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BULLETIN OF THE GEORGIAN NATIONAL ACADEMY OF SCIENCES, vol. 2, no. 3, 2008

Physics

Resistive-Viscous Suppression of Magnetorotational Instability in an Astrophysical Plasma

Jumber Lominadze[*], Ernest Pashitskii[**], Anatoly Mikhailovskii[***], Oleg Kharshiladze[#], Andrey Churikov[§], Victor Avsarkisov[§§]

* *Academy Member, E.Kharadze Abastumani National Astrophysical Observatory; M. Nodia Institute of Geophysics, Tbilisi*
** *Institute of Physics, NAS of Ukraine, Kiev, Ukraine*
*** *Institute of Nuclear Fusion, Russian Research Centre Kurchatov Institute; Institute of Astronomy of RAS, Moscow, Russia*
\# *M. Nodia Institute of Geophysics, Tbilisi*
§ *Syzran Branch of Samara Technical University, Syzran, Russia*
§§ *E.Kharadze Abastumani National Astrophysical Observatory; M. Nodia Institute of Geophysics, Tbilisi*

ABSTRACT. Resistive-viscous suppression of magnetorotational instability in an astrophysical plasma is studied. A criterion of such suppression is derived. *© 2008 Bull. Georg. Natl. Acad. Sci.*

Key words: *magnetorotational instability, resistive-viscous effect.*

The magnetorotational instability (MRI), [1, 2], became an important issue of both astrophysical and laboratory studies due to the paper [3] referring it to the problem of anomalous viscosity in accretion disks [4]. It was assumed in [1-3] that the medium subjected to the MRI is ideal. It seems to be important to elucidate dissipative effects on the MRI. The role of such effects can be played by the viscosity and resistivity of the medium. The viscosity effect has been studied in [5], while the resistivity - in [6]. The goal of the present paper is allowing for both the resistivity and viscosity. It is assumed that the medium is an astrophysical plasma of cylindrical geometry.

We take the Maxwell equations in the form

$$\partial \mathbf{B}/\partial t = \nabla\times[\nabla\times\mathbf{B}] + \mu\nabla^2\mathbf{B}, \tag{1}$$

$$\nabla\cdot\mathbf{B} = 0, \tag{2}$$

where $\mathbf{B}=\mathbf{B}_0+\widetilde{\mathbf{B}}$, $\mathbf{V}=\mathbf{V}_0+\widetilde{\mathbf{V}}$ are the total magnetic field and plasma velocity, $\mathbf{B}_0=(0,0,B_0)$, $\mathbf{V}_0=(0,0,V_0)$ are the equilibrium field and velocity, $\nabla B_0=0$, $V_0=r\Omega(r)$, $\Omega(r)$ is the angular rotation frequency, $\widetilde{\mathbf{B}}=(\widetilde{B}_r,\widetilde{B}_\theta,\widetilde{B}_z)$ and $\widetilde{\mathbf{V}}=(\widetilde{V}_r,\widetilde{V}_\theta,\widetilde{V}_z)$ are the perturbed magnetic field and velocity, $\mu=c^2/(4\pi\sigma)$, c is the light velocity, σ is the electric conductivity. We use the cylindrical coordinates (r,θ,z).

We assume the perturbations to be of the form $\exp(-i\omega t+ik_z z)\widetilde{F}(r)$, where ω is the oscillation frequency, k_z is the parallel wave number, the function $\widetilde{F}(r)$ is taken to be close to $\exp(ik_r r)$, where k_r is the radial wave number. Then Eqs. (1) and (2) yield

$$\widetilde{V}_r = -\omega_\mu \widetilde{B}_r/(k_z B_0), \tag{3}$$

$$\widetilde{V}_\theta = -\frac{\omega_\mu}{k_z B_0}\widetilde{B}_\theta + \frac{i\widetilde{B}_r}{k_z B_0}\frac{d\Omega}{d\ln r}, \tag{4}$$

$$\widetilde{B}_z = \frac{i}{k_z r}\frac{\partial}{\partial r}\left(r\widetilde{B}_r\right). \tag{5}$$

where $\omega_\mu = \omega + ik^2\mu$, $k^2 = \hat{k}_r^2 + k_z^2$, $\hat{k}_r$ is the operator $-i\partial/\partial r$.

