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Optimization of Delay Dynamic Systems with Mixed Initial Condition

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ABSTRACT. In this work delay optimal control problems with non-fixed initial moment and with a mixed initial condition are investigated. Necessary conditions of optimality are obtained. One of them, the essential novelty, is necessary condition of optimality for the initial moment, which contains the effect of mixed initial condition. The general results for linear time-optimal control problem are concretized.

Key words: *optimization, delay dynamic systems, necessary condition of optimality, mixed initial condition.*

1. Short Notices on the Development of the Delay Optimal Control Theory

Let two control systems be given and one of them be controlled by the influence at the time interval $[t_0, t_1]$, worked out by the second control system at the same time interval. With this, before the control influence affects the first control system, a definite time $\tau > 0$ elapses. The time τ is the so-called delay. A classical mathematical model corresponding to the above considered situation is the following system of control differential equations:

$$\begin{cases} L(p)x = y(t-\tau), \\ M(p)y = u(t). \end{cases}$$

Here $L(p)$ and $M(p)$ are given linear differential operators with constant coefficients; $u(t)$ is control function and $y(t)$ is control worked out by the second control system.

It is known that many real control objects can be described by such mathematical model. Optimal control problem for delay control differential equations was posed by R.Gamkrelidze. At first an analogue of Pontryagin's maximum principle for the optimal control problem with fixed initial moment

$$\dot{x}(t) = f(x(t), x(t-\tau), u(t)), t \in [t_0, t_1],$$

$$x(t) = \varphi(t), t \in [t_0 - \tau, t_0],$$

$$t_1 - t_0 \to \min,$$

was proved in [1].

The paper [1] became the basis for the development of the mathematical theory of optimal control with delayed arguments in many countries, including Georgia. Further development of this theory in Georgia is described in [2].

2. Problem Statement. Necessary Conditions of Optimality

2.1. Problem Statement. Let R_x^n be an n-dimensional vector space of points $x=(x^1,...,x^n)^T$, where T denotes transposition; the function $f(t,y_1,...,y_s,z_1,...,z_m,u)$ is continuous on $I\times O^s\times G^m\times V$ and continuously differentiable with respect to $y_i, i=\overline{1,s}$ and $z_j, j=\overline{1,m}$, where $I=[a,b]\subset R_t^1$ and $O\subset R_y^k, G\subset R_z^m$, $V\subset R_u^r$ are open sets, with this $n=k+m$ and $x=(y,z)^T$. Suppose also that scalar functions $\tau_i(t),\sigma_j(t),t\in R_t^1$ are absolutely continuous and satisfy the following conditions:

$$\tau_i(t)\le t,\dot{\tau}_i(t)>0,\sigma_j(t)\le t,\dot{\sigma}_j(t)>0.$$

Let Ω be a set of piecewise continuous control functions $u(t)\in U$ with discontinuity points of the first kind, where $U\subset V$ is an arbitrary set.

To each element $\mu=(t_0,t_1,u(\cdot))\in \mathrm{A}=I^2\times\Omega$ we assign the differential equation

$$\dot{x}(t)=(\dot{y}(t),\dot{z}(t))^T=f(t,y(\tau_1(t)),...,y(\tau_s(t)),z(\sigma_1(t)),...,z(\sigma_m(t)),u(t)) \tag{2.1}$$

with the initial condition

$$x(t)=(y(t),z(t))^T=(\varphi(t),g(t))^T,\ t\in[\hat{\tau},t_0),x(t_0)=(y_0,g(t_0))^T, \tag{2.2}$$

where $\varphi(t)\in O,g(t)\in G,t\in[\hat{\tau},b]$ are continuous initial functions and $\hat{\tau}=\min(\tau_1(a),..., \tau_s(a),\sigma_1(a),...,\sigma_m(a))$; $y_0\in O$ is a fixed point.

The condition (2.2) is said to be a mixed initial condition. It consists of two parts: the first part is $y(t)=\varphi(t),t\in[\hat{\tau},t_0),y(t_0)=y_0$, it is the so-called discontinuous part, because, in general, $y(t_0)\ne\varphi(t_0)$; the second part is $z(t)=g(t),t\in[\hat{\tau},t_0]$, it is the so-called continuous part because always $z(t_0)=g(t_0)$.

