Mechanics

Every Body will no Longer Remain at Rest or in Uniform Motion in a Straight Line if Compelled to Change its State by the Action of an External Force, but will Do it with a Delay

Guram Gabrichidze

Academy Member, K. Zavriev Institute of Structural Mechanics and Earthquake Engineering, Tbilisi

ABSTRACT. The text referred to in the heading is a periphrasis of Newton's famous Law with a difference that classical mechanics usually asserts that the reaction to an external force takes place instantly, while I draw attention to the fact that the reaction occurs with a delay in time. The opinion is advanced that the cause of the particle motion retardation, or dampening of vibrations, may not only be the factors being external with regard to the mass, but the nature of the mass itself. The article provides an explanation of the mechanism of realization of the delay effect after an example of the transformation of a simple differential equation of a particle motion into a differential equation with the time advanced argument. *© 2008 Bull. Georg. Natl. Acad. Sci.*

Key words: *Newton's Law, motion of a particle, damping of vibrations, time advanced argument.*

The text referred to in the heading is a periphrasis of Newton's famous law with a difference that classical mechanics usually asserts that the reaction to an external force takes place instantaneously, while I draw attention to the fact that the displacement of the mass, as a reaction to the external impact, occurs with a delay in time, which is the cause of retardation of the motion of a constrained particle or of the damping of vibrations.

Below I attempt to illustrate these propositions after a simple example of particle motion. The example has been taken from [1] and its summary follows the same source. Let point M with the mass m move in a straight line under the action of gravitational force F to the stationary center O (Fig 1). Suppose the force modulus $\boldsymbol{F}$ is proportional to the distance of the particle M from the center O, i.e. $\boldsymbol{F} = c \cdot OM$ where c is the constant aspect ratio. The law of motion of the particle M is to be found.

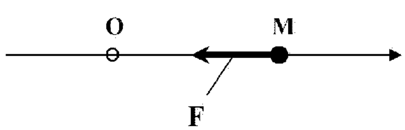

Fig. 1.

A corresponding differential equation of motion of the particle M will be recorded as follows:

$$\frac{d^2X(t)}{dt^2} + k^2X(t) = 0 \tag{1}$$

where $k^2 = \dfrac{c}{m}$.

The solution of the equation (1) is

$$X(t) = a \sin (kt + \alpha) \tag{2}$$

where a and α have arbitrary constant values.

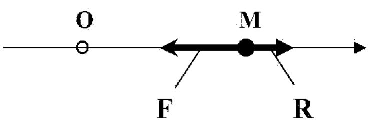

Fig. 2 .

The equation (2) is an equation of harmonic vibrations. Correspondingly, in the case of a straight-line motion under the action of the gravitational force proportional to the distance from the center of gravity, the particle will perform harmonic non-damped vibrations.

If the particles moves in a straight line along the axis x in the resisting medium, say in air, and the resistance force $\boldsymbol{R}$ is assumed to be proportional to the velocity of travel of the particle, then the differential equations of motion of the particle M could be recorded as follows (Fig. 2):

$$\frac{d^2X(t)}{dt^2}+2n\frac{dX(t)}{dt}+k^2X(t)=0\,, \qquad (3)$$

where μ is the constant aspect ratio characterizing the medium resistance, and $2\,n=\frac{\mu}{m}$.

The solution of the equation (3) is

$$X(t)=a\,e^{-nt}\cdot\sin\,(k_1\,t+\alpha), \qquad (4)$$

where $k_1^{\,2}=k^2-n^2$.

This equation differs from the equation (1) of harmonious vibrations by the factor e^{-nt}; therefore the M particle motions represent vibrations near the center O, but the peak-to-peak value of these vibrations no longer preserves its constant value; they tend to rapidly decrease with time. Therefore, these vibrations are called damped vibrations.

We have set forth the physical problem based on [1] where the resistance of medium, namely air, is referred to as the cause of the damping of vibrations. In classical mechanics, proposed are also other models explaining the fact of existence of damping of vibrations – the internal friction model, the energy dissipation model, etc., which does not alter the essence of the matter, for all of them consider that the cause of the damping of vibrations is beyond the particle.

