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Physics

Heat-Conductivity-Induced Instabilities in a Rotating Plasma

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ABSTRACT. Instabilities in a rotating cylindrical plasma with account of the finite heat conductivity are studied. The analysis is restricted to the local axisymmetric modes with parallel phase velocity smaller than the sound velocity in the plasma with $\beta >> 1$, where β is the ratio of the plasma pressure to the magnetic field pressure. Two approaches in studying such modes are discussed: the Balbus approach following the Boussinesq approximation and the regular one using the Frieman-Rotenberg technique. It is shown that the local dispersion relation obtained by the regular (rigorous) approach is more general than the Balbus dispersion relation. It is demonstrated that the latter misses the heat-conductivity-induced instability described here. Such instabilities can be revealed in both astrophysical and laboratory plasmas.

Key words: *heat-conductivity-induced instability, Frieman-Rotenberg technique, Balbus dispersion relation.*

1. Introduction

Recently the theory of MHD (magnetohydrodynamic) instabilities in rotating media [1, 2] has been advanced by allowing for the viscosity effects [3, 4]. As a result, a family of viscosity-induced instabilities has been revealed. The viscosity is a dissipative effect. A dissipation can be also produced by heat conductivity, and a natural question is whether heat-conductivity-induced instabilities may develop in a rotating plasma. Its analysis is the goal of the present paper. Finally, we show that such instabilities are possible.

Investigations performed in [3, 4] are based on an analytical approach going back to the Frieman-Rotenberg (FR) technique [5-10]. This means using a pair of first-order differential equations for the so-called FR variable (the sum of the perturbed pressures of the medium and the magnetic field) and the perturbed radial medium displacement. Excluding the FR variable from these equations one arrives at the mode equation which is a second-order differential equation for perturbed displacement. Taking the latter in the eikonal form $\exp(ik_r r)$, where r is the radial coordinate, k_r is the radial wave number, one obtains a local dispersion relation. This can be called regular or rigorous local dispersion relation.

Meanwhile, starting from [11] (see also [12, 13]) a different approach is often used for the local modes. In this case one takes the eikonal form $\exp(ik_r r)$ from the very beginning in basic dynamical equations, assuming this form to be valid for each perturbed function. The local dispersion relation thus found can be called "heuristic", in contrast to the rigorous one.

As a consequence, we have to deal with two different local dispersion relations, the heuristic and the rigorous. According to [4], these dispersion relations coincide only if: (1) the modes are axisymmetric; (2) the equilibrium pressure gradient is zero.

The effects of finite heat conductivity have been studied in [14] for the local axisymmetric modes in the scope of the heuristic local dispersion relation derived in the Boussinesq approximation. However, in this problem we have a nonvanishing equilibrium pressure gradient. Then a question arises: whether the results of [14] are adequate? This is discussed in the present paper. The answer is that the heuristic approach misses the heat-conductivity-induced instabilities.

2. Basic equations

We start with the standard MHD plasma motion equation

$$\rho \frac{d\mathbf{V}}{dt} = -\nabla p + \rho \mathbf{g} - \frac{1}{4\pi}\left\{\nabla \frac{\mathbf{B}^2}{2} - (\mathbf{B}\cdot\nabla)\mathbf{B}\right\}. \quad (1)$$

Here $\mathbf{V}$ is the plasma velocity, $\mathbf{B}$ is the magnetic field, p is the plasma pressure, ρ is the plasma mass density, $\mathbf{g}$ is the gravitation force, $d/dt = \partial/\partial t + (\mathbf{V}\cdot\nabla)$. We use the Ohm law in the form $\mathbf{E} + [\mathbf{V}\times\mathbf{B}]/c = 0$, where E is the electric field and c is the speed of light. This equation leads to the standard frozen-in condition $\partial\mathbf{B}/\partial t - \nabla\times[\mathbf{V}\times\mathbf{B}] = 0$. In addition, we use the Maxwell equation $\nabla\cdot\mathbf{B} = 0$, the plasma continuity equation $d\rho/dt + \rho\nabla\cdot\mathbf{V} = 0$ and the heat conductivity equation

$$n dT/dt + (\Gamma - 1)p\nabla\cdot\mathbf{V} = -(\Gamma - 1)\nabla\cdot\mathbf{q}. \quad (2)$$

Here $\Gamma = 5/3$ is the adiabatic exponent, $n = \rho/M$ is the plasma number density, M is the ion mass, T is the plasma temperature related to the pressure and density by $p=nT$, $\mathbf{q} = -k_T \mathbf{b}(\mathbf{b}\cdot\nabla T)$ is the heat flux, where $\mathbf{b} = \mathbf{B}/B$ and k_T is the heat conductivity coefficient.

