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Phase Synchronization of Stick-Slip Process by Periodical Mechanical Forcing

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ABSTRACT. In the present study the character of stick-slip process in laboratory spring-slider system under weak external periodical (tangential and normal) mechanical forcing has been investigated. We report the experimental evidence of phase synchronization in a slip dynamics, induced by the external periodic mechanical impact. In the spring-slider system at certain conditions we have a stick-slip effect. In our experiments the slip events are distinguished by acoustic emission bursts, which are generated by slider displacement. In addition to the drag the weak variable mechanical forcing was superimposed either tangential or normal to the slip plane. With increasing the external forcing one can see increasing phase synchronization of the first arrivals (onsets) of stick-slip generated acoustic pulses. The grouping of the onsets in a certain phase of the external periodic forcing is considered as a hallmark of phase synchronization. The onsets of stick-slip pulses in the case of normal mechanical forcing are shifted relatively to onsets in case of tangential forcing.

Key words: *stick-slip, phase synchronization, periodical mechanical forcing.*

1. Introduction

The additional mechanical or electromagnetic forcing, which can be much smaller than the main driving force, may provoke triggering and synchronization during stick-slip process, which means that these phenomena are connected with nonlinear interaction of objects, namely with initiation of instability in systems that are close to the critical state [1-4]. Earlier similar studies considered mainly the effect of forcing on the friction coefficient [5].

Understanding of triggering and synchronization effects can be obtained in controllable experiments. We carried out laboratory experiments on the spring-slider system with periodic mechanical forcing, which is weak in comparison with the main dragging force of the spring. In the previous paper the effect on stick-slip dynamics of weak mechanical forcing applied tangentially and normal to the slip plane was studied [6, 7].

2. Experimental Setup

We investigated (mechanical) triggering and synchronization of instabilities in an experimental spring-slider system by recording acoustic emission, accompanying the slip events [1, 2, 8, 9].

Experimental set up represents a system of two horizontally oriented plates of roughly finished basalt. A constant dragging force of order of 4N was applied to the upper (sliding) plate; in addition, the system was subjected to periodic mechanical perturbations of various amplitudes. The mechanical forcing was much weaker compared to the driving force. The mechanical forcing was realized by the vibrator "CB-5" for normal directed forcing and by "CB-20" for tangential directed forcing. Slip events were recorded as acoustic emission (AE) bursts. The intensity of mechanical vibration was regulated by the voltage applied to the vibrator.

3. Results

The experiments were conducted for two modes of forcing: I. when the forcing load is applied normal to the slip surface and II. when the forcing load is applied parallel to the slip surface; for brevity we will refer to them as normal forcing and tangential forcing.

In the case of normally directed forcing we calculated the maximum value of mechanical forcing, which corresponds to the maximum measured voltage applied to mechanical vibrator (i.e. when the voltage applied to the vibrator equals 6.5 V). The mass of the oscillating element of the vibrator m is ≈20g, so we obtain for the natural frequency f of the oscillating element of the vibrator, $f = \sqrt{\frac{k}{m}} = 5\,\text{Hz}$ where k is the stiffness of the vibrator spring. From this expression we obtain, $k = 25m = 0.5$ N/m.

The maximum deflection x_{max} of the oscillating element at the applied voltage 6.5 V equals $\approx 4\cdot10^{-3}$m, so the corresponding (maximal) intensity of forcing F_{max} is:

$$F_{\max} = kx_{\max} \approx 2.10^{-3}\,N. \qquad (1)$$

As the forcing is periodic, its current value F is: $F = F_{\max}\cos(2\pi\varpi t)$. The maximum rate of vibrator force change is:

$$\left(\frac{dF}{dt}\right)_{\max} = 2\pi\,\varpi\, f_{\max} \approx 0{,}25 N/s. \qquad (2)$$

The friction (dragging) force is $F_{fr} = Kl$, where K is the dragging spring stiffness and l is the spring elongation. The rate of dragging shear force change in our experiments is $\frac{dF_{fr}}{dt} = (Kl)' = Kv$, where v is the dragging velocity. In our experiment $v \approx 0.9 mm/s$, $K \approx 140 N/m$, i.e. $\frac{dF_{fr}}{dt} \approx 0.12 N/s$.