Definition 2.1. Let $\mu=(t_0,t_1,u(\cdot))\in\mathrm{A}$, where $t_0<t_1<b$. A function $x(t;\mu)=(y(t;\mu),z(t;\mu))^T$, $t\in[\tau,t_1]$, where $y(t;\mu)\in O,z(t;\mu)\in G$ is called a solution corresponding to the element μ, if it satisfies condition (2.2) on the interval $[t_0,t_1]$, is absolutely continuous and satisfies Eq. (2.1) almost everywhere.

Definition 2.2. An element $\mu=(t_0,t_1,u(\cdot))\in\mathrm{A}$ is said to be admissible if the corresponding solution $x(t)=x(t;\mu)$ satisfies the conditions

$$\begin{cases} q^i(t_0,t_1,x(t_1))\le 0, i=\overline{1,l_1},\\ q^i(t_0,t_1,x(t_1))=0, i=\overline{l_1+1,l}.\end{cases} \tag{2.3}$$

We denote the set of admissible elements by A_0.

Definition 2.3. An element $\mu_0=(t_{00},t_{10},u_0(\cdot))\in\mathrm{A}_0$, where $t_{00},t_{10}\in(a,b)$, is said to be optimal if

$$q^0(t_{00},t_{10},x_0(t_{10}))\le q^0(t_0,t_1,x(t_1)) \tag{2.4}$$

for any $\mu\in\mathrm{A}_0$, where the functions $q^i(t_0,t_1,x),i=\overline{0,l}$ are continuously differentiable with respect to $t_0,t_1\in I$ and $x=(y,z)^T\in\{(y,z)^T\in R_x^n:y\in O,z\in G\}$; $x_0(t)=(y_0(t),z_0(t))^T=x(t;\mu_0)$.

Here and in what follows, it is assumed that $u_0(t)=u_0(t_{00}+)$ for $t\le t_{00}$ and $u_0(t)=u(t_{10}-)$ for $t\ge t_{10}$. It is obvious that these suppositions do not affect the optimality of the element μ_0.

The problem (2.1)-(2.4) is called an optimal control problem with mixed initial condition. It consists in finding an optimal element μ_0.

2.2 Formulation of the main results. To state the main results we need the following notations:

$$f_{0i} = f(t_{00}, \underbrace{y_0,\dots,y_0}_{i}, \underbrace{\varphi(t_{00}),\dots,\varphi(t_{00})}_{p-i}, \varphi(\tau_{p+1}(t_{00})),\dots,\varphi(\tau_s(t_{00})), g(\sigma_1(t_{00})),\dots,g(\sigma_m(t_{00})), u_0(t_{00})), \quad i=\overline{0,p}.$$

Further $\gamma_i(t)$ is an inverse function to $\tau_i(t)$, $\gamma_i^0=\gamma_i(t_{00})$ and $\rho_j(t)$ is an inverse function to $\sigma_j(t)$;

$$f_{0i}(t) = f(\gamma_i^0, y_0(\tau_1(\gamma_i^0)),\dots,y_0(\tau_{i-1}(\gamma_i^0)), y_0, \varphi(\tau_{i+1}(\gamma_i^0)),\dots,\varphi(\tau_s(\gamma_i^0)), z_0(\sigma_1(\gamma_i^0)),\dots,$$

$$z_0(\sigma_m(\gamma_i^0)), u_0(t)), \quad f_{1i}(t) = f(\gamma_i^0, y_0(\tau_1(\gamma_i^0)),\dots,y_0(\tau_{i-1}(\gamma_i^0)), \varphi(t_{00}), \varphi(\tau_{i+1}(\gamma_i^0)),\dots,\varphi(\tau_s(\gamma_i^0)),$$

$$\dots, z_0(\sigma_1(\gamma_i^0)),\dots,z_0(\sigma_m(\gamma_i^0)), u_0(t)), i=\overline{p+1,s}.$$

Theorem 2.1. *Let* $\mu_0=(t_{00},t_{10},u_0(\cdot))$ *be an optimal element and* $x_0(t)=(y_0(t),z_0(t))^T$ *be corresponding solution and the following conditions hold:*

2.1. $\gamma_i^0 = t_{00}, i=\overline{1,p}$ *and* $\gamma_{p+1}^0<\dots<\gamma_s^0<t_{10}$;

2.2. *There exist finite limits* $\dot\gamma_i^- = \dot\gamma_i(t_{00}-), i=\overline{1,s}$ *and a number* $\delta>0$ *such that*

$$\gamma_1(t)\le\dots\le\gamma_p(t), t\in(t_{00}-\delta,t_{00});$$

2.3. *The function* $g(t), t\in[t_{00}-\delta,t_{00}]$ *is absolutely continuous and there exists a finite limit* $\dot g^- = \dot g(t_{00}-)$.

