საქართველოს მეცნიერებათა ეროვნული აკადემიის მოამბე, ტ. 2, №1, 2008
BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 2, no. 1, 2008

Mathematics

On some Approximate Properties of Multidimensional Fejer-Type Kernel Integral Operators

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ABSTRACT. Some approximate properties of a *d*-dimensional Fejer-type operator in the sense of the norm of the space $L^p(R^d)$, $p\in[1,+\infty[$ are established. © *2008 Bull. Georg. Natl. Acad. Sci.*

Key Words. *d*-dimensional Fejer-type kernel integral operator.

1. Points of Euclidian space R^d, as usual, are denoted by the symbols $x = (x_1, x_2,\dots, x_n)$, $y = (y_1, y_2,\dots, y_n)$, $t = (t_1, t_2,\dots, t_n)$,... . Below we consider functions $f: R^d \to R$, $R \equiv R^1 =]-\infty,+\infty[$. Let $p\in[1,+\infty[$ be a number and $f\in L^p(R^d)$. Then we mean that

$$\|f\|_p = \left\{ \int_{R^d} |f(x)|^p \right\}^{1/p}.$$

If $(e_i)_{i=1}^d$ are coordinate vectors, $h > 0$, $\delta > 0$, $d \geq 2$, then the expression

$$\omega_i(\delta, f)_p = \sup_{0<h\leq\delta} \|f(\cdot + e_i h) - f(\cdot)\|_p \quad (i=\overline{1,d})$$

is named partial L^p–modulus of continuity of function f.

2. Below by the symbol K_i ($i = \overline{1,d}$) we denote [see [1], p.145] a Fejer-type one-dimentional kernel with respect to coordinate i. The main properties of the kernel K_i are:

1) $\int_R K_i(u)du = 1$,

2) $K_i(-u) = K_i(u)$, $u\in R$,

3) there exists a positive finite number A such that

$$|K_i(u)| \leq A,\ u \in [-1,1],$$

4) there exists a positive finite number A_1 such that

$$|u^2 K_i(u)| \leq A_1,\ u \in R\ (i=\overline{1,d}).$$

Suppose

$$K(x) = \prod_{i=1}^{d} K_i(x_i).$$

We call it *d*-tuple Fejer-type kernel. If $p \in [1,+\infty[$ is a number, $\lambda_1 > 0$ ($i=\overline{1,d}$) and $f \in L^p(R^d)$ is a function, then we call the following expression

$$T_\lambda(f)(x) \equiv \prod_{i=1}^{d} \lambda_i \int_{R^d} f(t)K[\lambda(x-t)]dt \equiv \prod_{i=1}^{d} \lambda_i \int_{R^d} f(t)\prod_{i=1}^{d} K_i[\lambda_i(x_i-t_i)]dt$$

d-dimensional Fejer-type integral operator.

3. In the case of one dimension in the paper [2] some approximate properties of Fejer-type integral operator were established for functions of the class $L^p(R)$. An analogous problem for d-dimensional Fejer-type integral operators is solved in this paper.

Theorem. Let $p \in [1,+\infty[$ be a number and a function $f \in L^p(R^d)$. Then

$$\|T_\lambda(f)-f\|_p \le A(p,d)\left[\sum_{i=1}^{d}\frac{1}{\lambda_i}\int_{\lambda_i}^{\infty}\frac{\omega_i(u,f)_p}{u^2}du\right],$$

where $A(p,d)$ is a positive finite number depending on only the indicated parameters.

მათემატიკა

ფეიერის ტიპის მრავალგანზომილებიანი ინტეგრალური ოპერატორის ზოგიერთი აპროქსიმაციული თვისების შესახებ

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სტატიაში დადგენილია ფეიერის ტიპის d-განზომილებიანი ინტეგრალური ოპერატორების ზოგიერთი აპროქსიმაციული თვისება $L^p(R^d)$, $p \in [1,+\infty[$ სივრცის ნორმის თვალსაზრისით.

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Received December, 2006