Let

$$(2A - 2(a_1 + a_2) + 1)e_1^{-1} e_2^{-1}|_{t=t_0} = 0 \Rightarrow \left[2e^{\int (a_1 - a_2)dt}\left(\int e^{\int (a_1 - a_2)dt} dt\right)^{-1} - 2a_1 + 1\right]_{t=t_0} = 0 \Rightarrow$$

$$e^{\int (a_1 - a_2)dt} - \left(a_1 - \frac{1}{2}\right)\int e^{\int (a_1 - a_2)dt} dt = 0, \; \text{if } t = t_0 \Rightarrow \dot{z} - \left(a_1 - \frac{1}{2}\right)z = 0, \; \textit{if } t = t_0,$$

where $z=\int e^{\int(a_1-a_2)dt}dt\Rightarrow$

$$ce^{\int(a_1-\frac{1}{2})dt}=\int e^{\int(a_1-a_2)dt}dt,\ \text{if}\ t=t_0\Rightarrow c\left(a_1-\frac{1}{2}\right)=e^{\int(\frac{1}{2}-a_2)dt},\text{if}\ t=t_0\Rightarrow a_1\neq\frac{1}{2},\ \text{if}\ t=t_0.$$

Consequently, we have

$$\frac{1}{a_1-\frac{1}{2}}e^{\int(a_1-a_2)dt}-\int e^{\int(a_1-a_2)dt}dt=0,\text{if}\ t=t_0\Rightarrow\left(\frac{1}{a_1-\frac{1}{2}}\right)^{\bullet}+(a_1-a_2)\frac{1}{a_1-\frac{1}{2}}-1=0,\text{if}\ t=t_0.$$

Denote $y=\dfrac{1}{a_1-\frac{1}{2}}$. Then we have:

$$y=e^{-\int(a_1-a_2)dt}(\widetilde{c}+\int e^{\int(a_1-a_2)dt}dt),\ \text{if}\ t=t_0,$$

where $\widetilde{c}-\forall$const., that is impossible.

Hence, $K\neq 0$ and $c_2=M/K$.

From formula (13) it follows

$$Y(t_0)=x_1-A\big|_{t=t_0}x_0-\frac{M}{K}B\big|_{t=t_0}=D.$$

From formulae (12) it follows

$$c_1=Y(t_0)e_1^{-1}s\big|_{t=t_0}-\frac{M}{K}\int e_1^{-2}e_2^{-1}sdt\big|_{t=t_0}=E$$

and

$$Y=e_1s^{-1}\left(E+\frac{M}{K}\int e_1^{-2}e_2^{-1}sdt\right),\ t\in I.$$

Finally, from formulae (12) it follows

$$c_0=x_0e_2^{-1}s^{-1}\Big|_{t=t_0}-\int e_2^{-1}s^{-1}Ydt\Big|_{t=t_0}=F.$$

Here D, E, F are known constants.

Theorem 3 is proved. □

Let $\dot{x}=yx,\ t\in I\Rightarrow x=ce^{\int ydt},\quad \dot{x}=cye^{\int ydt},\quad x^{(2)}=c(\dot{y}+y^2)e^{\int ydt},\ x^{(3)}=c[(\dot{y}+y^2)^{\bullet}+y(\dot{y}+y^2)]e^{\int ydt}$,

$$x^{(4)}=c\left\{[(\dot{y}+y^2)^{\bullet}+y(\dot{y}+y^2)]^{\bullet}+y[(\dot{y}+y^2)^{\bullet}+y(\dot{y}+y^2)]\right\}e^{\int ydt},\quad t\in I.$$

From formula (10) it follows

$$y^{(2)}+3y\dot{y}+y^3=(\dot{a}_2-a_1a_2)^{\bullet}+(a_1+a_2)(\dot{a}_2-a_1a_2),\quad t\in I. \tag{14}$$