The ratio of rate of the periodic (vibrator) force to the dragging force rate is:

$$\frac{dF_{fr}/dt}{(dF/dt)_{\max}} \approx 0.5. \qquad (3)$$

The forcing rate is larger than the dragging rate, so synchronization is possible [10].

In the case of tangentially directed forcing we calculated the ratio of the rate of periodic (vibrator) force to the dragging force rate:

$$\frac{(dF/dt)_{\max}}{dF_{fr}/dt} \approx 0.13. \qquad (4)$$

This means that in this case the synchronization condition is also achieved.

From the presented results for the normal forcing (Fig. 1) it is evident that synchronization of various strength is observed for the range of forcing voltages from 1 to 6 V. Results for the normal forcing at 0.1, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0 and 6.5V electrical voltage, applied to vibrator, are presented for several cases – Fig.1. The above calculation shows that for 6.5 V the forcing is $\sim 2\cdot10^{-3}N$ and our assessment of F for 1V is $\sim 5\cdot10^{-4}N$. For the tangential directed forcing, for 6 V the forcing F is $\sim 4\cdot10^{-3}N$ and for 1V is $\sim 5\cdot10^{-4}N$.

In Fig. 1 are presented distributions of the onsets relative to the phase (in decimals) of the forcing period

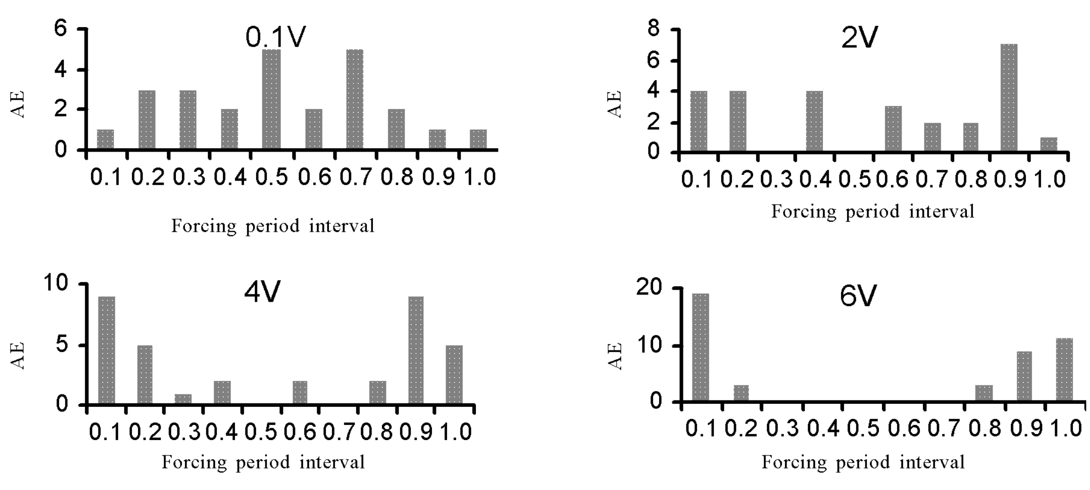

Fig. 1. Distribution of acoustic emission onsets relative to forcing period phases (in decimals) for different intensities of normal forcing

