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Cybernetics

Mathematical Models of Some Control Problems of Power Engineering

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ABSTRACT. In this paper the problem of optimal performance of a power system is considered. The problem is posed in various aspects within the frames of the theory of optimal control of stores. Mathematical models are presented by means of the recurrent equations of a dynamic programming. In general case the method of disposal from the "Damnation dimension" is offered. *© 2008 Bull. Georg. Natl. Acad. Sci.*

Key words: *inventory control, dynamic programming, optimization, energetics.*

During the functioning of power systems a significant role is played by hydroelectric power stations with water reservoirs. Reasonable control of water store makes it possible to satisfy the demand for electric power with the least expenditures. For any period for which the decision is made, the share of hydroelectric energy, which is assumed to be free-of-charge in comparison with expensive thermal energy, in total demand should be established. Because of the specific character of river flow and demand of electric power this problem is dynamic and stochastic in general.

Many authors, we among them, investigated the problem of optimal performance of a power system with a large number of various power plants by means of models of mathematical programming. For a single hydroelectric power station this problem under different assumptions was investigated by the method of dynamic programming by J. Little [1], S. Karlin [2], J. Gessford [3], and others. They investigated the problem from the point of view of the theory of optimal control of stores. Here we make an effort to apply a refined method of functional equations to investigate more general problems and to remove some assumptions made in the papers cited above.

We divide the horizon of scheduling (year, as a rule) into n periods (fortnight, monthly, etc.); during one period the flow of water, production and demand for electric power are considered uniform in time.

We introduce the following notations:

x_i - inflow of water in water reservoir during the period i,

u_i - outflow of water from water reservoir to turbines during the period i,

r_i - demand for electric power during the period i,

c - price of the unit of electric power from thermal stations,

K - maximal energy produced by a thermal station during a period,

p - penalty for deficiency per unit of electric power ($p > c$),

Q_i - store of water in water reservoir at the beginning of ith period ($Q_i \le \overline{Q}$).

The parameters x_i, u_i, Q_i can be understood both as the amount of water and the amount of energy, therefore it is possible to measure them in terms of volume (cubic meters) as well as in terms of electric power (kilowatt-hours). Conversion of the amount of water to the corresponding amount of energy is performed using the well-known formula

N=9.8$\eta u H$, where η - is the efficiency factor, and H - full flow. H=$H(Q)$ certainly is the function of the amount of water in the reservoir and varies in time. For all water reservoirs existing in Georgia power engineers, using topographic maps, have established the relations H=$H(Q)$ by means of curves. Applying these graphs to our problem, it is possible to design tables with appropriate step in advance and calculate the energy N_i for period i in kilowatt-hours by the formula

$$N_i = 9{,}8\eta \sum_{t=1}^{\tau} H\left(\min[Q_i + t\cdot\frac{x_i - u_i}{\tau}, \overline{Q}]\right)\cdot\frac{u_i}{\tau},$$

where τ is the number of days in a period.

Below all parameters are measured by equivalent amounts of electric power. So we use u_i instead of N_i.

Due to our notations the expenditure during one (ith) period will be

$$c\cdot\min[K, r_i - u_i] + p\cdot\max[0, r_i - u_i - K].$$

If we denote the minimum summarized expenditures in expenditures for the periods from i to n (including n) during optimal control of water store, when there is Q_i amount of water in the reservoir at the beginning of ith period, by $f_i(Q_i)$, then we obtain the following recurrent relation for $i = 1,2,\ldots,n-1$:

$$f_i(Q_i) = \min_{u_i}\{c\cdot\min[K, r_i - u_i] + p\cdot\max[0, r_i - u_i - K] + f_{i+1}(\min[Q_i + x_i - u_i, \overline{Q}])\}. \quad (1)$$

Here $0 \le u_i \le \min[r_i, Q_i + x_i]$. This restriction seems natural to us, since consumption of water x_i together with Q_i at ith period occurs permanently. It is possible to calculate $f_n(Q_n)$ easily in view of the fact that the best value for u_n is $\overline{u}_n = \min[r_n, Q_n + x_n, \overline{Q}]$.

As for the factor of indeterminacy, demand for electric power in periods is anyhow steady and can be predicted. We shall consider them as constants. Regarding the flows two hypotheses are usually made – random variables x_i are independent or correlated and they represent a Markov chain (simple or complicated). In both cases hypothetical distribution functions are obtained by means of rich statistical data and the relation (1) can be altered in an appropriate way. Respectively $[0; \min(r_i, Q_i + \overline{\delta}(x_i))]$ should be considered as the interval of variation of parameter u_i, where $\overline{\delta}(x_i)$ denotes the upper bound of the interval of fidelity of the random variable x_i with significance level given in advance. Also, below all cases of recurrent relations are written only for a determined case.

Now consider a cascade system of ℓ hydroelectric power stations when the water reservoir is available only for the upper station and between stations j-1 and j $(j = \overline{2,\ell})$ there is an additional inflow ξ_i^j (it is the most commonly used case of cascade). Assume that the jth station can pass I^j amount of water. Then this station in the ith period can produce N_i^j energy from the $\min[I^j, u_i + \xi_i^1 + \xi_i^2 + \cdots + \xi_i^j]$ amount of water. We obtain the following recurrent relation

$$f_i(Q_i) = \min_{\substack{0\le u_i \le Q_i + x_i \\ \sum_{j=1}^{\ell} N_i^j \le r_i}} \{c\cdot\min[K, r_i - \sum_{j=1}^{\ell} N_i^j] + p\cdot\max[0, r_i - \sum_{j=1}^{\ell} N_i^j - K] + f_{i+1}(\min[Q_i + x_i - u_i, \overline{Q}])\} \quad (2)$$

Here $f_i(Q_i)$ $(i = 1,\ldots,n-1)$ represents the minimum summarized expenditures in periods from i to n during optimal control of the cascade when there is Q_i amount of water in a reservoir at the beginning of the ith period. $f_n(Q_n)$ can be calculated after $\overline{u}_n$ is calculated.