Plasma is assumed to be incompressible,

$$\nabla \cdot \mathbf{V} = 0, \tag{6}$$

while its velocity satisfies the motion equation

$$\rho \frac{d\mathbf{V}}{dt} = -\nabla p + \rho \mathbf{g} - \frac{1}{4\pi}\left\{\nabla \frac{\mathbf{B}^2}{2} - (\mathbf{B}\cdot\nabla)\mathbf{B}\right\} + \rho\nu\nabla^2\mathbf{V}. \tag{7}$$

Here $\rho = \rho_0 + \widetilde{\rho}$ is the total plasma mass density, $p = p_0 + \widetilde{p}$ is the total plasma pressure, ν is the coefficient of kinematic viscosity, $d/dt = \partial/\partial t + (\mathbf{V}\cdot\nabla)$. From (7) one finds the equilibrium condition

$$r\Omega^2 = g. \tag{8}$$

Equation (6) leads to

$$ik_z\widetilde{V}_z + \frac{1}{r}\frac{\partial}{\partial r}\left(r\widetilde{V}_r\right) = 0. \tag{9}$$

It follows from (7) that

$$-i\omega_\nu \widetilde{V}_r - 2\Omega\widetilde{V}_\theta - \frac{iv_A^2 k_z}{B_0}\widetilde{B}_r + \frac{1}{\rho_0}\frac{\partial}{\partial r}\widetilde{p} + \frac{v_A^2}{B_0}\frac{\partial \widetilde{B}_z}{\partial r} = 0, \tag{10}$$

$$-i\omega_\nu \widetilde{V}_\theta + \frac{\kappa^2}{2\Omega}\widetilde{V}_r - \frac{iv_A^2 k_z}{B_0}\widetilde{B}_\theta = 0, \tag{11}$$

$$-i\omega_\nu \widetilde{V}_z + \frac{ik_z}{\rho_0}\widetilde{p} = 0, \tag{12}$$

where $\omega_\nu = \omega + ik^2\nu$, $v_A^2 = B_0^2/(4\pi\rho_0)$ is the Alfven velocity squared, $\kappa^2 = 4\Omega^2 + d\Omega^2/d\ln r$ is the square of epicyclic frequency. We take ρ_0 to be constant, so that $\widetilde{\rho} = 0$.

By means of (3), (9) and (12) one finds

$$\widetilde{V}_z = -\frac{i}{k_z^2 B_0}\frac{1}{r}\frac{\partial}{\partial r}\left(\omega_\mu r\widetilde{B}_r\right), \tag{13}$$

$$\widetilde{p} = -\frac{i\omega_\nu\rho_0}{k_z^3 B_0}\frac{1}{r}\frac{\partial}{\partial r}\left(\omega_\mu r\widetilde{B}_r\right). \tag{14}$$

In turn, (3), (4) and (11) result in

$$\widetilde{V}_\theta = \frac{i}{k_z B_0 D_{\mu\nu}}\left(2\Omega\omega_\mu^2 + D_{\mu\mu}\frac{d\Omega}{d\ln r}\right)\widetilde{B}_r, \tag{15}$$

where

$$D_{\mu\nu} = \left(\omega + i\mu k^2\right)\left(\omega + i\nu k^2\right) - k_z^2 v_A^2, \tag{16}$$

$$D_{\mu\mu} = \left(\omega + i\mu k^2\right)^2 - k_z^2 v_A^2. \tag{17}$$

Substituting (5), (14), (15) into (10), we arrive at the mode equation

$$\frac{\partial}{\partial r}\left[\frac{\omega_\nu}{r}\frac{\partial}{\partial r}\left(\omega_\mu r\widetilde{B}_r\right) - \frac{k_z^r v_A^2}{r}\frac{\partial}{\partial r}\left(r\widetilde{B}_r\right)\right] -$$

$$-k_z^2\left[D_{\mu\nu} - \frac{1}{D_{\mu\nu}}\left(4\Omega^2\omega_\mu^2 + D_{\mu\mu}\frac{d\Omega^2}{d\ln r}\right)\right]\widetilde{B}_r = 0. \tag{18}$$