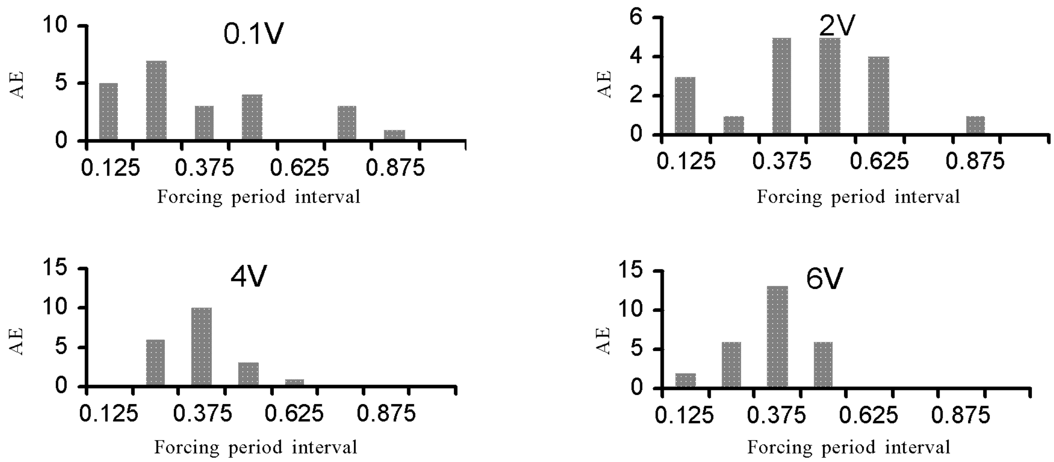

Fig. 2. Distribution of acoustic emission onsets number relative to the forcing period phase (in decimals) for different intensities of tangential forcing

for the normal forcing. At low voltages (up to 1V) the onsets are more or less randomly distributed in the decimals of the forcing period. Voltage increase results in concentration of the onsets at a definite part of forcing period, namely in the first and the last decimals of forcing phase.

In Fig. 2 distributions of the onsets relative to the phase of the forcing period for the tangential forcing are presented. Results for the case of tangential forcing at 0.1, 0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, and 6.0V electrical voltages, applied to vibrator, are presented for four voltages Fig. 2. Again, the increase of forcing leads to phase synchronization, which in this case is most pronounced in the interval of 0.25-0.5 decimals of the forcing period and fills the gap observed for normal forcing (Fig.1).

The interval from 0.6 to 0.8 decimals of the forcing period seems a prohibited zone for both kinds of forcing.

In Fig.3a,b the standard deviation of AE offsets distribution relative to the normal (a) and tangential (b) forcing phase versus forcing voltage is presented. It is evident that with increasing the forcing voltage the standard deviation decreases significantly in both cases, i.e. concentration of the onset times at a definite phase of external forcing period takes place. This phase segment corresponds to the rising part of the external forcing period amplitude for the tangential forcing and to the minimum area of the external forcing sinusoid for the normal forcing.

It is interesting that the dependence of synchronization effectiveness on the intensity of forcing in both forcing modes is almost the same.

4. Theoretical Background

There are several mathematical models related to the stick-slip process. For example, we can, using the rate- and state-friction law, write down the equation of motion for non modified stick-slip [11]:

$$\sigma_n(\mu_0 + A\ln\dot{\delta} + B\ln\theta_0 - \frac{B}{D_c}\delta) = -k\delta + k\delta_0, \quad (5)$$

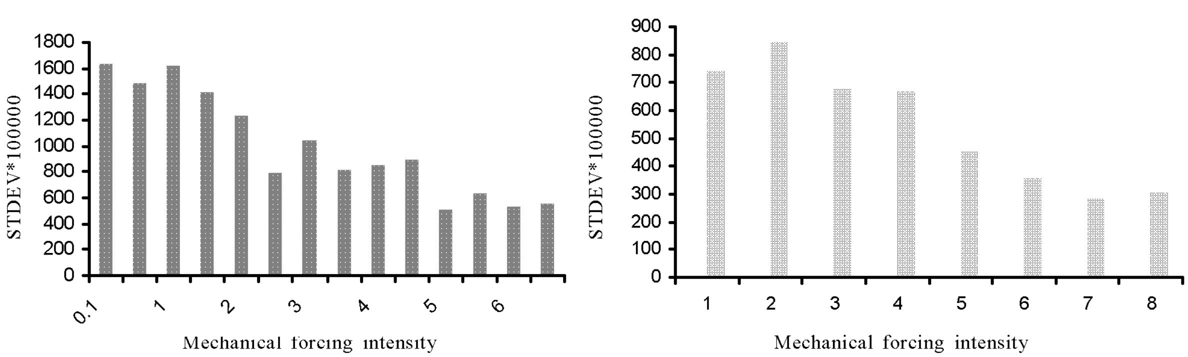

Fig. 3. The standard deviation of offsets distribution for different intensities (applied voltages) of external forcing: a) normal forcing, b) tangential forcing