Then, there exists a vector $\pi=(\pi_0,\dots,\pi_l)\neq 0$, *where* $\pi_0\le 0$, *and a solution* $\Psi(t)=(\psi(t),\chi(t))=(\psi_1(t),\dots,\psi_k(t),\chi_1(t),\dots,\chi_m(t))$ *of the equation*

$$\begin{cases} \dot\psi(t) = -\sum_{i=1}^{k}\Psi(\gamma_i(t))f_{0y_i}[\gamma_i(t)]\dot\gamma_i(t), \\ \dot\chi(t) = -\sum_{j=1}^{m}\Psi(\rho_j(t))f_{0z_j}[\rho_j(t)]\dot\rho_j(t), t\in[t_{00},t_{10}], \\ \Psi(t)=0, t>t_{10}, \end{cases} \tag{2.5}$$

such that the conditions listed below hold:

a) the maximum principle for the control $u_0(t)$, *that is,*

$$\Psi(t)f_0[t] = \max_{u\in U}\Psi(t)f(t,y_0(\tau_1(t)),\dots,y_0(\tau_s(t)),z_0(\sigma_1(t)),\dots,z_0(\sigma_m(t)),u)), t\in[t_{00},t_{10}];$$

b) the condition for the function $\Psi(t)$ *and vector* π: $\Psi(t_{00})=\pi Q_{0x}, \pi_i q^i(t_{00},t_{10},x_0(t_{10}))=0, i=\overline{1,l_1}$;

c) the condition for the final moment t_{10}: $\pi Q_{0t_1} = -\Psi(t_{10})f_0[t_{10}]$;

d) the condition for the initial moment t_{00}:

$$\pi Q_{0t_0} \ge \Psi(t_{00})f_{00} - \Psi(t_{00})[\dot\gamma_1^- f_{00} + \sum_{i=1}^{p-1}(\dot\gamma_{i+1}^- - \dot\gamma_i^-)f_{0i} - \dot\gamma_p^- f_{0p}] +$$

$$\sum_{i=p+1}^{s}\Psi(\gamma_i^0)[f_{0i}(\gamma_i^0-) - f_{1i}(\gamma_{0i}^0-)]\dot\gamma_i^- - \chi(t_{00})\dot g^-.$$

Here $f_0[t]=f(t,y_0(\tau_1(t)),\dots,y_0(\tau_s(t)),z_0(\sigma_1(t)),\dots,z_0(\sigma_m(t)),u_0(t))$,

$$Q=(q^0,\dots,q^l)^T, \; Q_{0x} = \frac{\partial}{\partial x}Q(t_{00},t_{10},x_0(t_{10})).$$

Some comments: the expression

$$-\Psi(t_{00})[\dot{\gamma}_1^- f_{00} + \sum_{i=1}^{p-1}(\dot{\gamma}_{i+1}^- - \dot{\gamma}_i^-)f_{0i} - \dot{\gamma}_p^- f_{0p}] + \sum_{i=p+1}^{s}\Psi(\gamma_i^0)[f_{0i}(\gamma_i^0-) - f_{1i}(\gamma_i^0-)]\dot{\gamma}_i^- - \chi(t_{00})\dot{g}^-$$

is the effect of the mixed initial condition; the addend

$$-\Psi(t_{00})[\dot{\gamma}_1^- f_{00} + \sum_{i=1}^{p-1}(\dot{\gamma}_{i+1}^- - \dot{\gamma}_i^-)f_{0i} - \dot{\gamma}_p^- f_{0p}] + \sum_{i=p+1}^{s}\Psi(\gamma_i^0)[f_{0i}(\gamma_i^0-) - f_{1i}(\gamma_i^0-)]\dot{\gamma}_i^-$$

is the effect of the discontinuous part of the mixed initial condition; the term $\chi(t_{00})\dot{g}^-$ is the effect of the continuous part of the mixed initial condition; if $\varphi(t_{00}) = y_0$ then $f_{00} = \ldots = f_{0p}$ and $f_{0i}(\gamma_i^0-) = f_{1i}(\gamma_i^0-), i = \overline{p+1,s}$ because the effect of discontinuity is equal to zero; if the conditions $\dot{\gamma}_p^- < \ldots < \dot{\gamma}_1^-$ are fulfilled, then the second part of condition 2.2) is superfluous; Theorem 2.1 corresponds to the case when at the point t_{00} left-sided variation takes place. It is proved by the method given in [3,4].