We consider a cylindrical plasma rotating in azimuthal direction θ with the angular frequency $\Omega = \Omega(r)$, where r is the radial coordinate. The equilibrium magnetic field $\mathbf{B}_0$ is assumed to be uniform and directed along the cylinder axis z, $\mathbf{B}_0 = (0,0,B_0)$, and we assume the gravitational force $\mathbf{g}$ with only the radial component, $\mathbf{g} = (g,0,0)$. In accordance with the Ohm law, there is an equilibrium electric field $\mathbf{E}_0 = (E_0,0,0)$ related to the rotation frequency $\Omega = V_0/r$ by $E_0 = -r\Omega B_0/c$. Here $V_0 = V_0(r)$ is the azimuthal equilibrium plasma velocity.

It follows from the equilibrium part of the radial component of the plasma motion equation (1) that

$$r\rho_0\Omega^2 = p_0' - \rho_0 g. \quad (3)$$

Here ρ_0 and p_0 are the equilibrium plasma mass density and the equilibrium plasma pressure, respectively, the prime is the radial derivative.

We assume each perturbed function in the basic MHD equations to depend on t, z as $\exp(-i\omega t + ik_z z)$, where ω is the oscillation frequency, k_z is the parallel projection of the wave vector. We introduce the FR variable [5-8] $p_* = \tilde{p} + \tilde{B}_z B_0/(4\pi)$, where $\tilde{p}$ is the perturbed pressure, $\tilde{B}_z$ is the z-th projection of the perturbed magnetic field. Then, similarly to [3, 4, 9, 10], we arrive at the pair of equations

$$D\tau = C_1 X - 4\pi C_2 p_*, \quad (4)$$

$$4\pi D p_*' = -4\pi C_1 p_* + C_3 X. \quad (5)$$

Here $\tau = (rX')/r$, X is the perturbed radial plasma displacement. These equations can be called canonical ones. Each particular case is characterized by a particular choice of the constants D, C_1, C_2, and C_3. In the case considered these constants are

$$D = D_0\left(1 + \frac{4\pi p_0}{B_0^2\alpha_s^T}\frac{\Gamma\omega + i\Delta_T}{\omega + i\Delta_T}\right), \quad (6)$$

$$C_1 = -\frac{4\pi p_0 D_0}{B_0^2\alpha_s^T}\frac{d\ln p_0}{dr}, \quad (7)$$

$$C_2 = D_0/B_0^2, \quad (8)$$

$$C_3 = 4\pi\rho_0 D_0\left[\left(1 + \frac{4\pi p_0}{B_0^2\alpha_s^T}\frac{\Gamma\omega + i\Delta_T}{\omega + i\Delta_T}\right)\left(D_0 - \kappa^2 - \frac{p_0'}{\rho_0}\frac{d\ln\rho_0}{dr} - \frac{4\Omega^2 k_z^2 v_A^2}{D_0}\right) + \right.$$

$$\left. + \frac{D_0 p_0' p_0}{\rho_0^2 v_A^2 \alpha_s^T \omega^2}\frac{d\ln p_0}{dr}\right]. \quad (9)$$

Here $D_0 = \alpha_A \omega^2$, $\alpha_A = 1 - k_z^2 v_A^2 / \omega^2$, $\kappa^2 = 4\Omega^2 + d\Omega^2 / d\ln r$, the value α_s^T is the sound propagator defined by

$$\alpha_s^T = 1 - \frac{p_0 k_z^2 (\Gamma\omega + i\Delta_T)}{\rho_0 \omega^2 (\omega + i\Delta_T)}, \tag{10}$$

where $v_A^2 = B_0^2 / (4\pi\rho)$ is the Alfven velocity squared, $c_s^2 = \Gamma p_0 / \rho$ is the sound velocity squared, $\Delta_T = \kappa_T (\Gamma - 1) k_z^2 / n_0$ is the characteristic heat-conductivity-induced decay rate.