In contrast to the above, I accentuate the proposition that the cause of the damping of vibrations and the cause of the retardation of displacements is the mass of the particle itself which, as a result of the external action starts to move but with a delay; therefore, the reaction in a constraint also occurs with a delay. In our specific case, this statement will sound as follows: if the external force $F\,(t)$ and the inertia force $m\frac{d^2X(t)}{dt^2}$ act on the mass m at the instant t, then the motion of the mass m originates with a certain delay, i.e. at the instant $(t+\tau)$.

If we accept this point of view, the equation (1) (without resistance of the medium) could be rewritten as follows:

$$\frac{d^2X(t)}{dt^2}+k^2X(t+\tau)=0\,. \qquad (5)$$

We obtain a differential equation with the time advanced argument [2], which enables to treat many phenomena from a new standpoint and, repeating the words of the author of the monograph, “opens a field full of attractive facts that so resembles and does not resemble the theory of ordinary differential equations”.

While pursuing limited objectives, let us expand $X(t+\tau)$ in the Taylor series

$$X(t+\tau)=X(t)+\tau\frac{dX(t)}{dt}+\;.\;.\;. \qquad (6)$$

and, preserve in the expansion (6) the first two members. Inserting this in (5), we obtain:

$$\frac{d^2X(t)}{dt^2}+k^2\tau\frac{dX(t)}{dt}+k^2X(t)=0\,. \qquad (7)$$

The equation (7) is identical to the equation (3), i.e. it produces the same damped vibrations of the particle M.

Thus, we have attempted to demonstrate that the retarding action of the inertia forces of the mass, i.e. the reaction delay effect in a constraint, results in damped vibrations and can be written down as a differential equation with the time advanced argument.

The reaction delay effect in a constraint was noticed by me when constructing a numerical algorithm for solving a dynamic problem of determining displacements of the systems of particles incorporated by constraints. The numerical algorithm was based on a counting process sampling in time and constituted a stage when the system of particles used to be released for a short time of constraints and be displaced as a system of free particles (particles without constraints), while the reaction forces of constraints were being applied with a delay. In such case, we obtain the same effect as upon the action of the medium resistance forces. For example, one-mass oscillator subjected to the action of the instantaneously applied constant force used to cease oscillations rather than doing it endlessly. This made me try to seek an analytical explanation of the mechanism of such an effect, led me to differential equations with the time advanced argument and suggested an idea to me that *the phenomenon of non-instantaneous reaction proper contained in itself a possibility of retardation and that it is inherent in the particle itself.* I have been encouraged to make such generalizations challenging the general principles of classical mechanics by the monographs [2] and [3], and especially by the works of S.M. Oganesyan [4], who has been raising the question of constructing a new mass model since 1998.

მექანიკა

თუ გარე ძალა იწყებს ზემოქმედებას მატერიალურ წერტილზე, იგი შეიცვლის უძრაობის, ან თანაბარი სწორხაზოვანი მოძრაობის მდგომარეობას, მაგრამ დაგვიანებით

გ. გაბრიჩიძე

აკადემიის წევრი, კ. ზავრიევის სამშენებლო მექანიკისა და სეისმომედეგობის ინსტიტუტი, თბილისი

სათაურში მოყვანილი ტექსტი არის ნიუტონის ცნობილი კანონის პერიფრაზი იმ განსხვავებით, რომ კლასიკურ მექანიკაში, ჩვეულებრივ, იგულისხმება, რომ გარე ზემოქმედებაზე რეაქცია ხდება მყისიერად, ჩვენ კი მივუთითებთ მასზე, რომ რეაქცია ჩნდება დაგვიანებით. წერილში გამოთქმულია მოსაზრება, რომ მატერიალური წერტილის მოძრაობის შეფერხების, ან რხევების ჩაქრობის მიზეზი შეიძლება იყოს არა მარტო ის ფაქტორები, რომლებიც მასისთვის წარმოადგენს გარემოს, არამედ თვით მასის ბუნება. სტატიაში მოყვანილია ანალიტიკური ილუსტრაცია იმისა, თუ როგორ ხდება მოძრაობის შეფერხება რეაქციის დაგვიანების მიზეზით. მაგალითად გამოყენებულია მატერიალური წერტილის მოძრაობის ჩვეულებრივი დიფერენციალური განტოლება, რომელიც გარდაიქმნება დიფერენციალურ განტოლებად წინმსწრები არგუმენტით.

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