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Mechanics

Impact of Two Moving Stamps on the Stressed State of an Elastic Half-Plane

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ABSTRACT. Stress-deformed state of an elastic orthotropic half-plane is studied for the case when two rigid stamps, moving with constant velocity, press the boundary. The frictional force between the stamp and the elastic body is ignored. *© 2007 Bull. Georg. Natl. Acad. Sci.*

Key words: *tensor components, moving stamp.*

Let us consider a problem when a piece-wise orthotropic body occupies a lower half-plane (y<0) and two stamps moving with constant velocity, press on it. We have to define the stressed state of the body. To do this we must determine the stress tensor components satisfying differential equations of motion [1], zero conditions

$$u(x,y,o)=v(x,y,o)=0,$$
$$\tau_{xy}(x,y,o)=\sigma_x(x,y,o)=\sigma_y(x,y,o)=0$$

and the following boundary conditions

$$\begin{cases}(\tau_{xy})\big|_{y=0}=0, & -\infty<x<\infty\\ \left(\dfrac{\partial v}{\partial x}\right)_{y=0}=f_k'(x)+const, & x\in L_1.\end{cases}\tag{1}$$

$$(\delta_y)_{y=0}=0,\ \ x\in L_2,\ k=1,2.$$

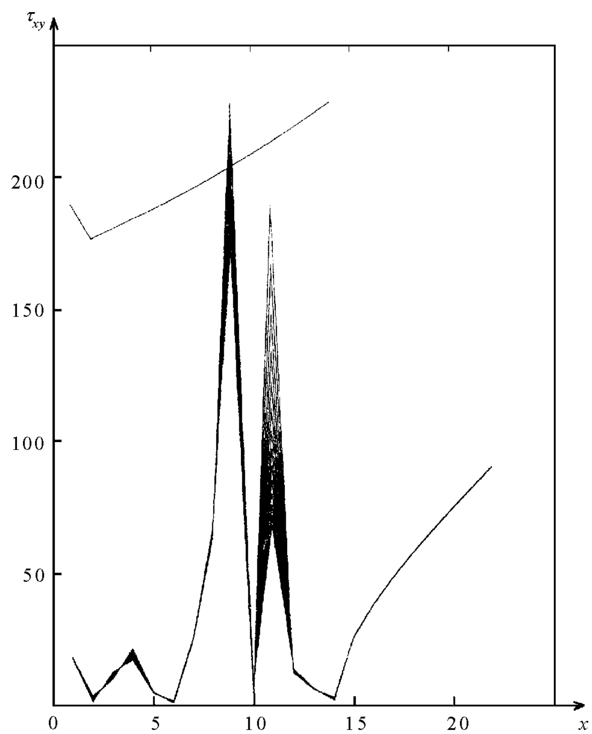
Here

$$L_1=[a_1,b_1]\cup[a_2,b_2],$$
$$L_2=(-\infty;a_1]\cup[b_1;a_2]\cup[b_2;\infty);$$
$$L=L_1+L_2,$$
$$f_k(x)=\begin{cases}f_1(x), & a_1\le x\le b_1,\\ f_2(x), & a_2\le x\le b_2.\end{cases}$$

where $f_1(x)$ and $f_2(x)$ are basis equations of the first and second stamps respectively.

Let us transform the variables $\xi=x-ct,\ \eta=y$, then the problem, set by us, will be reduced to the finding of the two analytic functions $F_1(x)$ and $F_2(x)$ on the lower half-plane [2], which on the boundary satisfy the conditions

$$\begin{cases} I_m[aF_1''(\xi)+bF_2''(\xi)]=0, & -\infty<\xi<+\infty, \\ 2n\,\mathrm{Re}[F_1''(\xi)+F_2''(\xi)]=0, & \xi\in L_2, \\ 2I_m[r_1F_1''(\xi)+\ell_1F_2''(\xi)]=f_k'(\xi), & \xi\in L_1,\ k=1,2. \end{cases} \quad (2)$$



Figure

If we assume that

$$aF_1''(\xi)+bF_2''(\xi)]=0,$$

then the first formula from (2) will be satisfied.

Thus, without loss of generality, we can write

$$F_2''(z)]=-\frac{aF_1''(z)}{b}. \quad (3)$$

If we consider the last formula in (2) and introduce a function $F(z) = i\,F_1''(z)$, which will be continuously prolonged on the upper half-plane in such a way that values of functions coincide on the unloaded part of the boundary, then for $F(z)$ we get the following problem. We have to find a piece-wise analytic function $F(z)$, which is continuously prolonged at all points of the boundary, possibly, except of the points a_1, a_2, b_1, b_2, but nearby it satisfies the condition [3]:

$$|F(z)|<\frac{const}{|z-c_j|^{\alpha}}; \quad const=\alpha<1,$$

and on the boundary it satisfies the condition

$$\begin{cases} \mathrm{Re}[F''(z)]_{y=0}=-\dfrac{bf_k'(\xi)}{2n(b-a)}, & \xi\in L_1,\ k=1,2, \\ I_m[F''(z)]_{y=0}=0, & \xi\in L_2. \end{cases} \quad (4)$$

Denote $F(z)\equiv V_1+iU_1$, if provided that

$$V_1=\mathrm{Re}\,F(z)=\frac{1}{2}\left[F(z)+\overline{F(z)}\right], \quad U_1=I_mF(z)=\frac{1}{2}\left[F(z)+\overline{F(z)}\right],$$

then conditions (4) give

$$\begin{cases} F^+(\xi)+F^-(\xi)=-\dfrac{1}{4}\dfrac{bf_k'(\xi)}{n(b-a)}, & \xi\in L_1, \\ F^+(\xi)+F^-(\xi)=0, & \xi\in L_1,\ k=1,2. \end{cases} \quad (5)$$

The solution of (5) will be written in the following form:

$$F(z)=-\frac{1}{8\pi i}\frac{b\mu g(z)}{n(b-a)}\int_{L_1}\frac{f_k'(\xi)d\xi}{g^+(\xi)(\xi-z)}+\frac{ip(z)}{g(z)}, \quad (6)$$

where

$$g(z)=\sqrt{(z-a_1)(z-b_1)(z-a_2)(z-b_2)}.$$

Note that under $g(z)$ we suppose that branch which is unique on the plane, cut along L_1, so that $g(z)\cdot z^{-2}\to 1$ when $z\to\infty$. In (6) formula $p(z)=c_0z+c,$ the constants c_0 and c_1 must be determined from the following condition [4]:

$$\int_{a_1}^{b_1} (\sigma_y)_{y=0}\, d\xi = p_1; \quad \int_{a_2}^{b_2} (\sigma_y)_{y=0}\, d\xi = p_2. \tag{7}$$

The analysis of the result obtained shows that boundary problems for an orthotropic body, similarly to an isotropic body, will be reduced to the determination of two analytic functions with mixed boundary conditions. The difference is in coefficients characterizing the anisotropy.

The Figure shows the tangential stress distribution diagrams in contact area for different intervals of time.

მექანიკა

ორი მოძრავი შტამპის გავლენა დრეკადი ნახევარსიბრტყის დაძაბულ მდგომარეობაზე

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REFERENCES

1. *R.D. Bantsuri* (1990), Proceedings of A.Razmadze Mathematical Institute, **100** (Russian)
2. *L.A. Galin* (1953), Kontaktnye zadachi teorii uprugosti. M. (Russian).
3. *N.I. Muskhelishvili* (1968), Singulyarnye integral'nye uravneniya. M. (Russian).
4. *S.G.Lekhnitskii* (1947), Anizotropnye plastinki. M. (Russian).

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