Excluding the variable p_* from Eqs. (4) and (5), one arrives at the canonical mode equation [3, 4, 9, 10]

$$D(D\tau_B / C_2)' + \Lambda X = 0 \,. \tag{11}$$

Here

$$\Lambda = a + b \tag{12}$$

$$a = C_3 - C_1^2 / C_2 , \tag{13}$$

$$b = -Dr[C_1 / (rC_2)]' \tag{14}$$

Taking $X \sim \exp(ik_r r)$ in (11), one has the canonical local dispersion relation

$$-k_r^2 D^2 / C_2 + \Lambda = 0 \,. \tag{15}$$

For Λ of the form (12) this local dispersion is the rigorous one. For $p_0' = 0$ one has from (7) that $C_1 = 0$. Then Eq. (12) turns into

$$\Lambda = C_3 \,. \tag{16}$$

For such Λ Eq.(15) takes the form

$$-k_r^2 D^2 / C_2 + C_3 = 0 \,. \tag{17}$$

3. Dispersion relation in the quasi-incompressible approximation

This approximation means that the sound propagator (10) is substituted by

$$\alpha_s^T \to -\frac{k_z^2 c_s^2 (\Gamma\omega + i\Delta_T)}{\omega^2 \Gamma (\omega + i\Delta_T)}, \tag{18}$$

Then, in accordance with (6)-(9), the parameter C_2 remains the same, while D, C_1, and C_3 reduce to

$$D = -D_0^2 / (k_z^2 v_A^2) , \tag{19}$$

$$C_1 = -\frac{D_0 \omega^2}{k_z^2 v_A^2} \frac{\omega + i\Delta_T}{\Gamma\omega + i\Delta_T} \frac{d\ln p_0}{dr} , \tag{20}$$

$$C_3 = -\frac{4\pi\rho_0 D_0^2}{k_z^2 v_A^2} \left(D_0 - \kappa^2 - \frac{4\Omega^2 k_z^2 v_A^2}{D_0} + \mu \right), \tag{21}$$

where

$$\mu = \frac{p_0'}{\rho_0} \left(\frac{\omega + i\Delta_T}{\Gamma\omega + i\Delta_T} \frac{d\ln p_0}{dr} - \frac{d\ln\rho_0}{dr} \right). \tag{22}$$

With (8), (19)-(21), Eqs. (13) and (14) yield

$$a = -\frac{4\pi\rho_0 D_0^2}{k_z^2 v_A^2} \left[D_0 - \kappa^2 - \mu - \frac{4\Omega^2 k_z^2 v_A^2}{D_0} + \right.$$

$$\left. + \frac{\omega^4}{D_0 k_z^2} \left(\frac{\omega + i\Delta_T}{\Gamma\omega + i\Delta_T} \right)^2 \left(\frac{d\ln p_0}{dr} \right)^2 \right], \tag{23}$$

$$b = \frac{4\pi\omega^2 D_0^2}{k_z^2 v_A^2} \frac{\omega + i\Delta_T}{\Gamma\omega + i\Delta_T} \frac{d}{d\ln r} \left(\frac{\rho_0}{r} \frac{d\ln p_0}{dr} \right). \tag{24}$$

Substituting (23) and (24) into (11), one finds

$$\Lambda = -4\pi\rho_0 D_0^2 (D_0 + \lambda) / (k_z^2 v_A^2) , \tag{25}$$

where

$$\lambda = -\kappa^2 + \mu - \frac{4\Omega^2 k_z^2 v_A^2}{D_0} +$$

$$+ \frac{\omega^4}{D_0 k_z^2} \left(\frac{\omega + i\Delta_T}{\Gamma\omega + i\Delta_T} \right)^2 \left(\frac{d\ln p_0}{dr} \right)^2 -$$

$$- \frac{\omega^2}{k_z^2 \rho_0} \frac{\omega + i\Delta_T}{\Gamma\omega + i\Delta_T} \frac{d}{d\ln r} \left(\frac{\rho_0}{r} \frac{d\ln p_0}{dr} \right). \tag{26}$$