where $\dot{\delta}$ is the sliding speed, θ is a state variable that accounts for the history of sliding and μ_0, A and B are constants. In order to account for periodic (normal) forcing we introduce the additional term :

$$\sigma_n(1+\varepsilon\cos\varpi t)\left(\mu_0 + A\ln\dot{\delta} + B\ln\theta_0 + \frac{B}{D_c}\delta\right) = \\ = -k\delta + k\delta_0. \quad (6)$$

Another approach to the analysis of experimental data is to use the Van der Pol equation:

$$\ddot{x} + \lambda(x^2-1)\dot{x} + x = 0, \quad (7)$$

where λ is a scalar parameter indicating the strength of the nonlinear damping; for $\lambda>>1$ equation (7) describes the accumulate and fire (relaxation) type of oscillations. Stick-slip process also belongs to the relaxation oscillators' class [12, 13]. From (7) we can calculate the relaxation period as,

$$T = \lambda[3 - 2\ln 3] \approx 1.6\lambda. \quad (8)$$

In our experiments the period of unmodified (natural) stick-slip is $T \approx 7s$, i.e. $\lambda \approx 4.35$; this agrees with the conditions for the relaxation process. Using equation (7) we can describe the natural stick-slip process. To model the forced relaxation stick-slip process we can use a forced Van der Pol equation. In our case the forcing period is much less than the natural stick-slip period.

As it is evident from Fig. 4 the increase of the forcing intensity from 0.1 V to 6.5 V leads to decrease of the pulse duration. The duration of pulse can be related to the radiated energy. Decreasing of stick-slip pulse recurrence period can be explained as a result of decreasing of friction coefficient imposed by normal mechanical periodical forcing. According to Bureau *et al.* [14], due to the normal periodical mechanical forcing the coefficient of the friction changes:

$$\Delta\bar{\mu} = -\frac{\bar{\mu}(\bar{\mu}+2A)}{4A}\varepsilon^2, \quad (9)$$

where $\bar{\mu} = \mu_0 - (B-A)\ln(V/V_0)$ and ε corresponds to F_{max} (2). From (9) it is evident that mechanical periodical normal forcing decreases the coefficient of friction and this leads to decreasing of stick-slip period. When μ decreases, λ also decreases (from the Van der Pol equation) and according to (8) the period of stick-slip $T \approx 1.6\,\lambda$ also decreases.

We can consider our experiment as the interaction of two oscillators: one of them is the natural stick-slip (slow process) and the other is forcing (fast process). In the simplest case of two periodic oscillators, synchronization is classically understood as a phase locking and for two coupled oscillators with nearly equal natural frequencies it implies that the phase difference remains constant ($\Delta\varphi$ = const). In some cases, even in the synchronous regime, $\Delta\varphi$ is not strictly constant, but fluctuates, although these fluctuations are bounded.

Synchronization may also be achieved in a more complicated way, when the frequencies of oscillators differ considerably. Obviously, there is no common frequency in this case, but the relation between the driving and forcing frequencies is a rational number called the winding or rotation number [3], defined as:

$$\rho = \frac{\Omega}{\varpi}, \quad (10)$$

In the case of a periodically kicked oscillator, this number is nothing other than the ratio between the observed frequency of oscillations of the system Ω and the frequency of the external forcing ϖ. This condition of the so-called high-order synchronization can be incorporated in the general framework of the frequency-locking as:

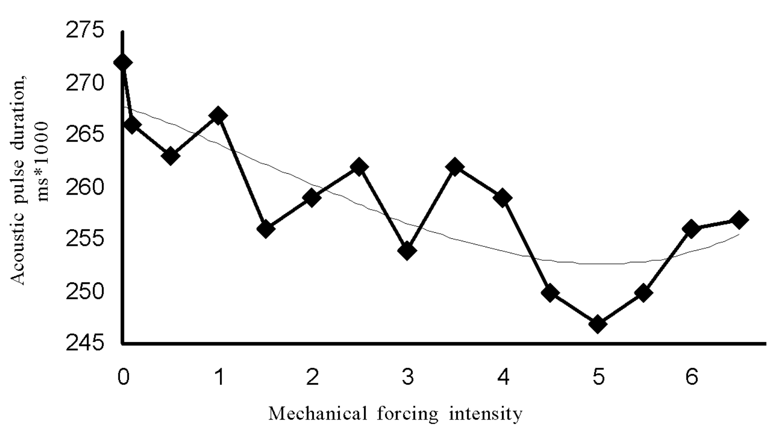

Fig. 4. Mean duration of stick-slip generated acoustic pulses for different intensity of normal forcing, with a trend line

$$n\varpi = m\Omega, \qquad (11)$$

In this case $n{:}m$ is the winding number: $\rho = n/m$. Phase-locking in this case can be expressed in terms of the oscillators' phases:

$$\left| n\varphi_f - m\varphi \right| < const, \qquad (12)$$

where φ_f is the phase of the force and φ is that of the kicked oscillator.

At normal forcing at applied voltage 0.1 V the ratio $n{:}m$ = 140:1, and when voltage is 6 V the ratio decreases to $n{:}m$ = 80:1. At tangential forcing the ratio $n{:}m$ = 210:1.

This means that the system needs to accumulate the energy of nearly 200 cycles of forcing in order to fire, i.e to give birth to a single event.

5. Conclusions

Experiments on the standard spring-slider system (fixed and sliding basalt samples), subjected to a constant pull and weak mechanical (normal and tangential) periodic force superimposed on it, in dry environment show that, at definite conditions, the system manifests the effect of phase synchronization of microslip events with weak periodic excitation.

The regular phase shift is observed between forcing sinusoid and the onsets of acoustic pulses; at the forcing normal to the slip plane, the initiation of motion is concentrated in the minimum area of the forcing sinusoid, which differs from the case of forcing parallel to the slip plane, when the concentration of AE pulses takes place on the rising section of forcing sinusoid.

In our experiments the high order synchronization was observed; the winding number, i.e., the ratio of forcing frequency to observed stick-slip average recurrence frequency is high. At weak forcing the ratio is $n{:}m$ = 140:1 for normal forcing and $n{:}m$ = 220:1 for tangential forcing. At relatively strong forcing the ratio is $n{:}m$ = 80:1 in the case of normal forcing and almost does not change for tangential forcing.

It was found also that an increase of forcing intensity causes shortening of (average) pulse duration.

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გეოფიზიკა

სტიკ-სლიპის პროცესის ფაზური სინქრონიზაცია პერიოდული მექანიკური დატვირთვის პირობებში

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ზამბარა-მცოცის სისტემაში, გარკვეულ პირობებში, ადგილი აქვს არათანაბარი ხახუნის (სტიკ-სლიპის) მოვლენას. ამ მოვლენის აღწერა შეიძლება სიჩქარეზე და მდგომარეობაზე დამოკიდებული ხახუნის კანონის საშუალებით. წარმოდგენილი ნაშრომი ეხება გარე პერიოდული (ნორმალური ან ტანგენსური) დატვირთვის ქვეშ მყოფი ლაბორატორიული ზამბარა-მცოცის სისტემის სრიალის რეჟიმის კვლევას. ზედა მცოცავ ფილაზე, მოძრაობის მართობულად ან ტანგენსურად, მოდებულია ცვლადი მექანიკური დატვირთვა, რომელიც მცირეა რეალურ გასწვრივ ძალებთან შედარებით. ჩვენს ექსპერიმენტებში სრიალის მომენტი ფიქსირდება აკუსტიკური ემისიის საშუალებით. გარე დატვირთვის ამპლიტუდის ზრდასთან ერთად შეინიშნება სტიკ-სლიპის აკუსტიკური იმპულსების პირველი შემოსვლების ფაზური სინქრონიზაციის ზრდა.

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