The theorem proposed below corresponds to the case when at the moment t_{00} right-hand variation takes place.

Theorem 2.2. *Let $\mu_0 = (t_{00}, t_{10}, u_0(\cdot))$ be an optimal element and condition 2.1) of the Theorem 2.1 and the following conditions hold:*

2.3. *There exist finite limits $\dot{\gamma}_i^+ = \dot{\gamma}_i(t_{00}+), i = \overline{1,s}$ and a number $\delta > 0$ such that*

$$\gamma_1(t) \le \ldots \le \gamma_p(t), t \in (t_{00}, t_{00}+\delta);$$

2.4. *The function $g(t), t \in [t_{00}, t_{00}+\delta]$ is absolutely continuous and there exists the finite limit $\dot{g}^+ = \dot{g}(t_{00}+)$.*

Then there exist a vector $\pi = (\pi_0, \ldots, \pi_l) \ne 0$, where $\pi_0 \le 0$, and a solution $\Psi(t) = (\psi(t), \chi(t))$ of Eq. (2.5) such that the conditions a), b), c) hold and for the optimal initial moment t_{00} the following inequality is fulfilled:

$$\pi Q_{0t_0} \le \Psi(t_{00})f_{00} - \Psi(t_{00})[\dot{\gamma}_1^+ f_{00} + \sum_{i=1}^{p-1}(\dot{\gamma}_{i+1}^+ - \dot{\gamma}_i^+)f_{0i} - \dot{\gamma}_p^+ f_{0p}] +$$

$$\sum_{i=p+1}^{s}\Psi(\gamma_i^0)[f_{0i}(\gamma_i^0+) - f_{1i}(\gamma_i^0+)]\dot{\gamma}_i^+ - \chi(t_{00})\dot{g}^+ .$$

Theorem 2.3. *Let $\mu_0 = (t_{00}, t_{10}, u_0(\cdot))$ be an optimal element and conditions of the theorems 2.1 and 2.2 hold and the function $g(t)$ is continuously differentiable in a neighborhood of the moment t_{00}. Moreover*

$$\dot{\gamma}_1^- f_{00} + \sum_{i=1}^{p-1}(\dot{\gamma}_{i+1}^- - \dot{\gamma}_i^-)f_{0i} - \dot{\gamma}_p^- f = \dot{\gamma}_1^+ f_{00} + \sum_{i=1}^{p-1}(\dot{\gamma}_{i+1}^+ - \dot{\gamma}_i^+)f_{0i} - \dot{\gamma}_p^+ f_{0p} = \hat{f}_0 ;$$

$$[f_{0i}(\gamma_i^0-) - f_{1i}(\gamma_i^0-)]\dot{\gamma}_i^- = [f_{0i}(\gamma_i^0+) - f_{1i}(\gamma_i^0+)]\dot{\gamma}_i^+ = \hat{f}_i, \quad i = \overline{p+1,s}.$$

Then, there exist a vector $\pi = (\pi_0, \ldots, \pi_l) \ne 0$, where $\pi_0 \le 0$, and a solution $\Psi(t) = (\psi(t), \chi(t))$ of Eq. (2.5), *such that the conditions a), b), c) hold and for the optimal initial moment t_{00} the following equality is fulfilled*

$$\pi Q_{0t_0} = \Psi(t_{00})[f_{00} - \hat{f}_0] + \sum_{i=p+1}^{s}\Psi(\gamma_i^0)\hat{f}_i - \chi(t_{00})\dot{g}(t_{00}).$$

This theorem corresponds to the case when at the point t_{00} doubly-sided variation takes place.

Remark. If a rank of the matrix $(Q_{0t_0}, Q_{0t_1}, Q_{0x})$ is equal to $1+l$, then the solution $\Psi(t)$ in Theorem 2.3 is nontrivial.