Then, with (8) and (19), dispersion relation (15) reduces to

$$k^2 D_0 / k_z^2 + \lambda = 0 , \tag{27}$$

where $k^2 = k_r^2 + k_z^2$. This is the rigorous dispersion relation for axisymmetric modes in the quasi-incompressible approximation. In contrast, in [14], based on the Boussinesq approximation, the dispersion relation (27) with $\lambda \to \lambda^B$ was studied, where

$$\lambda^B = -\kappa^2 + \mu - 4\Omega^2 k_z^2 v_A^2 / D_0, \quad (28)$$

the superscript "B'" points to the author's name of [14]. In the explicit form such a dispersion relation is

$$D_0 + \frac{k_z^2}{k^2}\left[-\kappa^2 + \frac{p_0'}{\rho_0}\left(\frac{\omega + i\Delta_T}{\Gamma\omega + i\Delta_T}\frac{d\ln p_0}{dr} - \frac{d\ln\rho_0}{dr}\right) - \frac{4\Omega^2 k_z^2 v_A^2}{D_0}\right] = 0. \quad (29)$$

This is the heuristic dispersion relation for the axisymmetric modes in the Boussinesq approximation (the Balbus dispersion relation).

4. Analysis

4.1 The Balbus approximation

For the case of weak heat conductivity, $k_z v_A >> \Delta_T$, the Balbus dispersion relation (29) predicts an aperiodical instability for the condition [14]

$$k^2 v_A^2 + d\Omega^2 / d\ln r + N_{BV}^2 < 0, \quad (30)$$

where N_{BV}^2 is the squared Brunt-Vaisala (BV) frequency:

$$N_{BV}^2 = -p_0'\left[\ln\left(p_0/\rho_0^\Gamma\right)\right]'/(\Gamma\rho_0). \quad (31)$$

In the opposite case of strong heat conductivity, $k_z v_A << \Delta_T$, one has another aperiodical instability from (29), if the following condition is satisfied [14]

$$k^2 v_A^2 + d\Omega^2 / d\ln r + N_{BV}^{2\infty} < 0, \quad (32)$$

where

$$N_{BV}^{2\infty} = -p_0'\left[\ln\left(p_0/\rho_0\right)\right]'/(\rho_0). \quad (33)$$

One can see that Eq. (30) transits formally to Eq.(32) at $\Gamma \to 1$.

Now let us take $k_r^2 >> k_z^2$ assuming that both (30) and (32) are not satisfied. Then one has roots with $D_0 \to 0$. We assume that these roots can be separated into large and small ones.

In the case of large roots Eq. (29) yields $\omega = \omega_{1,2} + i\gamma_{1,2}$ (γ is the growth rate), where

$$\mathrm{Re}\,\omega_{1,2} \approx \pm|k_z|v_A, \quad (34)$$

$$\gamma_{1,2} = -\frac{k_z^2}{2k_r^2}\frac{p_0'^2}{p_0\rho_0\Gamma^2}\frac{(\Gamma-1)\Delta_T}{(k_z v_A)^2 + \Delta_T^2/\Gamma^2}. \quad (35)$$

Thus, these roots describe damped oscillations, $\gamma_{1,2} < 0$. Note that the roots (34) correspond to the slow magnetoacoustic waves with $\omega^2 = k_z^2 c_s^2 / \left(1 + c_s^2 / v_A^2\right)$ in the limit $c_s^2 / v_A^2 \to \infty$.

In the case of small roots Eq. (29) reduces to

$$D_0 = -4\Omega^2 k_z^2 v_A^2 \frac{\omega + i\Delta_T/\Gamma}{\omega\left(\kappa^2 + N_{BV}^{2\infty}\right) + i\left(\kappa^2 + N_{BV}^{2\infty}\right)\Delta_T/\Gamma}, \quad (36)$$

which corresponds to

$$\lambda^B = 0. \quad (37)$$

It hence follows that $\omega = \omega_{3,4} + i\gamma_{3,4}$, where

$$\mathrm{Re}\,\omega_{3,4} \approx \pm|k_z|v_A, \quad (38)$$

$$\gamma_{3,4} = -\frac{p_0'^2}{p_0\rho_0}\frac{2\Omega^2 k_z^2 v_A^2(\Gamma-1)\Delta_T}{\Gamma^2\left[k_z^2 v_A^2(\kappa^2 + N_{BV}^2)^2 + (\kappa^2 + N_{BV}^{2\infty})^2\Delta_T^2/\Gamma^2\right]}. \quad (39)$$

These roots correspond to the Alfven modes. According to (39), $\gamma_{3,4} < 0$, so that, as in the case of large roots, one again deals with damped oscillations.