From Theorem 3 it follows

$$x=e_2s(c_0+\int e_2^{-1}s^{-1}Ydt),\quad Y=e_1s^{-1}(c_1+c_2\int e_1^{-2}e_2^{-1}sdt),\ t\in I\Rightarrow$$

$$\dot{x}=a_2x+\frac{\dot{s}}{s}x+Y,\ t\in I,\Rightarrow\ y=a_2+\frac{\dot{s}}{s}+\frac{\dot{s}}{s^2}\cdot\frac{c_1+c_2\int e_1^{-2}e_2^{-1}sdt}{c_0+\int\frac{\dot{s}}{s^2}(c_1+c_2\int e_1^{-2}e_2^{-1}sdt)},\ t\in I.$$

Consequently, the function

$$y = a_2 + \frac{\dot{s}}{s} + \frac{\dot{s}}{s^2} \cdot \frac{\tilde{c}_1 + \int e_1^{-2} e_2^{-1} s dt}{\tilde{c}_0 + \int \frac{\dot{s}}{s^2} (\tilde{c}_1 + \int e_1^{-2} e_2^{-1} s dt) dt}, \quad t \in I, \tag{15}$$

where $\tilde{c}_0, \tilde{c}_1 - \forall$const. is a solution of equation (14).

Let t_0 be an arbitrary fixed point of the interval I and y_0, y_1 arbitrary fixed constants. Let the following conditions hold: $y(t_0) = y_0$, $\dot{y}(t_0) = y_1$.

Then $y_0 = \left(a_2 + \frac{\dot{s}}{s} \right)_{t=t_0} + \frac{\dot{s}}{s^2} \cdot \frac{A}{\tilde{c}_0 + \int \frac{\dot{s}}{s^2} A dt} \Bigg|_{t=t_0}$,

where $A = \tilde{c}_1 + \int e_1^{-2} e_2^{-1} s dt$ and

$$y_1 = \left(a_2 + \frac{\dot{s}}{s} \right)^{\bullet} \Bigg|_{t=t_0} + \left(\frac{\dot{s}}{s^2} \right)^{\bullet} \cdot \frac{A}{\tilde{c}_0 + \int \frac{\dot{s}}{s^2} A dt} \Bigg|_{t=t_0} + \frac{\dot{s}}{s^2} \cdot \frac{\dot{A}}{\tilde{c}_0 + \int \frac{\dot{s}}{s^2} A dt} \Bigg|_{t=t_0} -$$

$$\left(\frac{\frac{\dot{s}}{s^2} A}{\tilde{c}_0 + \int \frac{\dot{s}}{s^2} A dt} \right)^2_{t=t_0} \Rightarrow \frac{\dot{s}}{s^2} \cdot \frac{A}{\tilde{c}_0 + \int \frac{\dot{s}}{s^2} A dt} \Bigg|_{t=t_0} = y_0 - \left(a_2 + \frac{\dot{s}}{s} \right)_{t=t_0} = B,$$

$$\frac{\dot{s}}{s^2} \cdot \frac{\dot{A}}{\tilde{c}_0 + \int \frac{\dot{s}}{s^2} A dt} \Bigg|_{t=t_0} = y_1 - \left(a_2 + \frac{\dot{s}}{s} \right)^{\bullet}_{t=t_0} - \left(\frac{\dot{s}}{s^2} \right)^{\bullet} \frac{s^2}{\dot{s}} \Bigg|_{t=t_0} B + B^2 = D,$$

where B, D are known constants.

Hence,

$$\tilde{c}_0 + \int \frac{\dot{s}}{s^2} A dt \Bigg|_{t=t_0} = \frac{1}{D} \left(\frac{\dot{s}}{s^2} \dot{A} \right)_{t=t_0} = \frac{1}{D} \left(\frac{\dot{s}}{s^2} e_1^{-2} e_2^{-1} \right)_{t=t_0} = E,$$

where E is a known constant.