It is possible to write similar equations for cases when in the cascade there are also other water reservoirs. Generally, when each hydroelectric power station has a reservoir, we write

$$f_i(Q_i^1, Q_i^2, \ldots, Q_i^\ell) =$$

$$= \min_{\substack{0\le u_i^j \le Q_i^j + \xi_i^j + u_i^{j-1} \\ \sum_{j=1}^{\ell} N_i^j \le r_i}} \left\{c\cdot\min\left[K, r_i - \sum_{j=1}^{\ell} u_i^j\right] + p\cdot\max\left[o, r_i - \sum_{j=1}^{\ell} u_i^j - K\right] + f_{i+1}(\min[Q_{i+1}^1, Q_{i+1}^2, \ldots, Q_{i+1}^\ell])\right\} \quad i = \overline{1, n-1}. \quad (3)$$

where $Q_{i+1}^j = \min[\overline{Q}^j, Q_i^j, + u_i^{j-1} - u_i^j + \xi_i^j]$. In order to simplify the calculations instead of restriction $\sum_{i=1}^{\ell} u_i^j \le r_i$ in braces we can add "penalty" term $\max[0, \sum_{i=1}^{\ell} u_i^j - r_i]$ with positive coefficient. When there is a possibility to sell the energy, we add the same term with negative coefficient $-a_i$, where $a_i \geq 0$ is the price of the unit electric energy at the ith period.

Obviously, in fact the relation (3) is suitable at most for the case $\ell = 3$. A similar difficulty arises in the general problem of optimal control of water resources in the power system. In order to avoid the so-called "damnation dimension", at least partially, we can introduce some "reasonable" assumptions for the policy of control. For example, we can apply one controlling parameter u_i for the ith period (total hydro power produced) and divide its value over stations proportionally to their potential possibilities $Q_i^j + x_i^j$, $j = 1, 2, \ldots, l$.

The recurrent relation for the general case looks like

$$f_i(Q_i^1, Q_i^2, \ldots, Q_i^m) = \min_{0 \leq u_i^j \leq M} \{c \cdot \min[K, r_i - u_i] + p \cdot \max[0, r_i - u_i - K] - a \cdot \max[0, u_i - r_i] +$$

$$+ f_{i+1}(Q_{i+1}^1, Q_{i+1}^2, \ldots, Q_{i+1}^{\ell})\}, \qquad i = \overline{1, n},$$

where $M = \sum_{j=1}^{m} (Q_i^j + x_i^j)$, $Q_{i+1}^j = Q_i^j + x_i^j - \frac{u_i}{M} \cdot (Q_i^j + x_i^j)$.

At each step it is necessary to perform one-dimensional minimization for $d^1 \cdot d^2 \cdot \ldots \cdot d^m$ sets of initial values of parameters $Q_i^1, Q_i^2, \ldots, Q_i^{\ell}$, where d^j specifies the number of possible water levels in the jth reservoir for the standard unit of volume Δq, $\overline{Q}^j = \Delta q \cdot d^j$.

The model suggested above was applied to the power system of Georgia. The system included one thermal electric station, three hydro electric stations with reservoirs (Enguri, Khrami, Zhinvali) and twenty five hydroelectric stations without actual reservoirs. The data collected 20 years ago was used. The results are close enough to the results obtained by us earlier by means of the model given in [4], which is adequate enough to the reality. 60 $(5 \cdot 3 \cdot 4)$ one-dimensional minimizations were performed for each step.

მათემატიკა

ენერგეტიკული სისტემის მართვის ზოგიერთი ამოცანის მათემატიკური მოდელი

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(წარმოდგენილია აკადემიკოს ნ. ვახანიას მიერ)

ოპერაციათა კვლევის ერთ-ერთი მიმართულება – მარაგთა მართვის თეორია შეისწავლის მომარაგება-გასაღების ექსტრემალურ ამოცანებს. ამ ამოცანებს შორის ძალზე პოპულარულია ჰიდროენერგეტიკასთან დაკავშირებული ამოცანები, კერძოდ, მდინარეთა ჩამონადენის რაციონალურად გამოყენების ამოცანა წყალსაცავების დონის რეგულირებით. დამუშავებულია სხვადასხვა ტიპის მათემატიკური მოდელები, მათ შორის აღსანიშნავია მოდელები დინამიკური პროგრამირების მეთოდის გამოყენებით. ცნობილი მიზეზის გამო, ისინი შედგენილია მხოლოდ ერთი წყალსაცავიანი სადგურისათვის.

ავტორთა მიერ შემოთავაზებულ მოდელებში დინამიკური პროგრამირების მეთოდის გამოყენებით აღიწერება კასკადური სისტემა საერთო წყალსაცავით და სისტემა მრავალი წყალსაცავიანი ჰიდროსადგურით ერთი ბუნებრივი დაშვების შემთხვევაში. ამავე დროს მოდელებში მოხსნილია ციტირებულ შრომებში დაშვებული ზოგიერთი არსებითი შეზღუდვა.

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