One can see that $\gamma_{3,4}/\gamma_{1,2} << 1$ for $\Omega^2 >> k_r^2 v_A^2$. This is the condition of the above separation of the roots.

4.2 Using rigorous dispersion relation

Aperiodical instabilities predicted by Eq. (27) have the same properties as those considered in Subsec. 4.1. Therefore, we restrict ourselves to finding out the properties of the oscillation branches (34), (38).

4.2.1 Large roots (the magnetoacoustic waves)

Turning to (27), one finds, instead of (35),

$$\gamma_{1,2} = -\frac{k_z^2}{2k_r^2\rho_0\Gamma^2}$$

$$\left[\frac{p_0'^2}{p_0}-v_A^2\frac{d}{d\ln r}\left(\frac{\rho_0 p_0'}{rp_0}\right)\right]\frac{(\Gamma-1)\Delta_T}{(k_z v_A)^2+\Delta_T^2/\Gamma^2}. \quad (40)$$

It hence follows that, in contrast to the Balbus dispersion relation predicting the damped modes, the large roots can describe unstable modes. For instance, when the effects squared with respect to p_0' and ρ_0' are neglected, Eq. (40) reduces to

$$\gamma_{1,2}=-\frac{k_z^2 p_0' v_A^2}{2k_r^2 r\rho_0\Gamma^2}\frac{(\Gamma-1)\Delta_T}{(k_z v_A)^2+\Delta_T^2/\Gamma^2}. \quad (41)$$

Then $\gamma_{1,2}>0$ for $p_0'<0$, so that the modes are unstable. This is one of the varieties of the heat-conductivity-induced instabilities.

4.2.2 Small roots (the Alfven modes)

For small roots Eq. (27) reduces to

$$\lambda=0, \quad (42)$$

or

$$D_0=-4k_z^2 v_A^2\Omega^2\lambda_1/\lambda_2. \quad (43)$$

Here

$$\lambda_1=\omega+i\Delta_T/\Gamma+\frac{v_A^2}{4\Gamma^2\Omega^2}\left(\frac{d\ln p_0}{dr}\right)^2\frac{(\omega+i\Delta_T)^2}{\omega+i\gamma_T/\Gamma}, \quad (44)$$

$$\lambda_2=\omega g_2+ih_2\Delta_T/\Gamma, \quad (45)$$

$$g_2=\kappa^2+N_{BV}^2+\frac{v_A^2}{\Gamma\rho_0}\frac{d}{d\ln r}\left(\frac{\rho_0}{r}\frac{d\ln p_0}{dr}\right), \quad (46)$$

$$h_2=\kappa^2+N_{BV}^{2\infty}+\frac{v_A^2}{\rho_0}\frac{d}{d\ln r}\left(\frac{\rho_0}{r}\frac{d\ln p_0}{dr}\right). \quad (47)$$

Then one has, instead of (39),

$$\gamma_{3,4}=-\frac{2k_z^2 v_A^2\Omega^2\Delta_T(\Gamma-1)}{\Gamma^2[(g_2\omega)^2+(h_2\Delta_T/\Gamma)^2]^2}$$

$$\frac{p_0'^2}{p_0\rho_0}\left(1+\frac{\rho_0 v_A^2}{4p_0\Omega^2}\sigma_p\right) \quad (48)$$

where $\omega=\omega_{3,4}$ and

$$\sigma_p=\frac{1}{\omega\Delta_T(\Gamma-1)}\mathrm{Im}\left[\frac{(\omega+i\Delta_T)^2}{\omega+i\Delta_T/\Gamma}\left(g_2\omega-ih_2\Delta_T/\Gamma\right)\right]. \quad (49)$$

In the limiting case of small p_0' and ρ_0' one has $g_2=h_2=\kappa^2$. Then (49) transits to

$$\sigma_p=\frac{\kappa^2}{\omega\Delta_T(\Gamma-1)}\mathrm{Im}\left[\frac{(\omega+i\Delta_T)^2}{\omega+i\Delta_T/\Gamma}\left(\omega-\frac{i\Delta_T}{\Gamma}\right)\right]. \quad (50)$$

In the particular case $\omega>>\Delta_T$ it hence follows that

$$\sigma_p=2\kappa^2/\Gamma. \quad (51)$$

Then Eq. (48) reduces to

$$\gamma_{3,4}=-\frac{2k_z^2 v_A^2\Omega^2 p_0'^2}{p_0\rho_0\Gamma^2\omega^2\kappa^4}\left(1+\frac{v_A^2\kappa^2}{2c_s^2\Omega^2}\right). \quad (52)$$

One can see that the modes considered are unstable for sufficiently large negative $d\Omega^2/d\ln r$:

$$-d\Omega^2/d\ln r\geq c_s^2/v_A^2>>1. \quad (53)$$

This is another variety of heat-conductivity-induced instabilities.