3. Linear Time-Optimal Control Problem

Let us consider the optimal control problem

$$\dot{x}(t) = A_1 y(t) + B_1 y(t-\tau) + A_2 z(t-\sigma) + Cu(t), t \in [t_0, t_1] \in I, u(\cdot) \in \Omega,$$

$$x(t) = (y(t), z(t))^T = (\varphi(t), g(t))^T, t \in [a-\tau_0, t_0), \ x(t_0) = (y_0, g(t_0))^T, x(t_1) = x_1$$

$$t_1 - t_0 \to \min,$$

where $\tau > 0, \sigma > 0$ are given numbers, $\tau_0 = \max(\tau, \sigma)$ and A_1, A_2, B_1, B_2, C are constant matrices with appropriate dimensions

Theorem 3.1. *Let* $\mu_0 = (t_{00}, t_{10}, u_0(\cdot))$ *be an optimal element and* $t_{00} + \tau < t_{10}$*; the function* $g(t)$ *is continuously differentiable in a neighborhood of the moment* t_{00}*. Then there exists a nontrivial solution* $\Psi(t) = (\psi(t), \chi(t))$ *of the equation*

$$\begin{cases} \dot{\psi}(t) = -\Psi(t)A_1 - \Psi(t+\tau)B_1, \\ \dot{\chi}(t) = -\Psi(t)A_2 - \Psi(t+\sigma)B_2, t \in [t_{00}, t_{10}], \\ \Psi(t) = 0, t > t_{00} \end{cases}$$

such that the conditions listed below hold:

3.1. *The maximum principle for the control* $u_0(t)$*, that is,*

$$\Psi(t)Cu_0(t) = \max_{u \in U} \Psi(t)Cu, \ t \in [t_{00}, t_{10}];$$

3.2. *The condition for the final moment* t_{10}

$$\Psi(t_{10})[A_1 y_0(t_{10}) + B_1 y_0(t_{10} - \tau) + A_2 z_0(t_{10}) + B_2 z_0(t_{10} - \sigma) + Cu_0(t_{10})] \geq 0;$$

3.3. *The condition for the initial moment* t_{00}

$$\Psi(t_{00})[A_1 y_0 + B_1 \varphi(t_{00} - \tau) + A_2 g(t_{00}) + B_2 g(t_{00} - \sigma) + Cu_0(t_{00})] +$$
$$\Psi(t_{00} + \tau)B_1[y_0 - \varphi(t_{00})] - \chi(t_{00})\dot{g}(t_{00}) = 0.$$

Theorem 3.1 follows from Theorem 2.3.

Now we consider a linear optimal control problem with a discontinuous initial condition

$$\dot{x}(t) = Ax(t) + Bx(t-\tau) + Cu(t), t \in [t_0, t_1] \subset I, u(\cdot) \in \Omega,$$

$$x(t) = v(t), t \in [t_0 - \tau, t_0), x(t_0) = x_0, x(t_1) = x_1,$$

$$t_1 - t_0 \to \min,$$

where A, B are constant matrices; $v(t), t \in [a-\tau, b]$ is a continuous function; x_0, x_1 are fixed points.

This problem is a particular case of the above provided problem.

Theorem 3.2. *Let* $\mu_0 = (t_{00}, t_{10}, u_0(\cdot))$ *be an optimal element and* $t_{00} + \tau < t_{10}$*. Then there exists a nontrivial solution* $\psi(t)$ *to the equation*

$$\begin{cases} \dot{\psi}(t) = -\psi(t)A - \psi(t+\tau)B, t \in [t_{00}, t_{10}], \\ \psi(t) = 0, t > t_{10} \end{cases}$$

such that the conditions listed below hold:

3.4. *The maximum principle for the control* $u_0(t)$

$$\psi(t)Cu_0(t) = \max_{u \in U} \psi(t)Cu, \ t \in [t_{00}, t_{10}];$$

3.5. *The condition for the final moment* t_{10}

$$\psi(t_{10})[Ax_0(t_{10}) + Bx_0(t_{10} - \tau) + Cu_0(t_{10})] \geq 0;$$

3.6. *The condition for the initial moment* t_{00}

$$\psi(t_{00})[Ax_0 + Bv(t_{00}-\tau) + Cu_0(t_{00})] + \psi(t_{00}+\tau)[x_0 - v(t_{00})] = 0.$$