Hence we obtain the local dispersion relation

$$D_{\mu\nu} - \frac{k_z^2}{k^2 D_{\mu\nu}}\left(4\Omega^2\omega_\mu^2 + D_{\mu\mu}\frac{d\Omega^2}{d\ln r}\right) = 0. \tag{19}$$

where $k^2 = k_z^2 + k_r^2$.

Taking $\omega = 0$ in (19), we arrive at the instability boundary

$$\frac{d\Omega^2}{d\ln r} + \frac{k^2}{k_z^2}\frac{\left(k_z^2 v_A^2 + k^4\mu\nu\right)^2}{k_z^2 v_A^2 + k^4\mu^2} + \frac{4\Omega^2 k^4\mu^2}{k_z^2 v_A^2 + k^4\mu^2} = 0. \tag{20}$$

Taking here $\mu = \nu = 0$, one finds

$$\frac{d\Omega^2}{d\ln r} + k^2 v_A^2 = 0. \tag{21}$$

This is the boundary of MRI in the ideal plasma [1-3]. It remains in force also for $\mu = 0$, $\nu \neq 0$. This means that only viscosity does not change the MRI boundary [5]. In turn, requiring $\kappa^2 > 0$ and $\mu \neq 0$, $\nu \neq 0$, one has that

$$k^4\mu\nu > k_z^2 v_A^2\left(2\frac{|\Omega|}{kv_A} - 1\right). \tag{22}$$

This is the criterion of resistive-viscous suppression of MRI in astrophysical plasma.

Acknowledgments

This work was supported by the International Science and Technology Center (ISTC) (grant N G-1217), the Science and Technology Center in Ukraine (grant N 3473), the Russian Foundation of Basic Researches (RFBR) (grant N 06-02-16767) and the Russian Program of Support of Leading Scientific Schools (grant N 9878.2006.2).

ფიზიკა

რეზისტიულ-ბლანტური მაგნიტო-ბრუნვითი არამდგრადობის სტაბილიზაცია ასტროფიზიკურ პლაზმაში

ჯ. ლომინაძე[*], ე. პაშიცკი[**], ა. მიხაილოვსკი[***], ო. ხარშილაძე[#], ა. ჩურიკოვი[§], ვ. ავსარქისოვი[§§]

** აკადემიკოსი, ე.ხარაძის საქართველოს ეროვნული ასტროფიზიკური ობსერვატორია, აბასთუმანი, მ.ნოდიას გეოფიზიკის ინსტიტუტი, თბილისი*
*** ფიზიკის ინსტიტუტი, უკრაინის ეროვნული მეცნიერებათა აკადემია, კიევი, უკრაინა*
**** ბირთვული სინთეზის ინსტიტუტი, რუსეთის კვლევითი ცენტრი, კურჩატოვის ინსტიტუტი, ასტრონომიის ინსტიტუტი, რუსეთის მეცნიერებათა აკადემია, მოსკოვი, რუსეთი*
 # მ.ნოდიას გეოფიზიკის ინსტიტუტი, თბილისი
§ სამარის ტექნიკური უნივერსიტეტის სიზრანის ფილიალი, სიზრანი, რუსეთი
§§ ე.ხარაძის საქართველოს ეროვნული ასტროფიზიკური ობსერვატორია, აბასთუმანი, მ.ნოდიას გეოფიზიკის ინსტიტუტი, თბილისი

განხილულია მაგნიტო-ბრუნვითი არამდგრადობის რეზისტულ-ბლანტური სტაბილიზაცია ასტროფიზიკურ პლაზმაში. დადგენილია ასეთი სტაბილიზაციის კრიტერიუმი.

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Received July, 2008