Consequently,

$$y_0 = \left(a_2 + \frac{\dot{s}}{s} \right)_{t=t_0} + \frac{1}{E} \left(\frac{\dot{s}}{s^2} A \right)_{t=t_0} \Rightarrow A\big|_{t=t_0} = E \left[y_0 - \left(a_2 + \frac{\dot{s}}{s} \right)_{t=t_0} \right] \frac{s^2}{\dot{s}} \Bigg|_{t=t_0} \Rightarrow$$

$$\tilde{c}_1 = E \left[y_0 - \left(a_2 + \frac{\dot{s}}{s^2} \right)_{t=t_0} \right] \frac{s^2}{\dot{s}} \Bigg|_{t=t_0} - \int e_1^{-2} e_2^{-1} dt \Bigg|_{t=t_0}.$$

We have

$$\tilde{c}_0 = E - \int \frac{\dot{s}}{s^2} A dt \Bigg|_{t=t_0} = E - \int \frac{\dot{s}}{s^2} (\tilde{c}_1 + \int e_1^{-2} e_2^{-1} dt) dt \Bigg|_{t=t_0}.$$

Consequently, the following theorem is valid.

Theorem 4. *If condition (9) holds, where* $a_i = a_i(t), i = 1,2, t \in I$ *are arbitrary admissible functions, then the general solution of equation* (14) *has the form* (15).

See, also [3].

Let $\dot{z} = yz, t \in I$ .Then equation (14) has the form

$$z^{(3)} - [(\dot{a}_2 - a_1 a_2)^{\bullet} + (a_1 + a_2)(\dot{a}_2 - a_1 a_2)]z = 0, \quad t \in I. \tag{16}$$

and $z = ce^{\int y dt}, t \in I,$ where $c - \forall$const.

From Theorem 4 it follows

Theorem 5. *If condition* (9) *holds, where* $a_i = a_i(t), i = 1,2, t \in I$ *are arbitrary admissible functions, then the general solution of equation* (16) *has the form:*

$$z = ce_2 s\left(\widetilde{c}_0 + \int \frac{\dot{s}}{s^2}(\widetilde{c}_1 + \int e_1^{-2} e_2^{-1} s dt)\right), \ t \in I,$$

i.e.
$$z = ce^{\int a_2 dt} \int e^{\int (a_1 - a_2) dt} dt \left[\widetilde{c}_0 + \int \frac{e^{\int (a_1 - a_2) dt}}{\left(\int e^{\int (a_1 - a_2) dt} dt\right)^2} \left(\widetilde{c}_1 + \int e^{-\int (2a_1 + a_2) dt} (\int e^{\int (a_1 - a_2) dt} dt) dt\right) dt\right], \quad t \in I \tag{17}$$

where $c, \widetilde{c}_0, \widetilde{c}_1 - \forall$const.

Remark. It is easy to see that

$$(\dot{a}_2 - a_1 a_2)^{\bullet} + (a_1 + a_2)(\dot{a}_2 - a_1 a_2) = \ddot{a}_2 + 3a_2 \dot{a}_2 + a_2^3, \quad t \in I.$$

See, also [4].

Example. Let $a_1 = \alpha a, a_2 = a, t \in I$, where $a = a(t),\ t \in I$ be an arbitrary admissible function, $\alpha - \forall$const.

From condition (9) it follows

$$\alpha \dot{a} + \alpha^2 a^2 + \alpha a^2 + 2\dot{a} + a^2 = 0, t \in I \Rightarrow \frac{1}{a} = \frac{1 + \alpha + \alpha^2}{2 + \alpha} t + \beta, \quad t \in I,$$

where $\beta - \forall$const. $\Rightarrow a = (\gamma t + \beta)^{-1},\ t \in I,$ where $\gamma = \dfrac{1 + \alpha + \alpha^2}{2 + \alpha} \Rightarrow$

$$a_1 = \alpha(\gamma t + \beta)^{-1}, \ a_2 = (\gamma t + \beta)^{-1}, \quad t \in I.$$