Example. In the space R_x^2 we consider an optimal control problem with discontinuous initial condition

$$\begin{cases} \dot{x}^1(t) = x^2(t-1) - 1, t \in [t_0, t_1] \in I = [0,5] \\ \dot{x}^2(t) = u(t) \in [-1,1], \end{cases}$$

$$x^1(t_0) = 0, x^2(t) = -t, t \in [t_0 - 1, t_0), x^2(t_0) = 0, x^1(t_1) = 1,\ x^2(t_1) = 4.$$

$$t_1 - t_0 \to \min.$$

Let $(t_{00}, t_{10}, u_0(\cdot))$ be an optimal element. For an arbitrary $t_0 \geq 0$ on the interval $(t_0, t_0+1]$ we have

$$x^1(t) = -\frac{t^2}{2} + \frac{t_0^2}{2}.$$

It is clearly seen that $x^1(t) \neq 1,\ t \in (t_0, t_0+1]$. Thus $t_{00} + 1 < t_{10}$. The assumption of theorem 4.2 holds.

According to theorem 4.2 there exists a nontrivial solution $\psi(t) = (\psi_1(t), \psi_2(t))$ of the equation

$$\begin{cases} \dot{\psi}_1(t) = 0, \\ \dot{\psi}_2(t) = -\psi_1(t+1), t \in [t_{00}, t_{10}], \\ \psi_1(t) = 0, t > t_{00} \end{cases} \tag{3.1}$$

such that the following conditions are fulfilled:

- the maximum principle $u_0(t) = sign\psi_2(t)$; the condition for the optimal initial moment t_{00}

$$-t_{00}\psi_1(t_{00}) + \psi_2(t_{00})u_0(t_{00}) + t_{00}\psi_1(t_{00}+1) = 0, (u_0(t_{00}) = u_0(t_{00}+)); \tag{3.2}$$

- the condition for the optimal final moment t_{10}

$$\psi_1(t_{10})[x_0^2(t_{10}-1) - 1] + \psi_2(t_{10})u_0(t_{10}) \geq 0.$$

The solution of the equation (3.1) is

$$\psi_1(t) = \begin{cases} \alpha, t \in [t_{00}, t_{10}], \\ 0,\ t > t_{00}, \end{cases} \tag{3.3}$$

$$\psi_2(t) = \begin{cases} \alpha(t_{00} - t - 1) + \beta, t \in [t_{00}, t_{10} - 1], \\ \beta,\ t \in [t_{10} - 1, t_{10}], \\ 0, t > t_{10}, \end{cases}$$

where α, β are arbitrary numbers .

From (3.2), taking into account $\psi(t_{00}) = \psi(t_{00}+1)$ (see (3.3)), we get $\psi(t_{00})u_0(t_{00}) = 0$. There exists a number $\varepsilon > 0$ such that $\psi_2(t) \neq 0$ for $t \in (t_{00}, t_{00} + \varepsilon)$ because we have either $u_0(t_{00}) = 1$ or $u_0(t_{00}) = -1$. Thus $\psi_2(t_{00}) = 0$. Consequently the optimal control $u_0(t), t \in [t_{00}, t_{10}]$ is constant and it is equal either to 1 or to -1.

Elementary computation shows that the optimal control is $u_0(t) = 1$, optimal moments are $t_{00} = 0, t_{10} = 4$ and the optimal trajectory is

$$x_0^1(t) = \begin{cases} -\frac{t^2}{2},\ t \in [0,1), \\ \frac{t^2}{2} - 2t + 1, [1,4], \end{cases}$$

$$x_0^2(t) = \begin{cases} -t, & t \in [-1,0) \\ t, & t \in [0,4]. \end{cases}$$

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ნაშრომში გამოკვლეულია დაგვიანებულარგუმენტიანი ოპტიმალური მართვის ამოცანები არაფიქსირებული საწყისი მომენტით და შერეული საწყისი პირობით. მიღებულია ოპტიმალურობის აუცილებელი პირობები, მათ შორის არსებითი სიახლეა საწყისი მომენტის ოპტიმალურობის აუცილებელი პირობა, რომელიც შეიცავს შერეული საწყისი პირობის ეფექტს. ზოგადი შედეგები დაკონკრეტებულია ოპტიმალური მართვის, სწრაფქმედების აზრით, წრფივი ამოცანისთვის.

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