5. Discussion

We have shown that the problem of axisymmetric instability in magnetized rotating cylindrical plasma with finite heat conductivity is described by the pair of first-order differential equations (4) and (5) for the FR variable and the perturbed displacement with the canonical parameters D, C_1, C_2 and C_3 given by (6)-(9). These equations lead to the second-order differential equation (11) for the displacement with Λ defined by (12) and *a* and *b* by (13) and (14). This mode equation yields the canonical local dispersion relation (15), which is a rigorous one, in contrast to the Balbus dispersion relation given by (17). We have explained that the rigorous dispersion relation turns into the heuristic if Λ is approximated by (16). This corresponds to the assumption $C_1=0$ which results in vanishing *a* and the term $-C_1^2/C_2$ in the expression for *a*, see Eqs. 13) and (14).

We restrict ourselves to analysis of only the quasi-incompressible modes in the high-β plasma. This corresponds to representation of the sound propagator α_s^T

in form (18), which is close to the Boussinesq approximation used in Ref. 14. The rigorous dispersion relation in the quasi-incompressible approximation is given by (27), while the Balbus dispersion relation by (29).

The heuristic dispersion relation predicts aperiodical instabilities for the conditions (30) and (32) representing the limits of weak and high heat conductivity. In addition, this dispersion relation shows that there are two pairs of damped waves with frequencies close to the Alfven ones, see Eqs. (34), (35), (38) and (39).

We have noted that the rigorous dispersion relation leads to the same results on the aperiodical instabilities as the Balbus one. At the same time, in contrast to the Balbus dispersion relation, it also predicts the possibility of dissipative excitation of slow magnetoacoustic and Alfven waves, thereby leading to heat-conductivity-induced instabilities. The growth rates of these instabilities are characterized by Eqs. 40), (41), (48) and (52). In the scope of the local axisymmetric modes, the conditions of these instabilities are rather strict, see Eqs. (41) and (53). This fact stimulates more general treatment of these instabilities allowing for nonaxisymmetry [4] and nonlocality [15].

We have restricted ourselves to the cylindrical geometry. An alternative of this equilibrium is that dependent on the *z* coordinate. The heat conductivity effects in the *z*-dependent equilibria were studied in a series of papers by Coppi and coauthors, see [16] and references therein. Then the thermo-rotational instability was shown. Further development of the theory of this phenomenon seems to be desirable.

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ნაშრომში შესწავლილია მბრუნავი ცილინდრული პლაზმა სასრული თბოგამტარობის შემთხვევაში. ანალიზი შემოიფარგლება ლოკალური აქსისიმეტრიული მოდებით, რომელთა პარალელური ფაზური სიჩქარე ნაკლებია ბგერის სიჩქარეზე პლაზმაში, როდესაც $\beta >> 1$, სადაც β არის პლაზმის წნევისა და მაგნიტური ველის წნევის ფარდობა. განხილულია მოდების შესწავლის ორი მიდგომა: ბუსინესკის მიახლოების შემდგომი ბალბუსის მიდგომა და რეგულარული ფრეიმან-როტენბერგის მეთოდი. ნაჩვენებია, რომ ლოკალური დისპერსიული თანაფარდობა, რომელიც მიღებული იქნა ჩვეულებრივი (მათემატიკურად მკაცრი) მიდგომით,

არის უფრო ზოგადი, ვიდრე ბალბუსის დისპერსიული თანაფარდობა. ასევე არის ნაჩვენები, რომ ბალბუსის დისპერსიული თანაფარდობა ვერ აღწერს თბოგამტარობით გამოწვეულ ზემოთ აღწერილ არამდგრადობას. ამგვარი არამდგრადობები შეიძლება გამოვლინდეს როგორც ასტროფიზიკურ, ასევე ლაბორატორიულ პლაზმაში.

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