In this case equation (16) has the form

$$z^{(3)} + (\alpha^2 + \alpha - \alpha\gamma + \gamma - 2\gamma^2)(\gamma t + \beta)^{-3} z = 0, \ t \in I.$$

i.e.

$$z^{(3)} - (2\gamma^2 - 3\gamma + 1)(\gamma t + \beta)^{-3} z = 0, \quad t \in I. \tag{18}$$

From Theorem 5 it follows that the general solution of equation (18) has the form (17), i.e.

$$z = ce^{\int a dt}\left(\int e^{(\alpha - 1)\int a dt} dt\right)\left[\widetilde{c}_0 + \int \frac{e^{(\alpha - 1)\int a dt}}{\left(\int e^{(\alpha - 1)\int a dt} dt\right)^2}\left(\widetilde{c}_1 + \int e^{-(2\alpha + 1)\int a dt} (\int e^{(\alpha - 1)\int a dt} dt) dt\right) dt\right], \quad t \in I.$$

As $\int a dt = \dfrac{1}{\gamma} \ln(\gamma t + \beta)$ and $\int (\gamma t + \beta)^{\frac{\delta}{\gamma}} dt = \dfrac{1}{\delta + \gamma}(\gamma t + \beta)^{\frac{\delta + \gamma}{\gamma}}$.

We have

$$z = \frac{c}{\alpha + \gamma - 1}(\gamma t + \beta)^{\frac{\alpha + \gamma}{\gamma}}\left[\widetilde{c}_0 + (\alpha + \gamma - 1)^2 \int (\gamma t + \beta)^{\frac{1 - \alpha - 2\gamma}{\gamma}}\left(\widetilde{c}_1 + \frac{1}{\alpha + \gamma - 1}\int (\gamma t + \beta)^{\frac{\gamma - \alpha - 2}{\gamma}} dt\right) dt\right], \ t \in I, \text{ i.e.}$$

$$z = c_1(\gamma t + \beta)^{\frac{\alpha + \gamma}{\gamma}} + c_2(\gamma t + \beta)^{\frac{1}{\gamma}} + c_3(\gamma t + \beta)^{\frac{2\gamma - \alpha - 1}{\gamma}},$$

where $c_1, c_2 c_3 - \forall$const.

If $\gamma = -1$, equation (18) has the form

$$z^{(3)} - 6(\beta - t)^{-3} z = 0, \qquad t \in I \,, \tag{19}$$

$\alpha = -1 \pm i\sqrt{2}$ and the general solution of equation (19) has the form

$$z = c_1 (\beta - t)^{2 \mp i\sqrt{2}} + c_2 (\beta - t)^{-1} + c_3 (\beta - t)^{2 \pm i\sqrt{2}}, \qquad t \in I. \tag{20}$$

If $\exists \ln(\beta - t), \ \forall t \in I,$ we have

$$(\beta - t)^{2 \mp i\sqrt{2}} = (\beta - t)^2 \left\{ \cos\left[\sqrt{2}\ln(\beta - t)\right] \mp i \sin\left[\sqrt{2}\ln(\beta - t)\right] \right\},$$

$$(\beta - t)^{2 \pm i\sqrt{2}} = (\beta - t)^2 \left\{ \cos\left[\sqrt{2}\ln(\beta - t)\right] \pm i \sin\left[\sqrt{2}\ln(\beta - t)\right] \right\}, t \in I$$

and from formula (20) it follows that the real general solution of equation (19) has the form

$$z = (\beta - t)^2 \left\{ A \cos\left[\sqrt{2}\ln(\beta - t)\right] + B \sin\left[\sqrt{2}\ln(\beta - t)\right] \right\} + C(\beta - t)^{-1}, t \in I,$$

where $A, B, C - \forall$const.

See also [5,6].

მათემატიკა

ჩვეულებრივი არაწრფივი დიფერენციალური განტოლებების კერძო კლასების ზოგადი ამონახსნების ფორმულების შესახებ

გ. ხარატიშვილი

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